Vortices in Self-gravitating Gaseous Discs

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Outline

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Dynamics of vortices in *non-self-gravitating discs* and their role in planet formation

- Vortex formation
  - nonlinear Kelvin-Helmholtz instability (Lithwick 2007)
  - baroclinic instability (Li et al. 2001, Klahr & Bodenheimer 2003, Petersen et al. 07)

- Anticyclonic vortices once formed grow in size via merging (in 2D case) due to an inverse cascade of energy. Cyclonic vortices quickly get sheared away (Bracco et al. 1999, Godon & Livio 1999, Umurhan & Regev 2004, Johnson & Gammie 2005)

- ***Vortices are closely coupled with/generate density waves (turning subsequently into shocks)*** , coupling is of ***linear origin***, that is, exists even in the linear theory and is primarily caused by background Keplerian shear (Johnson & Gammie 2005, Bodo et al. 2005, 2007, Mamatsashvili & Chagelishvili 2007)

- Vortices are believed to greatly accelerate planetesimal formation by trapping dust particles in their centres (Barge & Sommeria 1995, Johansen et al. 2004, Klahr & Bodenheimer 2006)

Self-gravitating discs

- Due to balance between heating and cooling, self-gravitating discs stay in a quasi-steady, self-regulated *gravitoturbulent* state (Boley et al. 2006, 2007; we consider non-fragmenting discs)

- **Density waves** – most commonly considered in self-gravitating discs.

Vortices – are left out !

Motivation

- ***The main goal of the present study is to understand the specific properties of vortex evolution in a quasi-steady gravitoturbulent state in light of the recently discovered (linear) coupling between vortices and density waves***

Implications for planetesimal formation

- In perspective, such a study will allow us to see if the vortex trapping mechanism can still be effective in the presence of self-gravity

Model description and numerical techniques

Local model – shearing sheet approximation

$$\frac{\partial \Sigma}{\partial t} + \nabla \cdot (\Sigma \mathbf{u}) - q\Omega x \frac{\partial \Sigma}{\partial y} = 0,$$

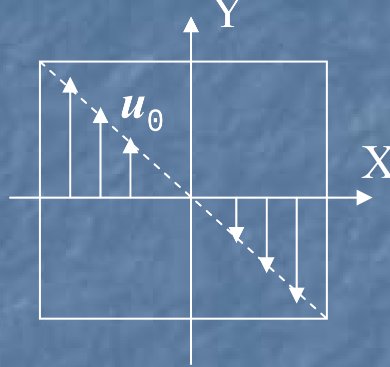
$$\frac{\partial u_x}{\partial t} + (\mathbf{u} \cdot \nabla) u_x - q\Omega x \frac{\partial u_x}{\partial y} = -\frac{1}{\Sigma}\frac{\partial P}{\partial x} + 2\Omega u_y - \frac{\partial \psi}{\partial x},$$

$$\frac{\partial u_y}{\partial t} + (\mathbf{u} \cdot \nabla) u_y - q\Omega x \frac{\partial u_y}{\partial y} = -\frac{1}{\Sigma}\frac{\partial P}{\partial y} + (q-2)\Omega u_x - \frac{\partial \psi}{\partial y}.$$

$$\frac{\partial U}{\partial t} + \nabla \cdot (U \mathbf{u}) - q\Omega x \frac{\partial U}{\partial y} = -P\nabla \cdot \mathbf{u} - \frac{U}{\tau_c},$$

$$P = (\gamma - 1)U, \; \gamma = 2$$

$$\Delta \psi = 4\pi G \Sigma \delta(z).$$



Basic Keplerian shear flow
$\boldsymbol{u}_0=(0, -q\Omega x)$, $q=1.5$.
Shearing sheet rotates with Ω
X-radial coordinate
Y-azimuthal coordinate

Numerical analysis in the local model allows higher numerical resolution than the global disc approach

$\boldsymbol{u}(u_x, u_y)$ – perturbed velocity relative to main Keplerian shear flow $\boldsymbol{u}_0$,
Σ – surface density, P – pressure, U – internal energy, ψ – gravitational potential,
Simple cooling law: constant cooling time $\tau_c=20\Omega^{-1} > 3\Omega^{-1}$ (**no fragmentation**, Gammie 2001)

Central quantity of this study – ***potential vorticity*** (PV)

$$I = \frac{1}{\Sigma}\left(\frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y} + (2-q)\Omega\right)$$

This is a basic quantity that is used to characterize vortex formation

Evolution equation for PV

$$\left(\frac{\partial}{\partial t} + \mathrm{u}\cdot\nabla - q\Omega x\frac{\partial}{\partial y}\right) I = \frac{1}{\Sigma^3}\left(\frac{\partial \Sigma}{\partial x}\frac{\partial P}{\partial y} - \frac{\partial \Sigma}{\partial y}\frac{\partial P}{\partial x}\right)$$

In what follows we concentrate on the specific properties of ***PV, Σ, P*** evolution in the quasi-steady *gravitoturbulent* state via numerically solving above hydrodynamic and Poisson equations in the shearing sheet

Numerical techniques

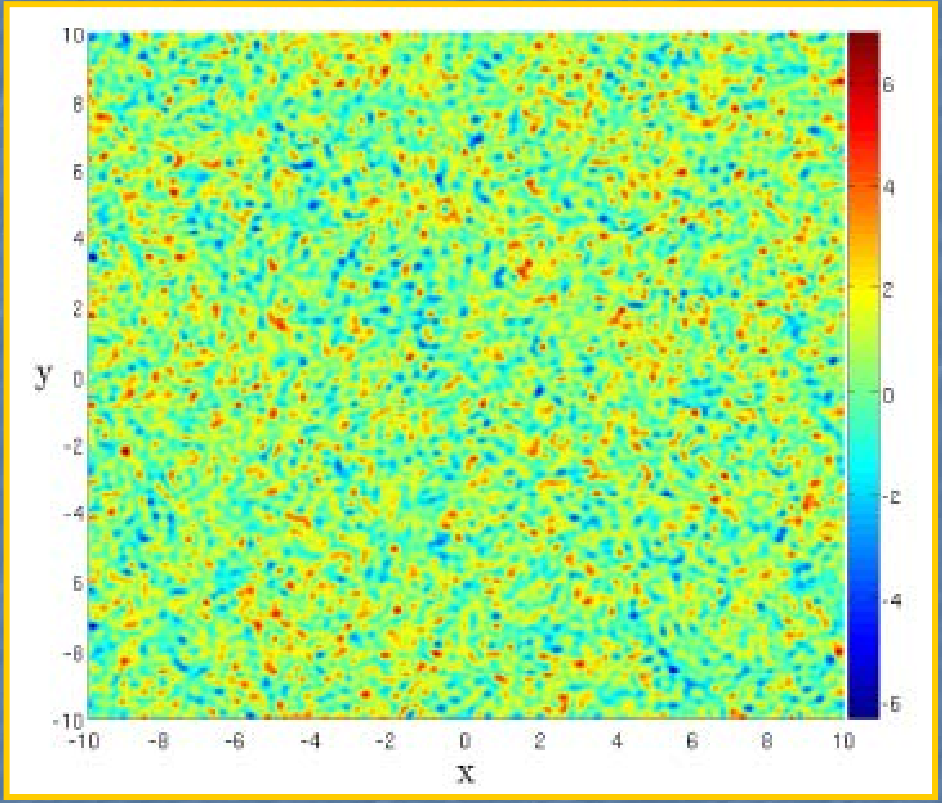
- ZEUS code suited for the shearing sheet (Gammie 2001, Johnson & Gammie 2003, 2005)
  — with modified treatment of advection by large Keplerian velocity (FARGO scheme, Masset 2000)

- Shearing sheet boundary conditions (Hawley et al. 1995). Shift in BC is done by means of FARGO scheme

- Poisson equation for self-gravity is solved via FFT technique modified for shearing coordinates

Influence of self-gravity on the vortex dynamics

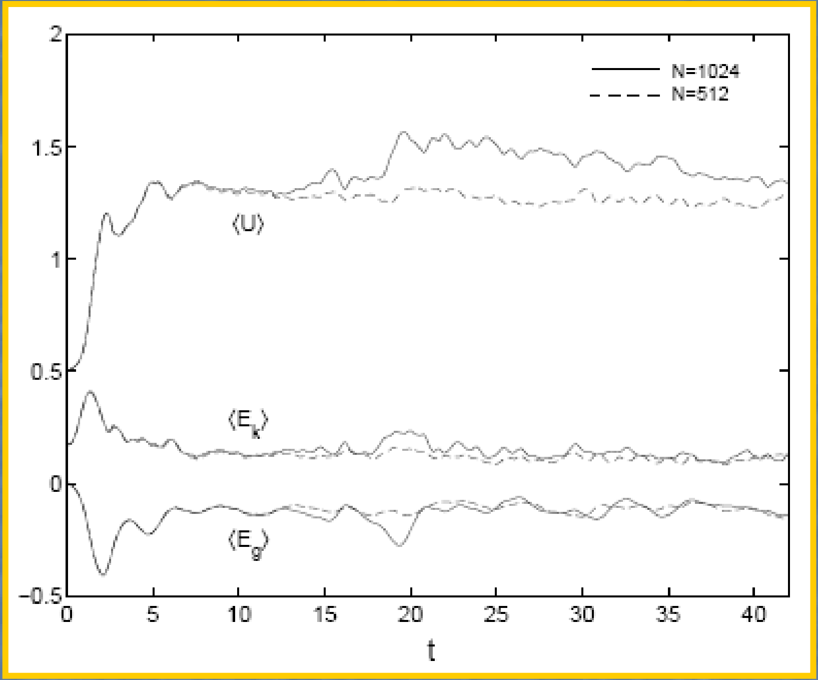
Initial conditions

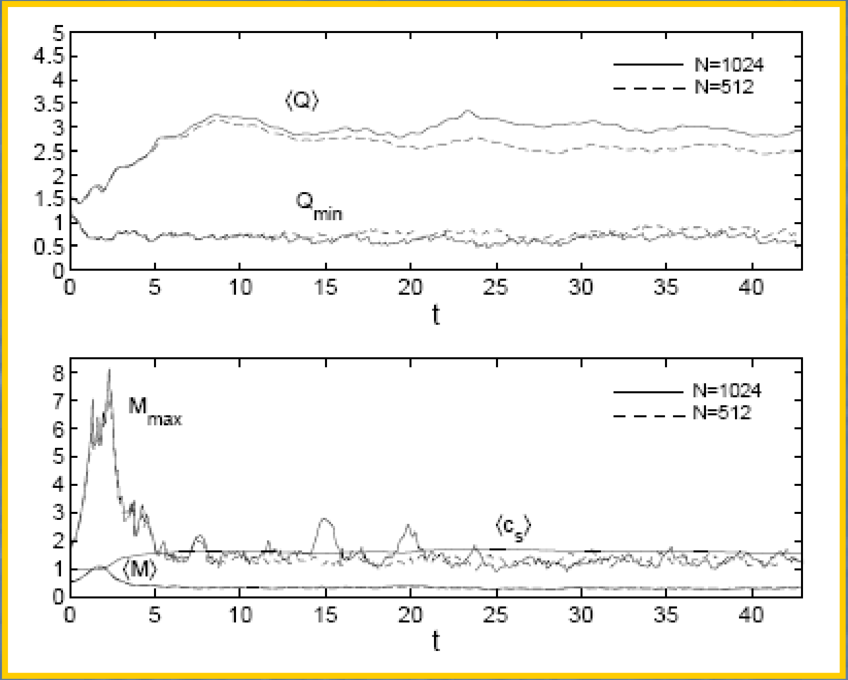


Initial random distribution of PV

Random/chaotic (Kolmogorov spectrum) velocity perturbations are imposed initially with nonzero potential vorticity (PV). Other variables are not perturbed initially.

Quasi-steady gravitoturbulence and evolution of vortices





- Average kinetic, internal and graviational energies as well as Toomre's Q ($=c_s\Omega/\pi G\Sigma$) and Mach number ($=u/c_s$) after initial transient (swing) amplification settle down to constant values signaling the onset of quasi-steady gravitoturbulence
- Saturated angular momentum transport parameter α is given by

$$\alpha = \frac{1}{q\gamma(\gamma-1)\Omega\tau_c},$$

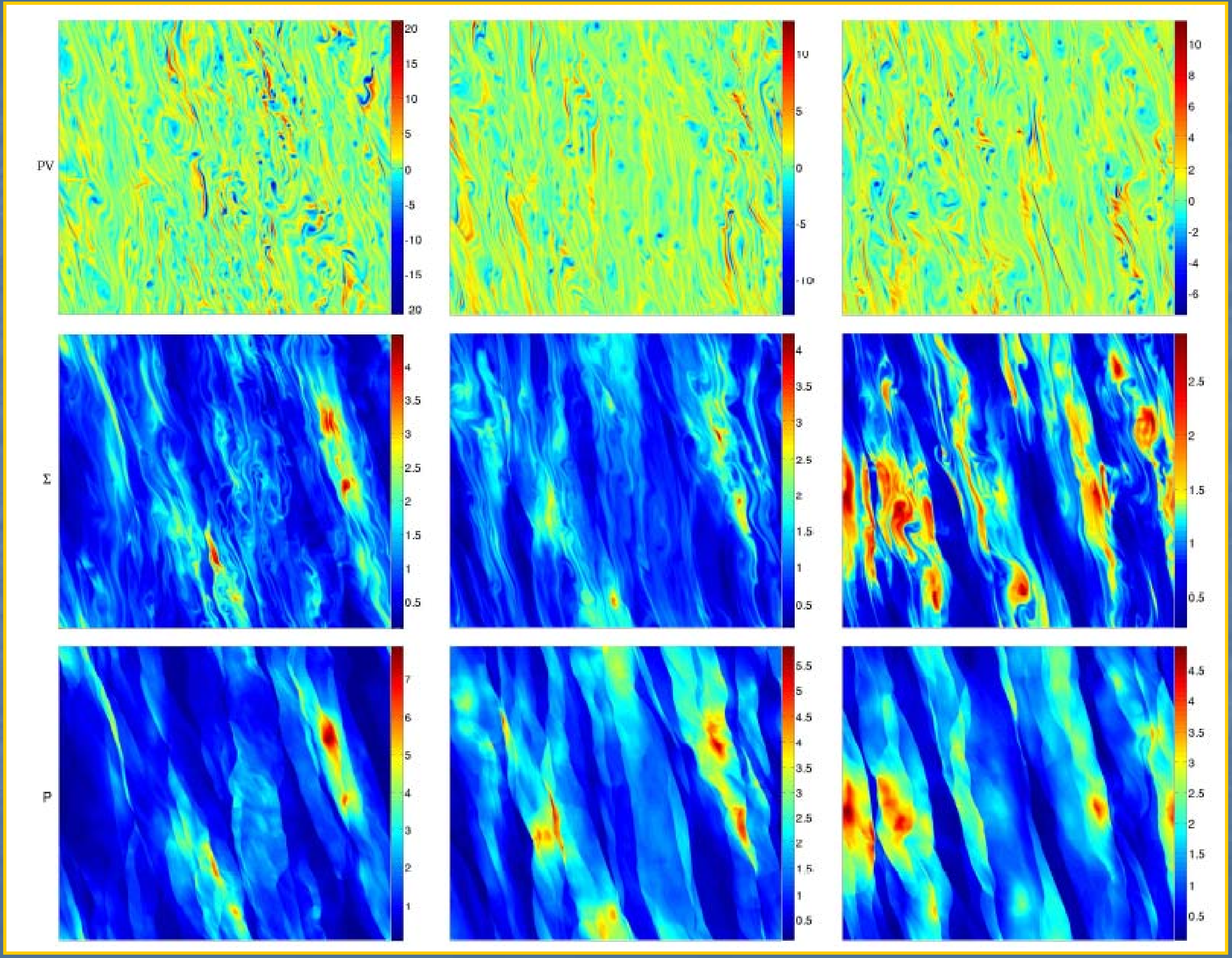
- **Note** minimum Q is small (0.6-0.7) and is associated with vortices

Evolution of vortices (potential vorticity) – snapshots at different time moments

PV

Σ

P=U



$t\Omega=14.7$ $t\Omega=29.3$ $t\Omega=44$

Evolution of vortices (potential vorticity) – 4 key evolutionary stages

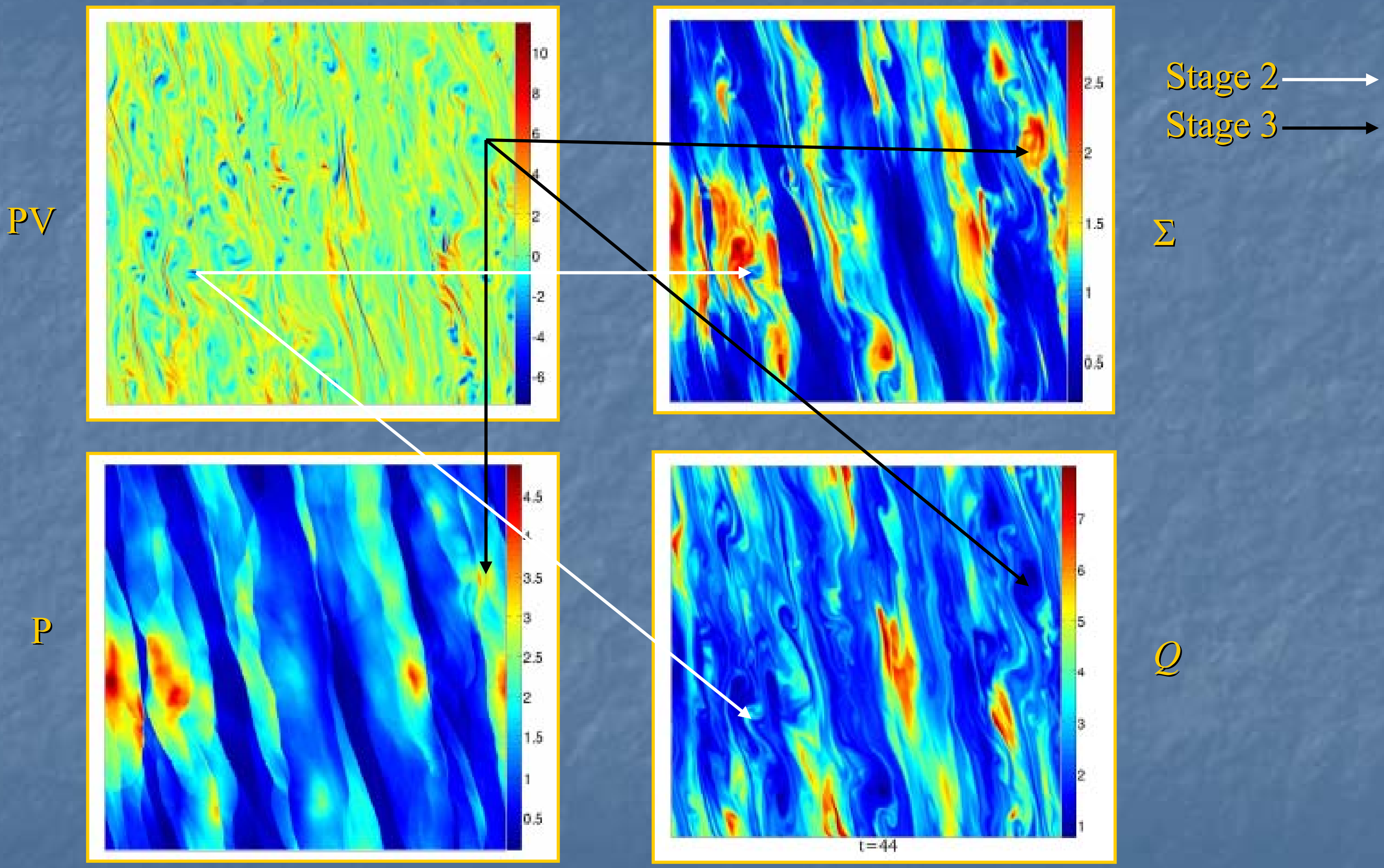
1. Formation of small-scale anticyclonic vortices from vortex strips

2. Gradual growth of vortices in size, characterized by underdense centre surrounded by overdense regions – sites of density wave emission

3. Vortices approach scales comparable to the local Jeans scale and at the same time self-gravity comes into play. Now vortices are more characterized by a single overdense region. PV is smaller by absolute value than that in the above case. Q gradually drops

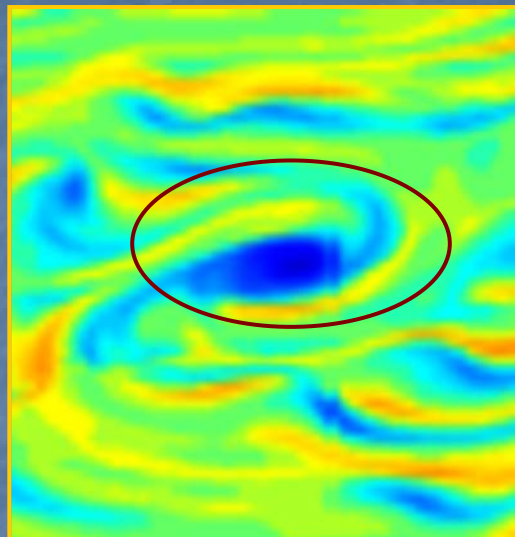
4. Q is sufficiently small (0.6-0.7) and vortices are in the process of shearing by self-gravity/gravitational instability and Keplerian shear

Evolution of vortices – snapshot at a single instant ($t\Omega=44$).
Correlations/correspondence among structures in PV, Σ, P and Q fields

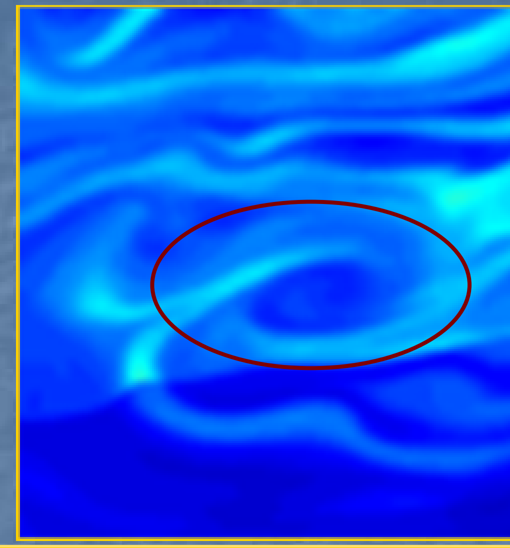


Evolution of vortices – analogy with other simulations (stage 2, underdense and overdense ring-like region – sites of coupling with waves)

PV

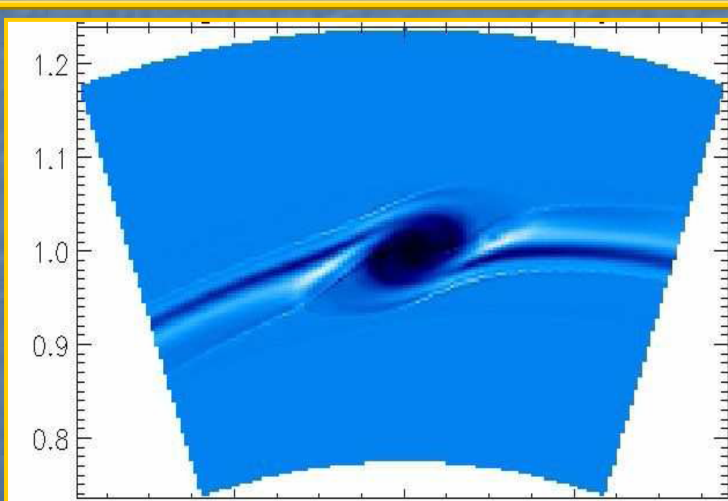


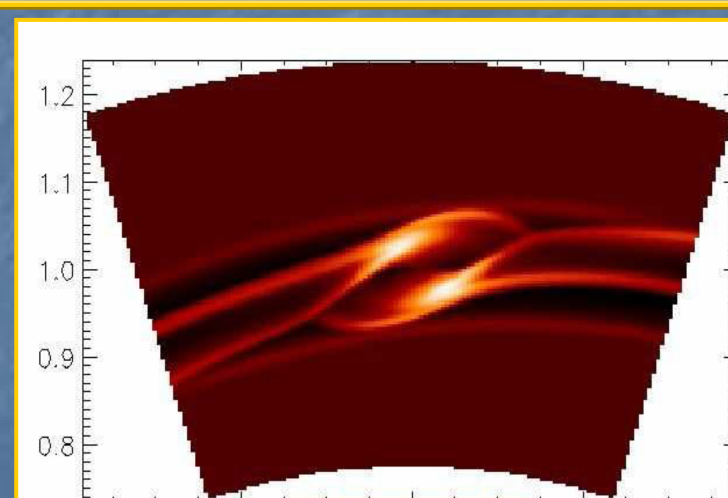
Our plots



Σ

PV



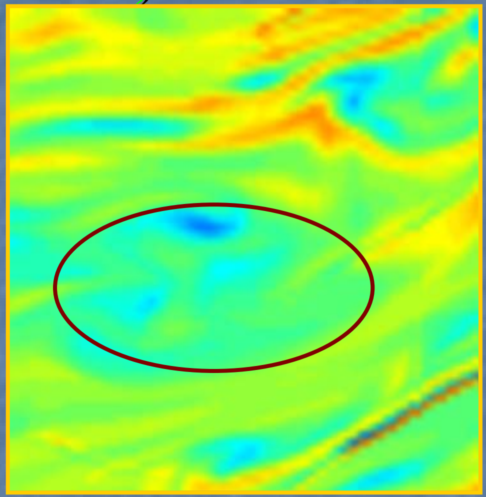


Σ

Simulations of the adjustment of a single vortex, **no** self-gravity (Bodo et al. 2007)

Evolution of vortices – analogy with other simulations (final stages 3-4, only stronger overdense region with lower *Q* is left and is gradually getting sheared)

PV



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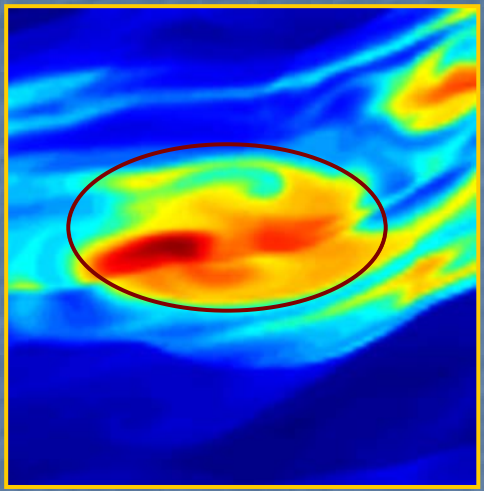
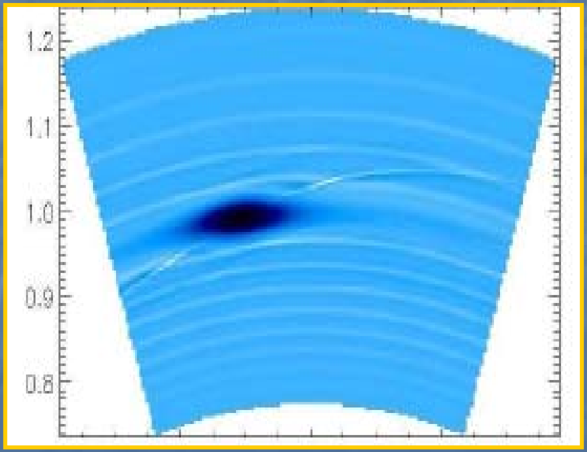
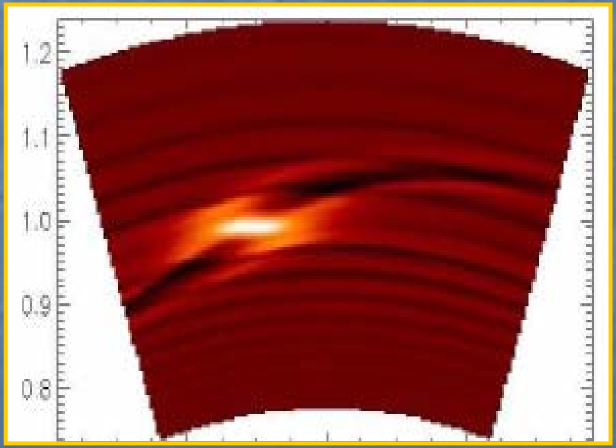


Σ

PV





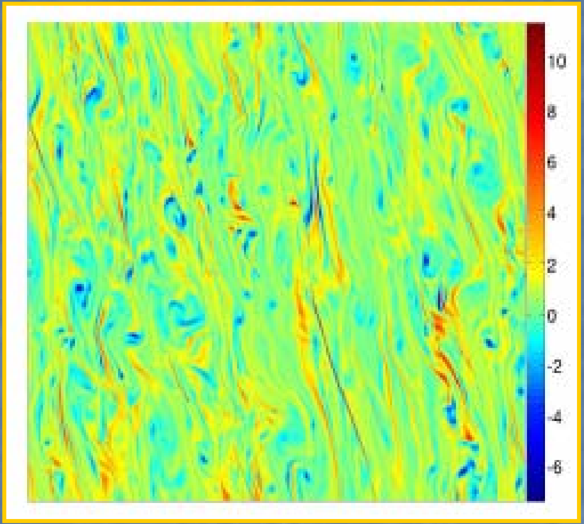
Σ

Simulations of the adjustment of a single vortex, **no** self-gravity (Bodo et al. 2007)

Contrast with non-self-gravitating case

With self-gravity

PV



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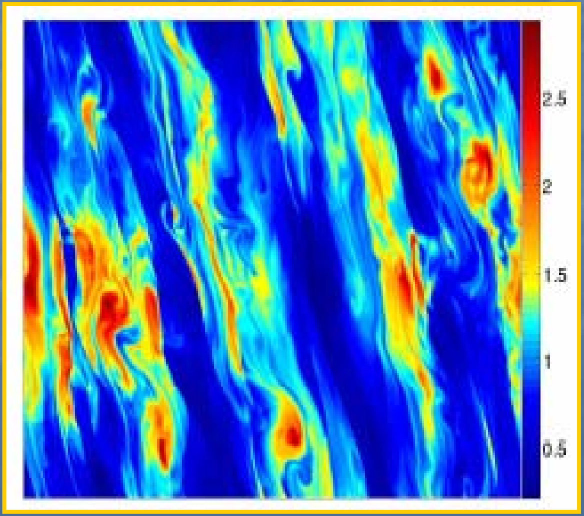
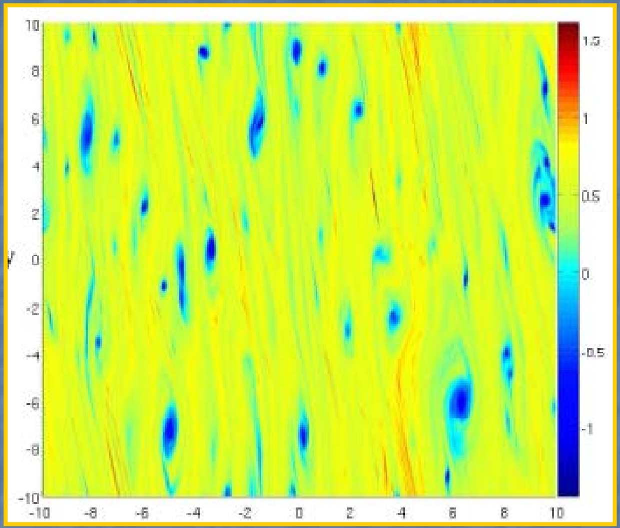
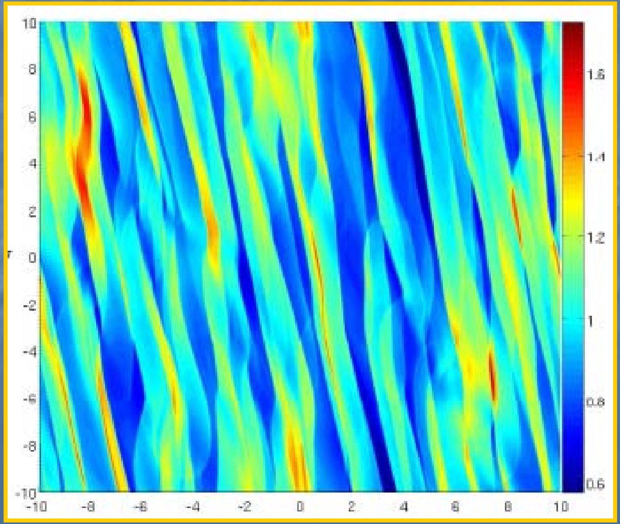


Without self-gravity

PV

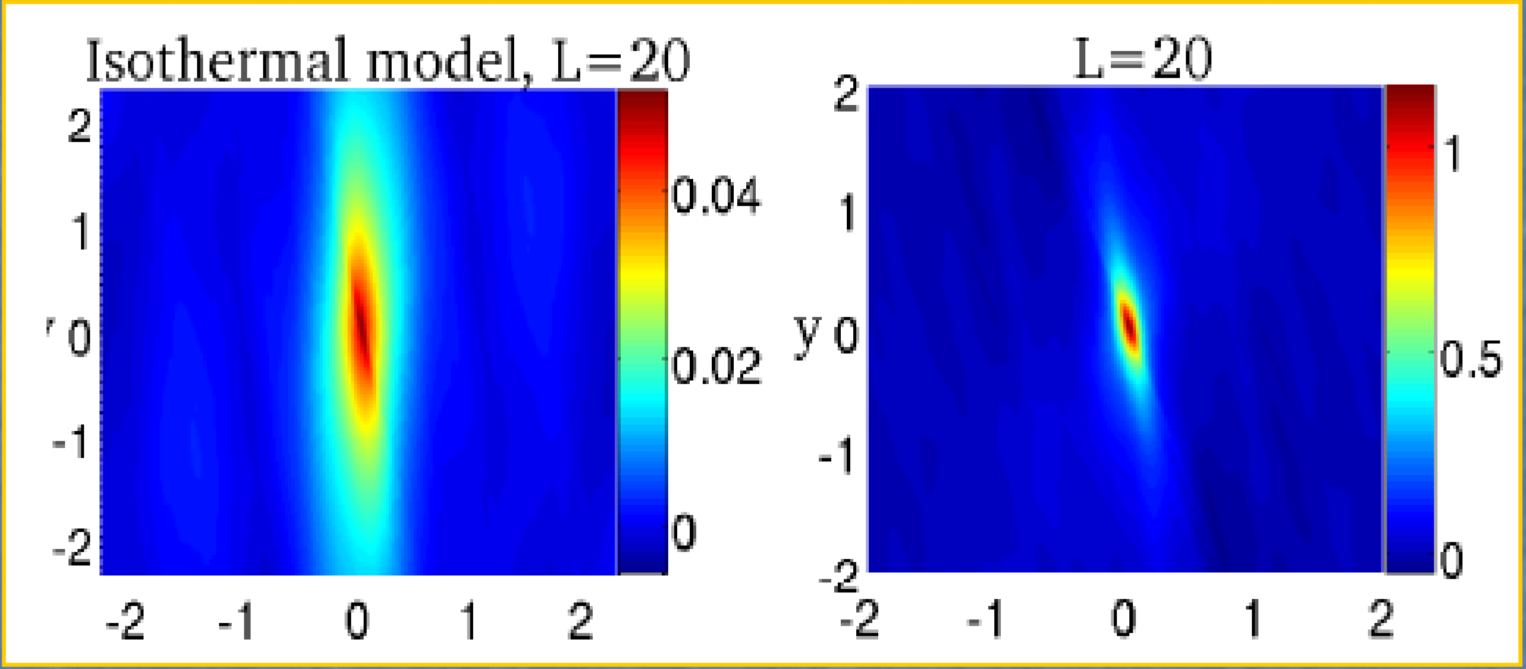


Σ



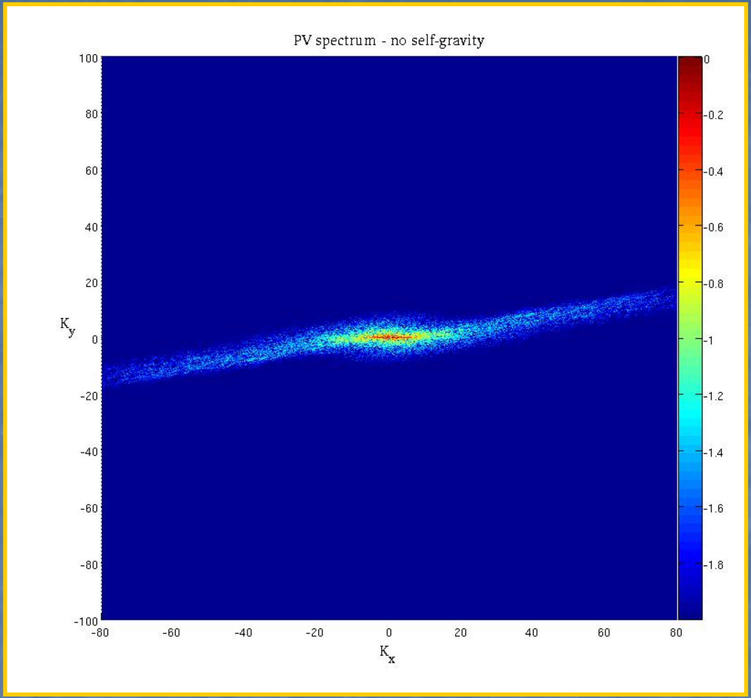
Contrast with non-self-gravitating case – autocorrelation functions

$$R_I(x,y) = \frac{\Sigma_0^2}{\Omega^2 L_x L_y} \int \delta I(x',y')\delta I(x+x',y+y')dx'dy',$$

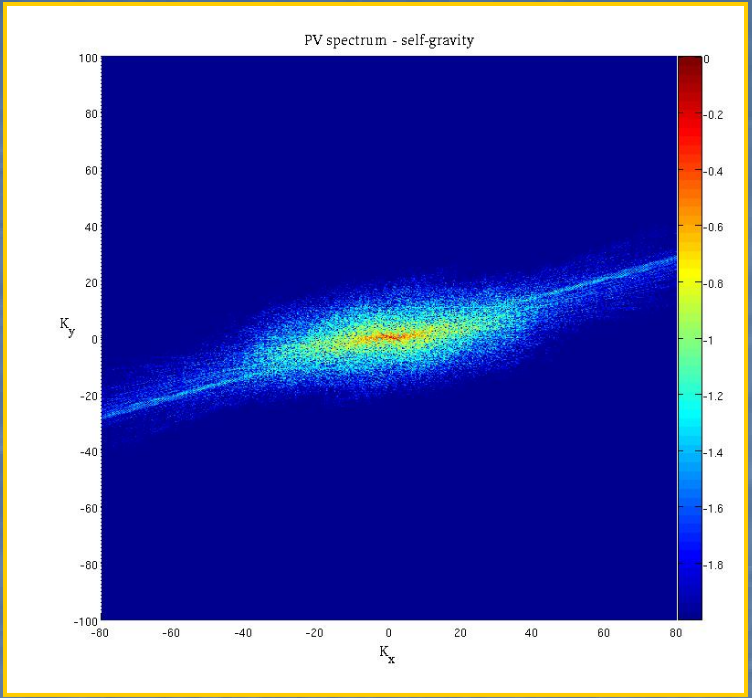


Non-self-gravitating **Self-gravitating**

Contrast with non-self-gravitating case – potential vorticity spectra



Non-self-gravitating



Self-gravitating

- PV (turbulent) spectra in both cases are strongly anisotropic due to the main Keplerian shear flow
- Spectrum in the self-gravitating case is broader than that in non-self-gravitating case – self-gravity opposes inverse cascade of power towards larger scales

Conclusions

- Self-gravity prevents the development of long-lived vortices, they instead are short-lived and transient structures.

- Vortices generate density waves (shocks). ***The dynamics of vortices and density waves are strongly coupled due to inhomogeneity (shear) of Keplerian rotation. Coupling persists even in the linear theory***

- Self-gravity opposes the inverse cascade energy to larger scales

Implications for planetesimal formation

- It seems difficult for such vortices to trap dust particles. If still trapped, particles should collapse out by their own self-gravity quickly before transient overpressure creating it gets sheared away and disperse (study in progress)

Thank you

Paper is available at:
arXiv:0901.1617 (MNRAS, 2009, in press)