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# Gravitational Interaction between a Low Mass Planet and a Viscous Disk and a Possible New Mode of Type I Planetary Migration

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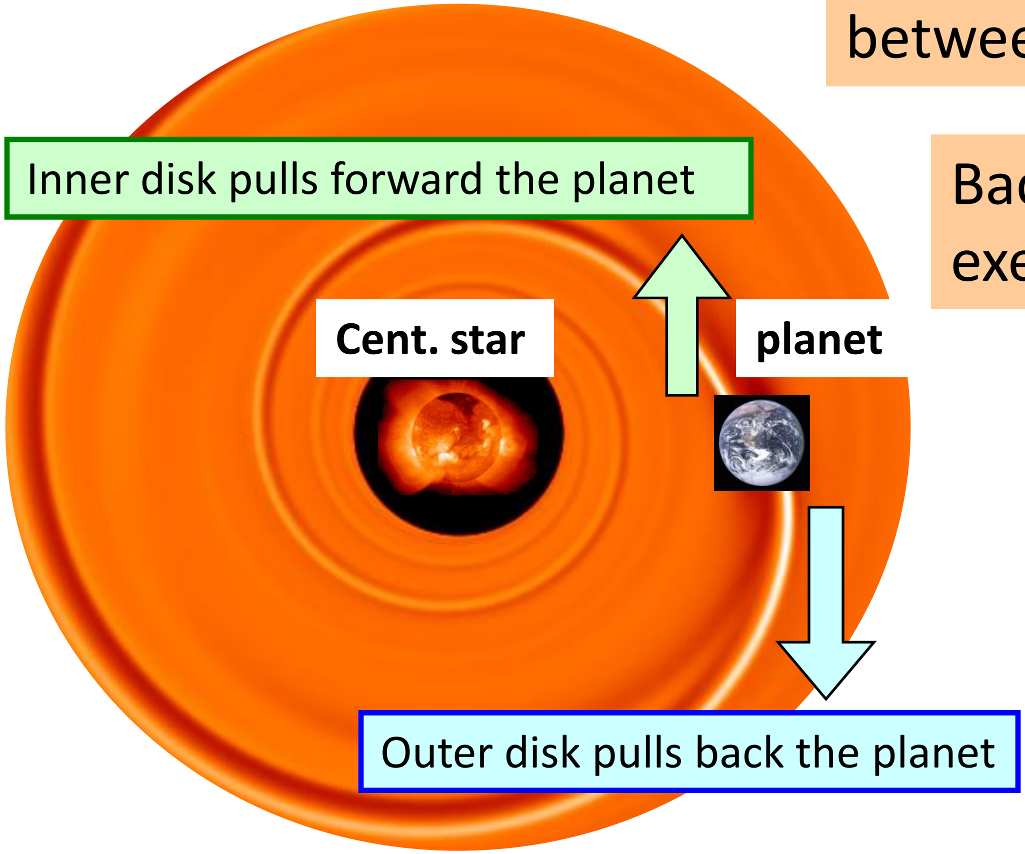
# Disk-Planet Interaction

Spiral density wave formation due to ***gravitational interaction*** between the disk and the planet




Back reaction of the wave exerts torque on the planet

After some complicated calculations…

***Protoplanets seems to fall into the central star***

Background figure is from F. Masset's webpage
http://www.maths.qmul.ac.uk/~masset/moviesmpegs.html

# Type I Migration Rate

**Protoplanets migrate inward due to disk-planet interaction**

‘Tanaka formula’: ***linear*** analysis, ***isothermal*** disk

$$\tau = (2.7 + 1.1\alpha)^{-1} \frac{M_c}{M_p} \frac{M_c}{\sigma_p r_p^2} \left(\frac{c}{r_p \Omega_p}\right)^2 \Omega_p^{-1}$$

For a protoplanet of 1 Mearth at 5AU embedded in a Minimum Mass Solar Nebula

$$\tau \sim 8 \times 10^5 \mathrm{yr} < \tau_{nebula} \sim 10^7 \mathrm{yr}$$

Protoplanets fall onto the central star ***BEFORE*** gas dispersal

Tanaka, Takeuchi and Ward (2002)

# Some Recent Studies

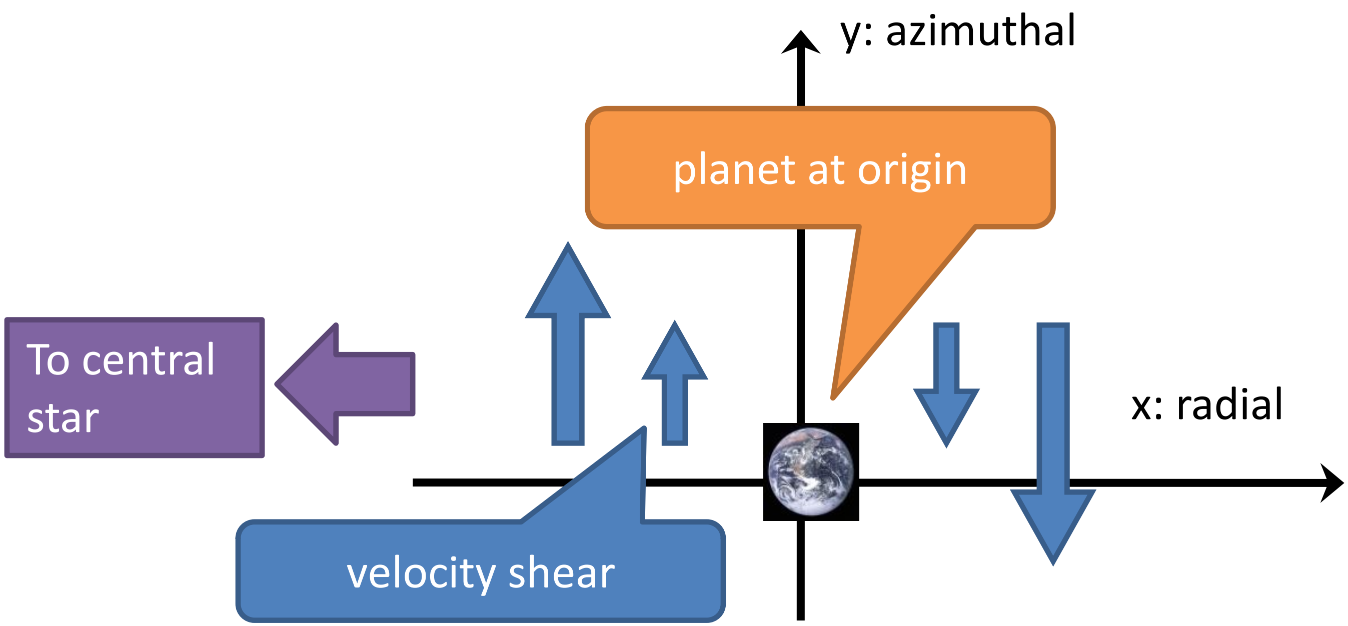
- ***Modification to migration rate and direction*** due to various physical processes in the disk
  - Viscosity
    - Masset (2001, 2002), Paardekooper and Papaloizou (2009)…
  - Self-gravity
    - Baruteau and Masset (2008)…
  - Thermal physics
    - Paardekooper and Mellema (2006), Baruteau and Masset (2008), Paardekooper and Paploizou (2008), Kley and Crida (2008), Bitsch and Kley(Poster 7.1)…
  - Turbulence
    - Nelson and Papaloizou (2004), Oishi et al. (2007)…
  - Ordered (stable) magnetic field
    - Terquem (2003), Fromang et al. (2005), Muto et al. (2008)…
- Necessary to calculate how different physical processes affect type I migration rate
  - ***What are 'additional terms' to Tanaka formula?***

# This Work

- Linear study of viscous disk-planet interaction revisited
  - Local linear analysis of wake generation/structure
    - One-sided *Lindblad torque*
  - Different formulation is used
    - Easy to extend to other cases
  - Wide range of viscous parameter is studied
  - Qualitatively different behaviour at high viscosity

# Setup of Calculation

- Local shearing-sheet approximation
  - Possible to look at detailed structure in the vicinity of the planet
  - Cannot calculate differential torque between outer/inner disk
  - Necessary calculation before differential torque is calculated (in modified local approx.)



# Basic Equations

- Navier-Stokes equation with bulk viscosity=0 (for simplicity)

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{v}) = 0$$

$$\frac{\partial \boldsymbol{v}}{\partial t} + \boldsymbol{v} \cdot \nabla \boldsymbol{v} + 2\Omega_{\mathrm{p}} \boldsymbol{e}_z \times \boldsymbol{v} = -\frac{c^2}{\rho} \nabla \rho + 3\Omega_{\mathrm{p}}^2 x \boldsymbol{e}_x + \nu \nabla^2 \boldsymbol{v} + \frac{1}{3} \nu \nabla (\nabla \cdot \boldsymbol{v}) - \nabla \psi_{\mathrm{p}}$$

Coriolis force

tidal force

Viscous terms

Perturbation by planet

- Background state: linear Kepler shear

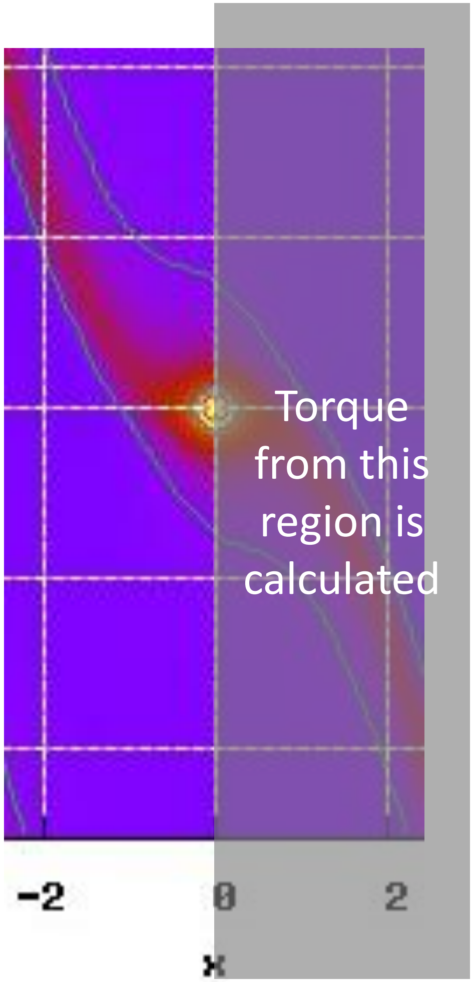
$$\rho = \rho_0 = \mathrm{const} \qquad \boldsymbol{v}_0 = -\frac{3}{2} \Omega_{\mathrm{p}} x \boldsymbol{e}_y .$$

# Linear Analysis

1. Planet potential is treated as a source of perturbation

$$\left(\frac{\partial}{\partial t}-\frac{3}{2}\Omega_{\mathrm{p}}x\frac{\partial}{\partial y}\right)\frac{\delta\rho}{\rho_0}+\nabla\cdot\delta\boldsymbol{v}=0$$

$$\left(\frac{\partial}{\partial t}-\frac{3}{2}\Omega_{\mathrm{p}}x\frac{\partial}{\partial y}\right)\delta\boldsymbol{v}-2\Omega_{\mathrm{p}}\delta v_y\boldsymbol{e}_x+\frac{1}{2}\Omega_{\mathrm{p}}\delta v_x\boldsymbol{e}_y$$
$$=-c^2\nabla\frac{\delta\rho}{\rho_0}+\nu\nabla^2\delta\boldsymbol{v}+\frac{1}{3}\nu\nabla\left(\nabla\cdot\delta\boldsymbol{v}\right)-\nabla\psi_{\mathrm{p}}$$




2. Density structure (in steady state) is calculated

3. Torque exerted by one side of the disk is calculated

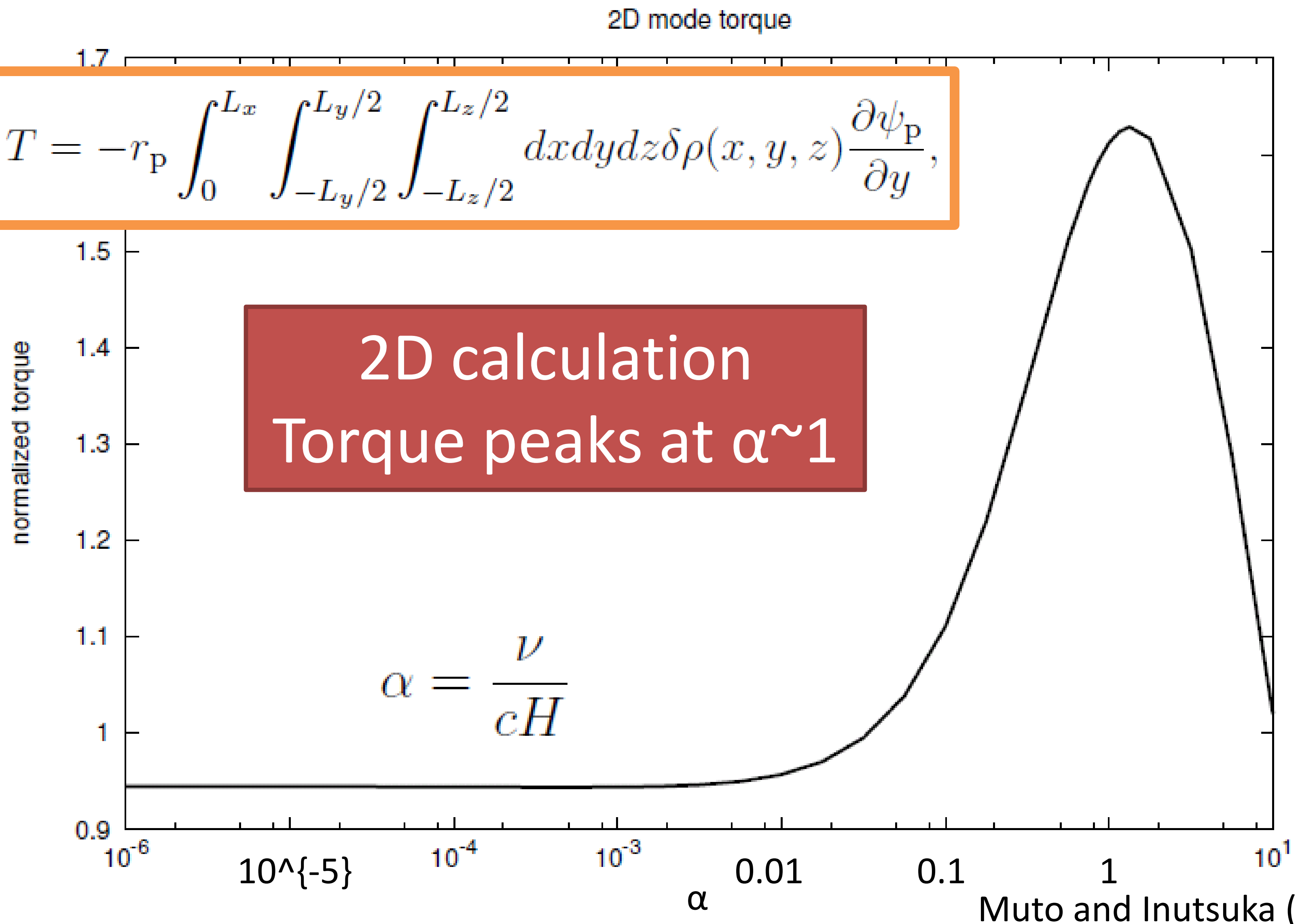
$$T=-r_{\mathrm{p}}\int_0^{L_x}\int_{-L_y/2}^{L_y/2}\int_{-L_z/2}^{L_z/2}dxdydz\delta\rho(x,y,z)\frac{\partial\psi_{\mathrm{p}}}{\partial y}$$

# Some Technical Notes

- Technical difficulties in obtaining stationary solution with viscosity
  - Highest-rank derivative comes from viscous terms
  - Boundary condition? Limit to $\nu \rightarrow 0$?
- ***Time-dependent method***
  - Fourier analysis in 'sheared coordinate'
    - Originally, Narayan et al. (1987)
  - Non-axisymmetric modes reach the steady state as a result of time evolution
  - Simple system of ODEs with respect to time
  - Easy to include the effects of various physical processes

# Magnitude of one-sided torque

$$T = -r_{\rm p} \int_0^{L_x} \int_{-L_y/2}^{L_y/2} \int_{-L_z/2}^{L_z/2} dx dy dz \delta\rho(x,y,z) \frac{\partial \psi_{\rm p}}{\partial y},$$

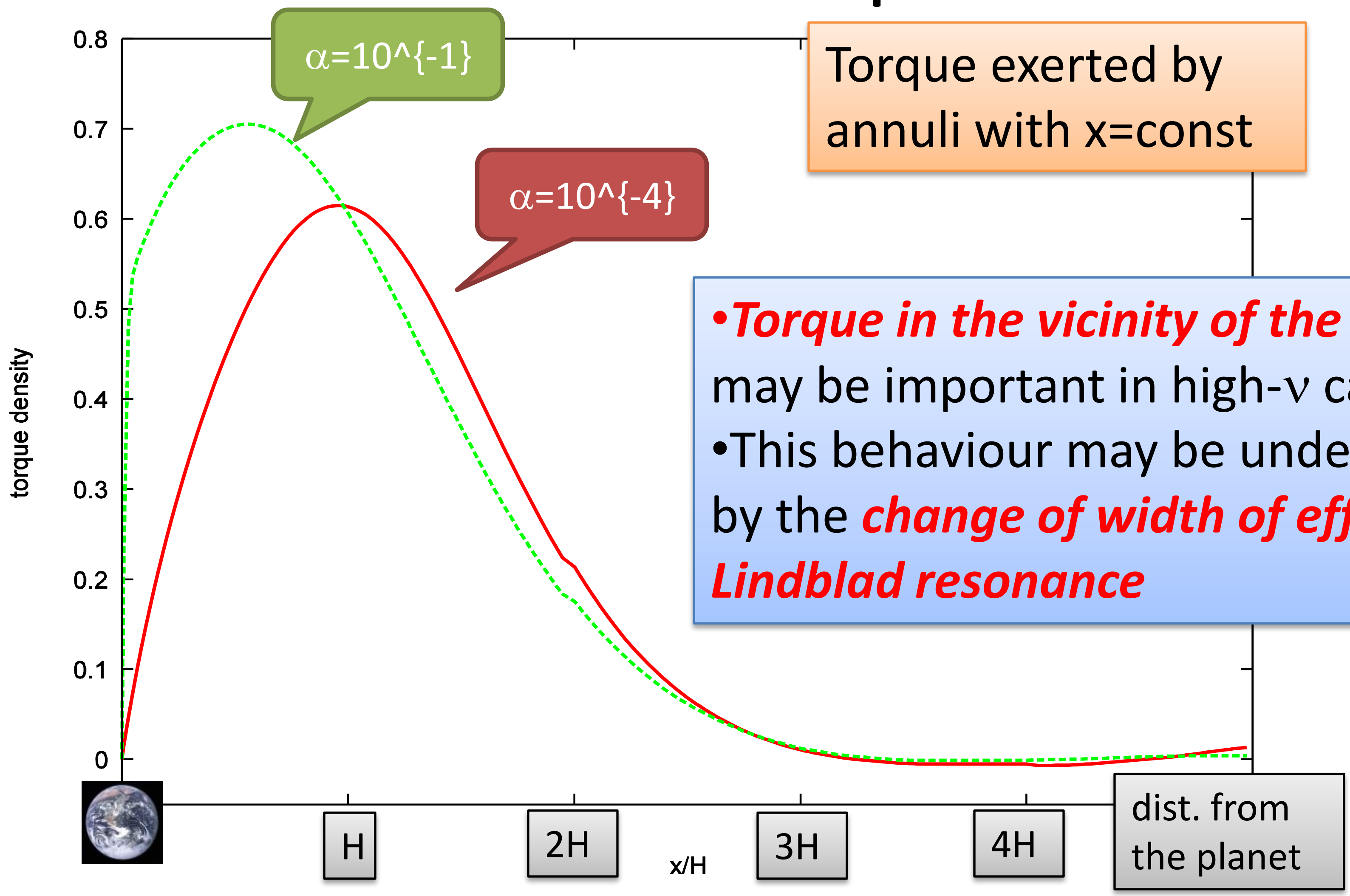



2D calculation
Torque peaks at α~1

$$\alpha = \frac{\nu}{cH}$$

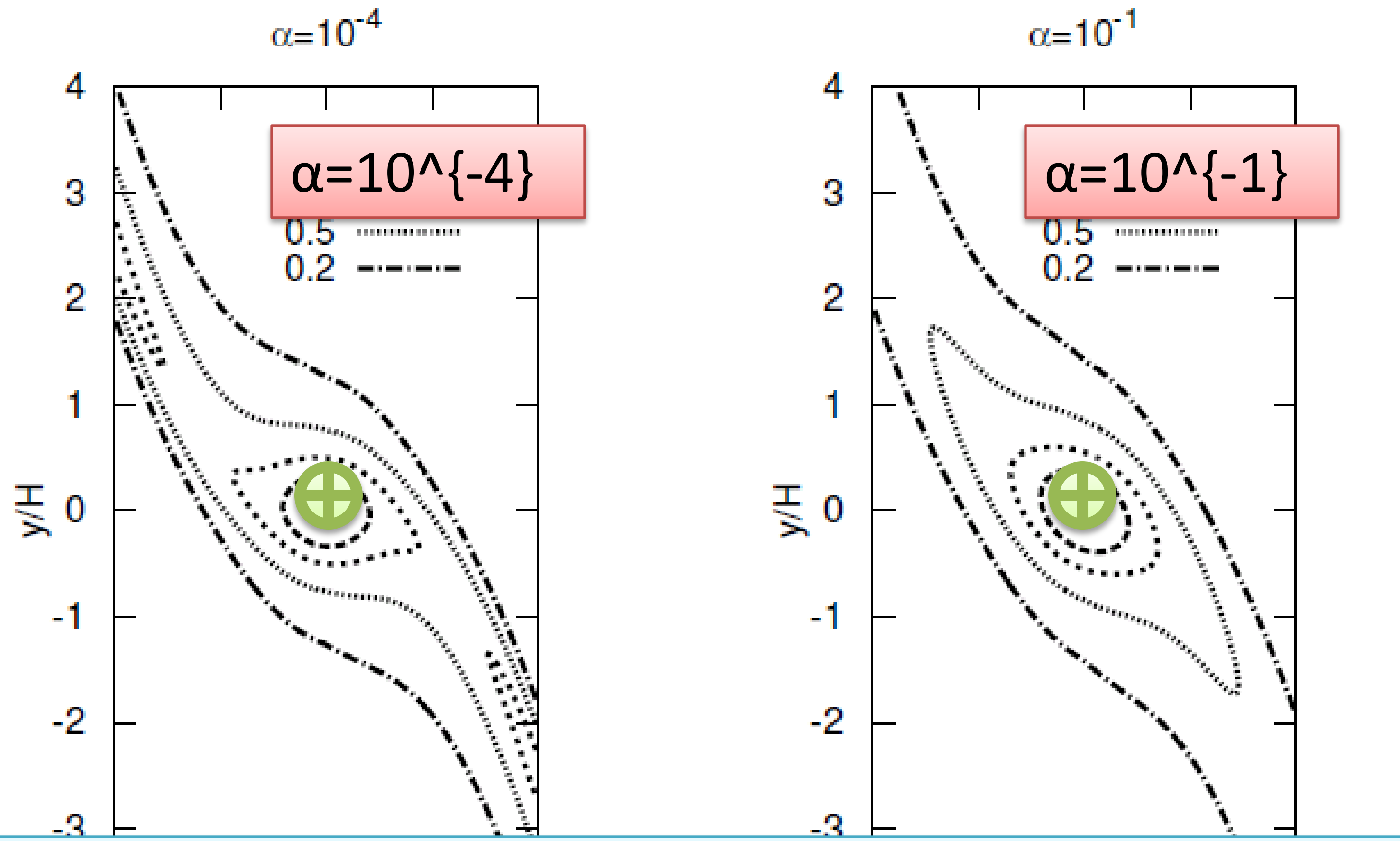
Muto and Inutsuka (2009)
arXiv:0902.1887

# Location of the Disk that Contributes to the Torque



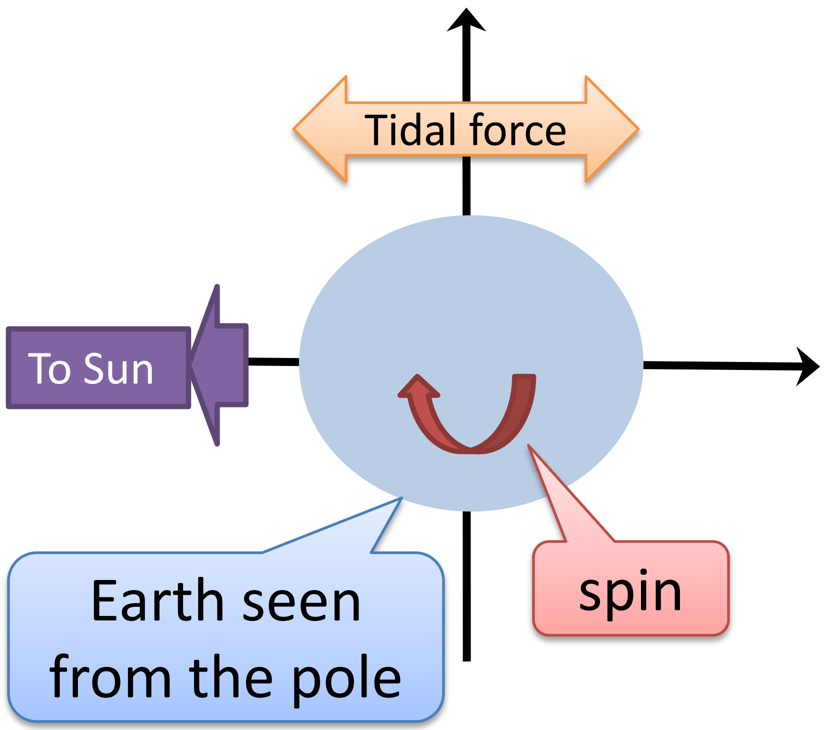

- ***Torque in the vicinity of the planet*** may be important in high-ν cases
- This behaviour may be understood by the ***change of width of effective Lindblad resonance***

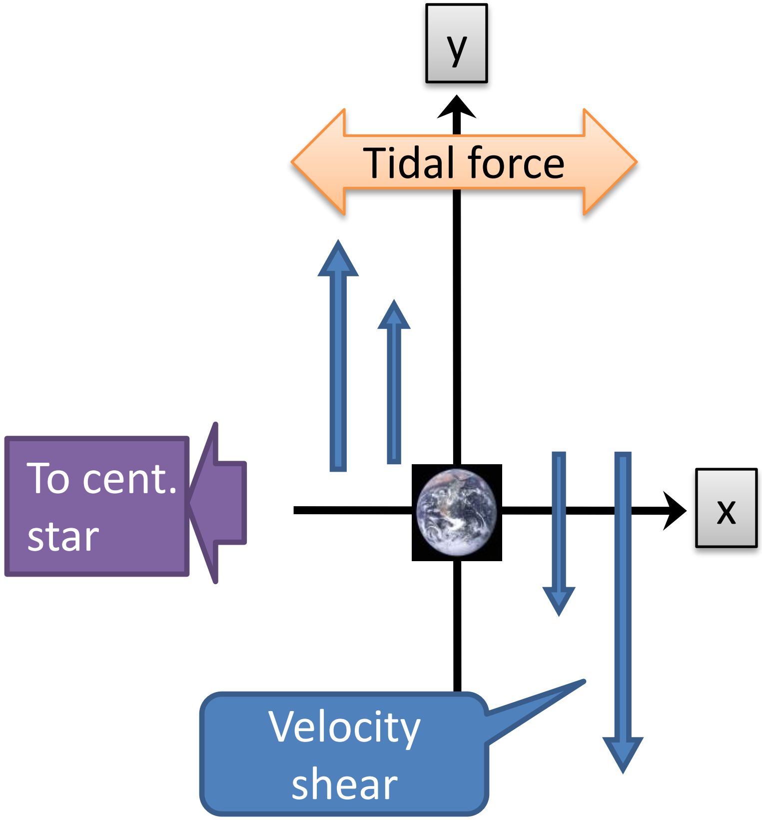
# Density Structure at Disk Midplane




- ***Tilted spheroidal density structure*** in the vicinity of the planet
  - Torque imbalance in y-direction (azimuth)
  - Density structure in the vicinity of the planet is important

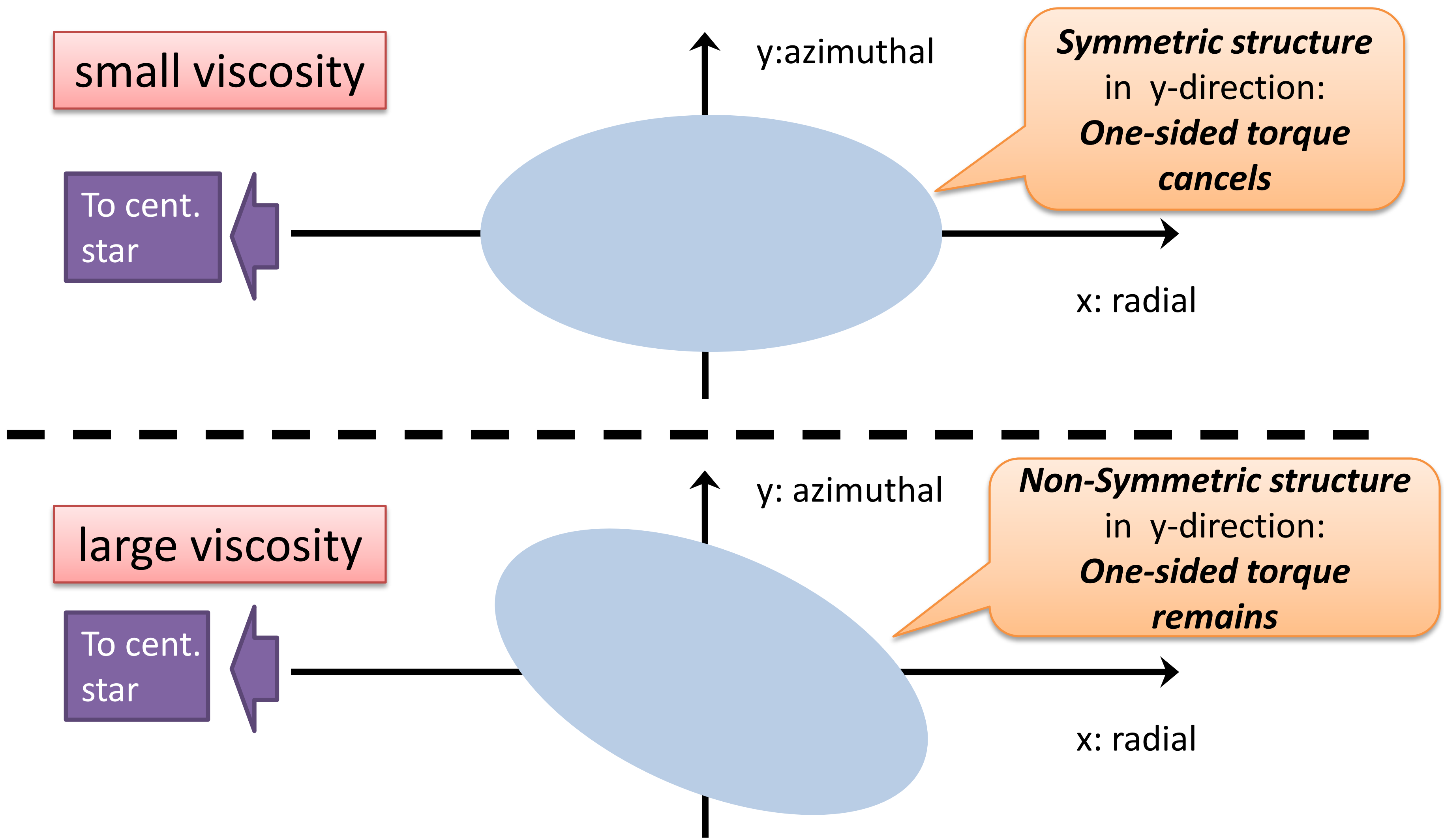
# Analogy with the Oceanic Tide



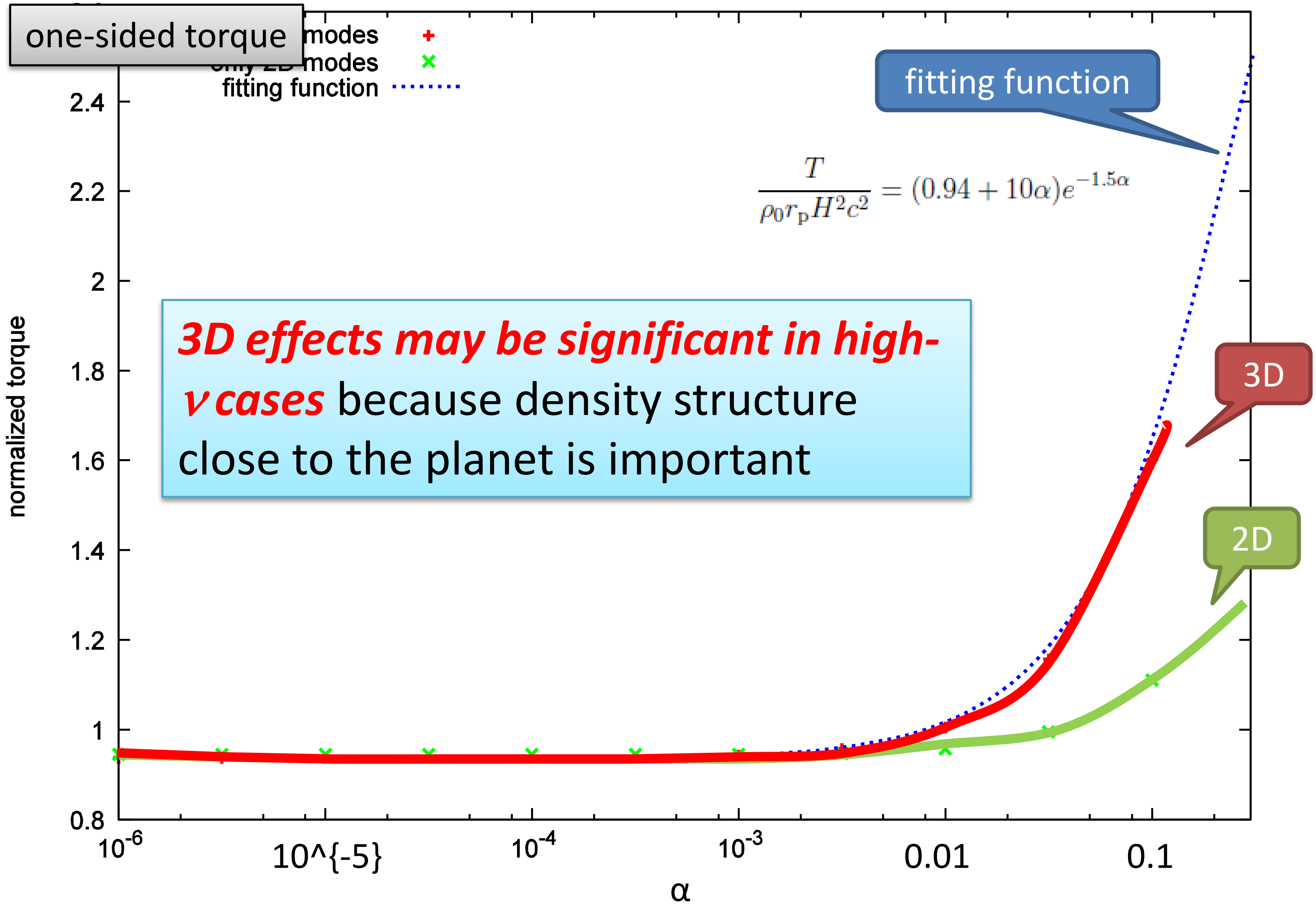




- Tidal force is the same
- Velocity shear corresponds to the spin of the Earth

# Density structure: qualitative view



# Importance of 3D structure




***3D effects may be significant in high-ν cases*** because density structure close to the planet is important

# Summary and Future Work

- Local linear analysis of viscous disk-planet interaction is performed with various viscosity using a different formulation from that used before
- Density structure in the vicinity of the planet may play an important role in highly viscous disk
- Three-dimensional, high resolution calculation is essential in highly viscous disk

- Extend formulation to 'modified' local approx.
  - calculate differential torque
  - compare with corotation torque
- Non-linear simulations