Runaway growth in protoplanetary disks

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Outline

- Characteristics of Runaway Growth (RG)
- Modeling RG
  - Continuous vs discrete methods
  - High dynamic range Monte Carlo method
- Planetesimal accretion in the Kuiper Belt

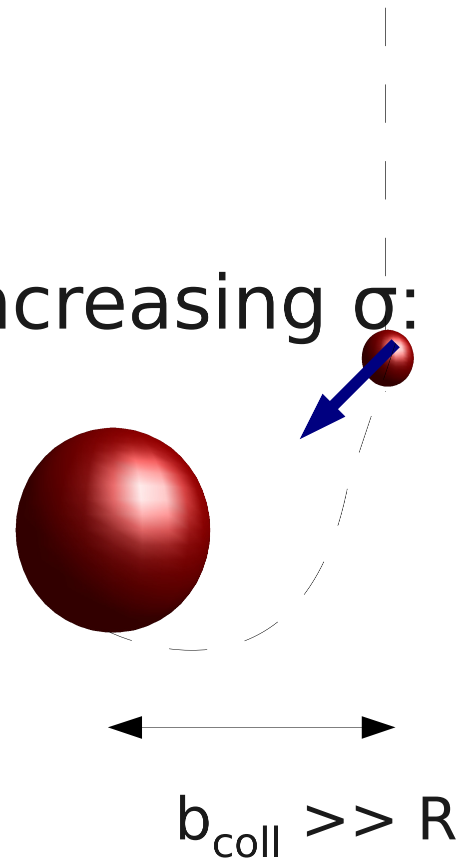
Terminology

- Usage of “runaway growth”
  - Positive feedback (e.g., thermal runaway)
  - Accretion of gas on planet
  - Within a single population due to coagulation
    e.g., dust grains, planetesimals, stellar systems
  - Mathematically, referred to as “gelation”
    Instantaneous!

Gravitational focussing

- Massive particles (planetesimals) gravitationally attract each other, increasing σ:

$$\sigma_{ij} = \sigma_{\rm geom}\left[1 + \left(\frac{v_{\rm esc}}{\Delta v}\right)^2\right]; \qquad v_{\rm esc}^2 = \frac{GM_{\rm big}}{R}$$



Thus, $\sigma \propto (M_{big})^{4/3}$ if $\Delta v < v_{esc}$

- Potential for RG in grav.-dominated regime.

Runaway growth (RG)

- Define RG as $$\frac{d(M_1/M_2)}{dt} > 0$$
  - Particles **separate** in mass
  - Condition independent of absolute rate of growth
- dM/dt determined by the collision kernel

$$K_{ij} = \sigma_{ij} \Delta v_{ij}; \quad \text{if} \quad K_{ij} \propto M_i^{\nu} m_j^{\beta-\nu} \quad (M_i \gg m_j) \quad \text{then}$$

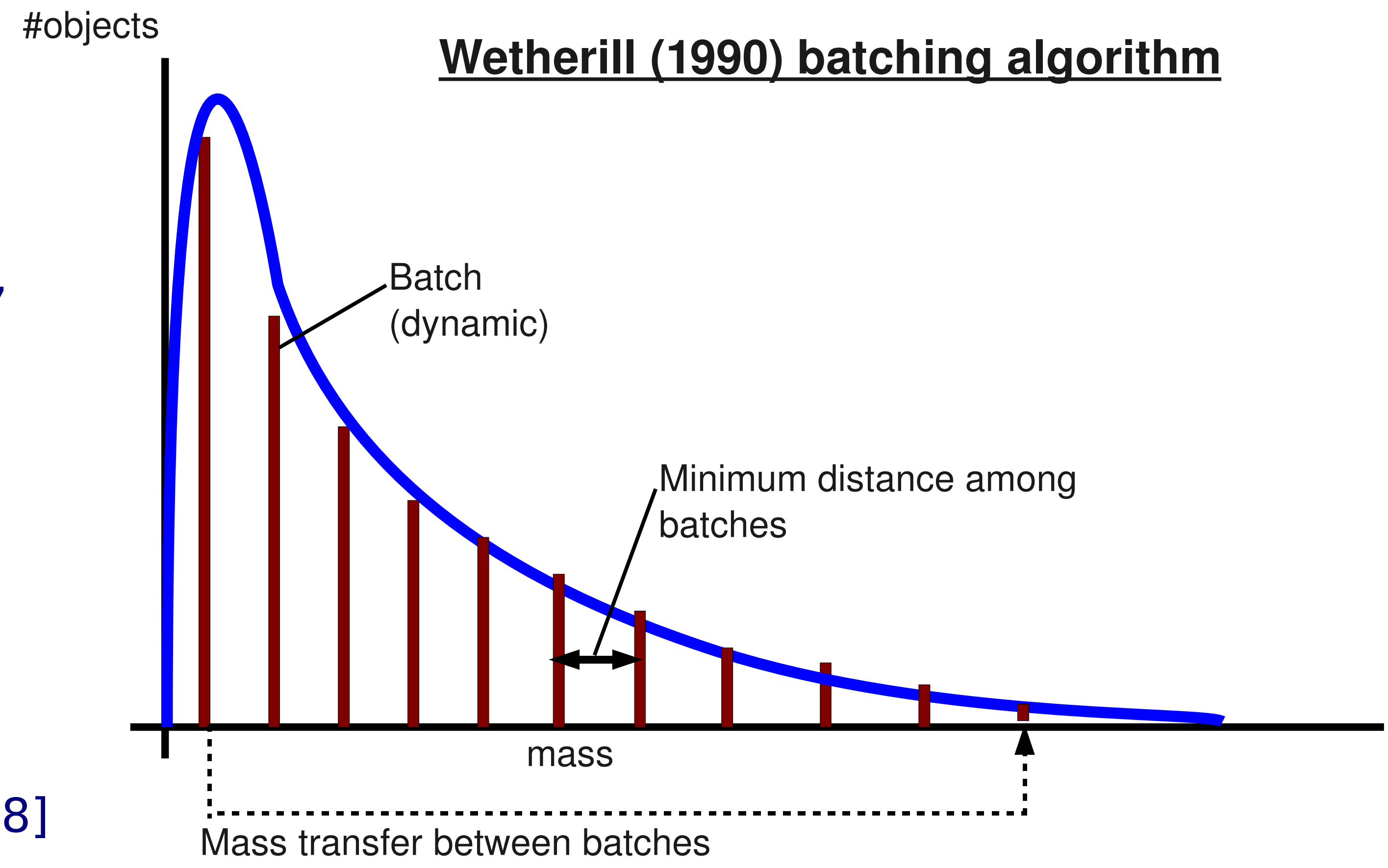
  - It follows [e.g., Lee 2000; Ormel & Spaans (2008)]:
    - If ν>1: RG, instantaneous (!) in the limit N -> ∞
    - If $\nu \leq 1$, β>1: RG after a time t_0
  - Gravitational focusing $\sigma \propto M^{4/3}$; ν>1 possible

Modelling RG

- Difficult for continuous methods

  But see Lee (2000, 2001)

- Enforce discretization
  [e.g., Wetherill 1990, Kenyon & Luu (1999), Inaba et al. (2001)]

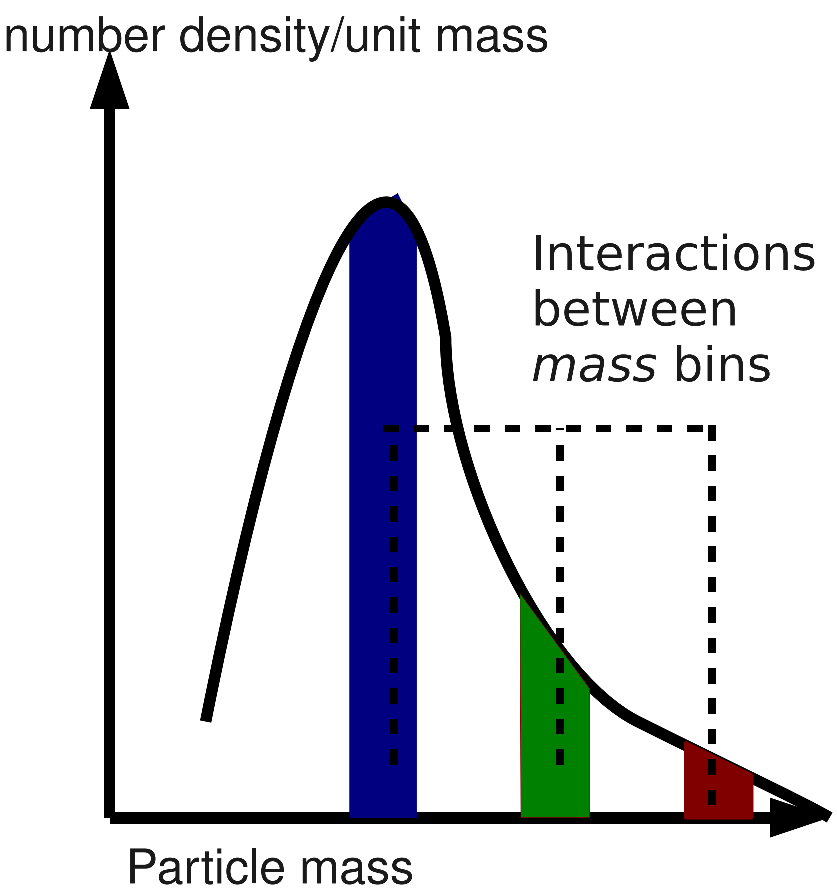


- Use a Monte Carlo method
  [Ormel & Spaans 2008]

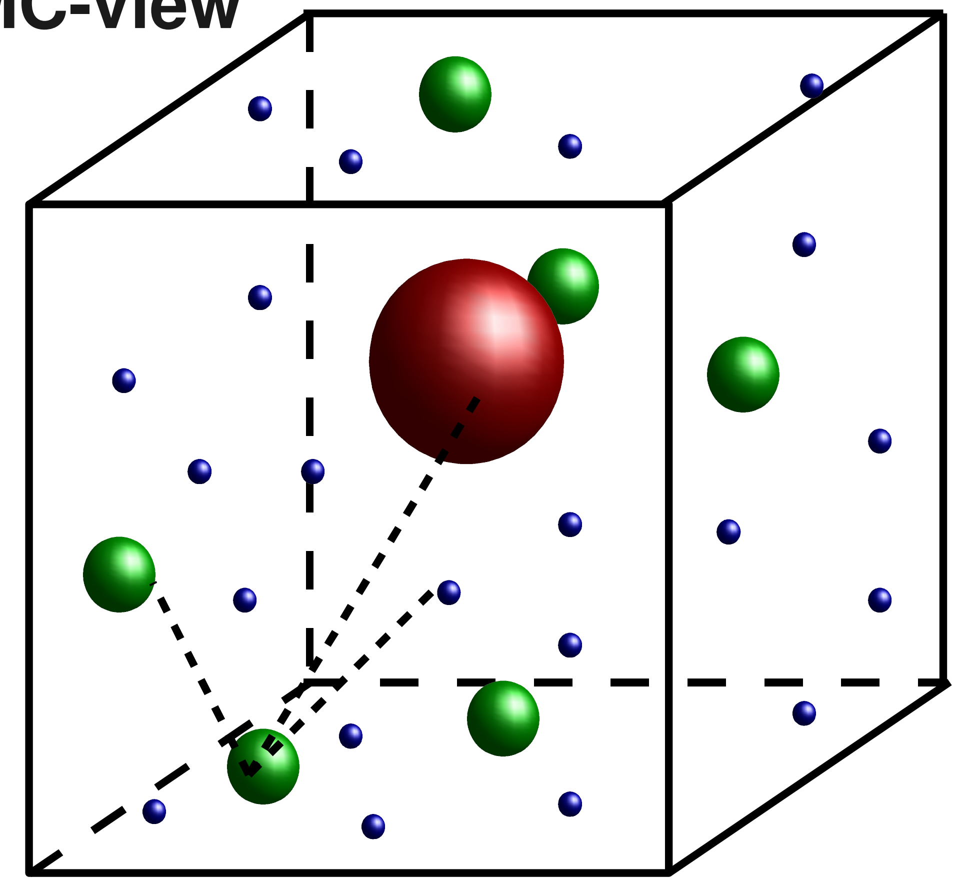
Continuous vs Monte Carlo (MC)

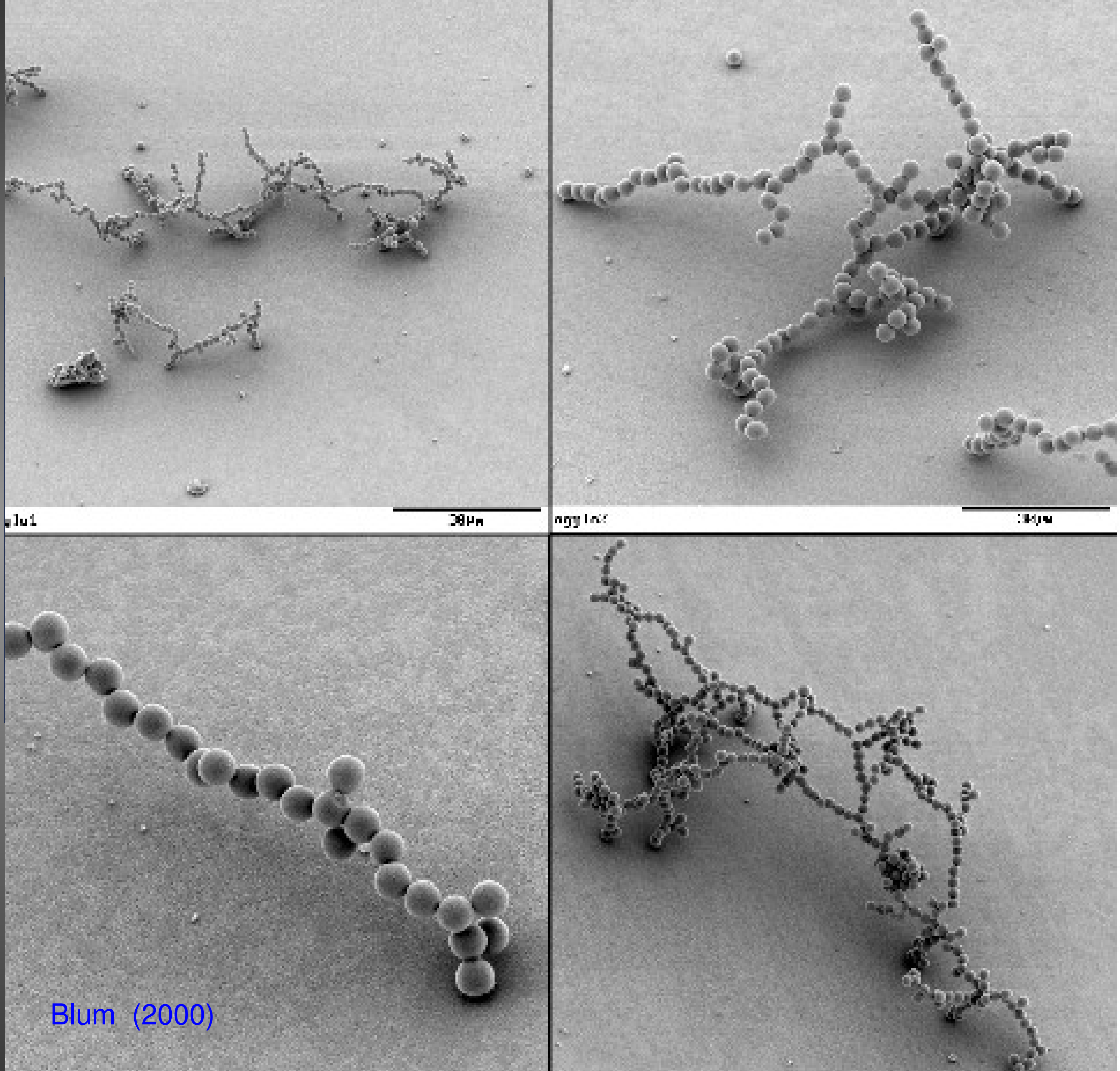
Continuous view

(distribution method)



MC-view





Blum (2000)

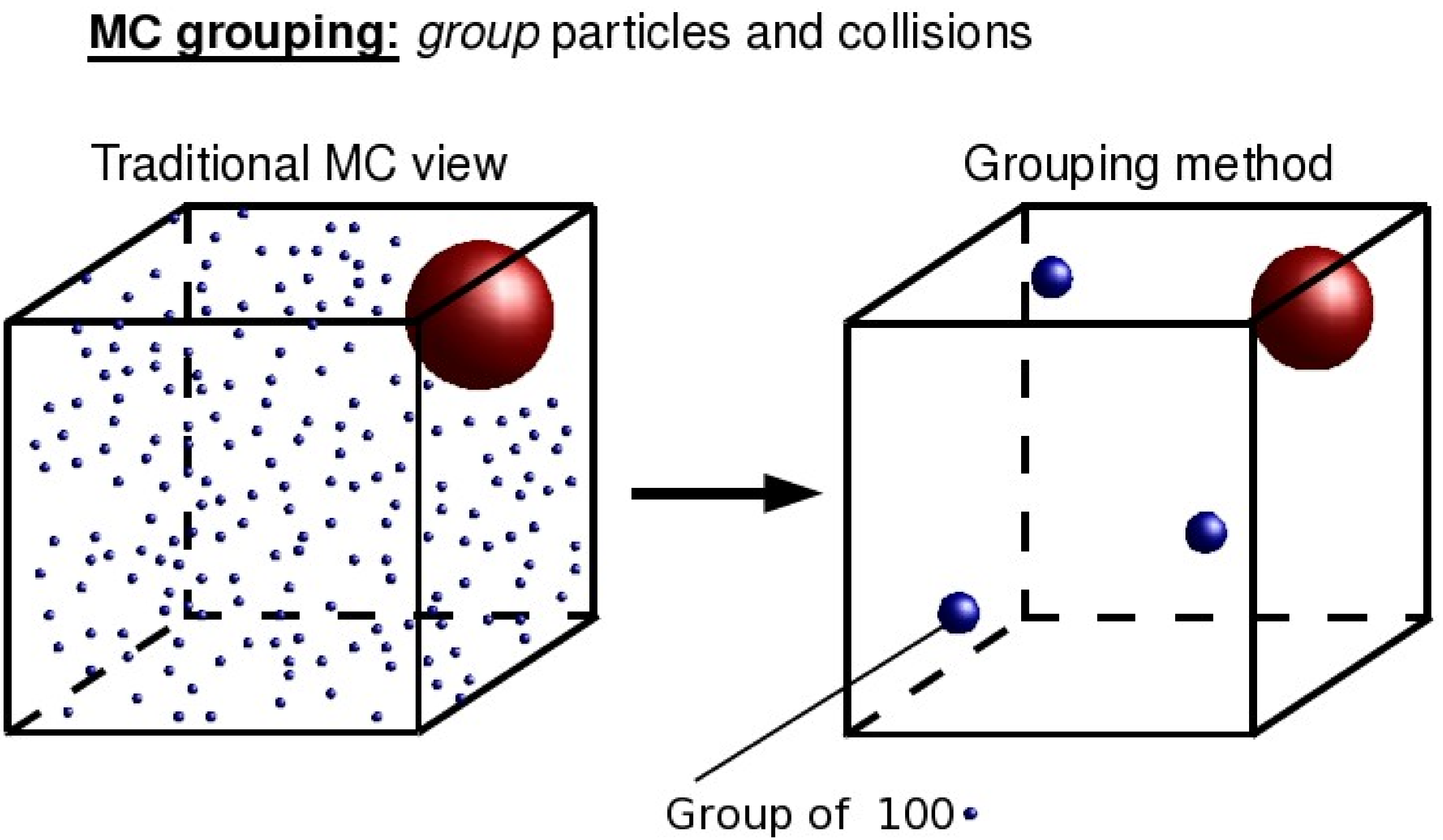
Monte Carlo coagulation

- Main advantage: include “particle” properties
  - E.g., porosity, composition, eccentricity, etc.
  - See poster P5.5 by A. Zsom et al.

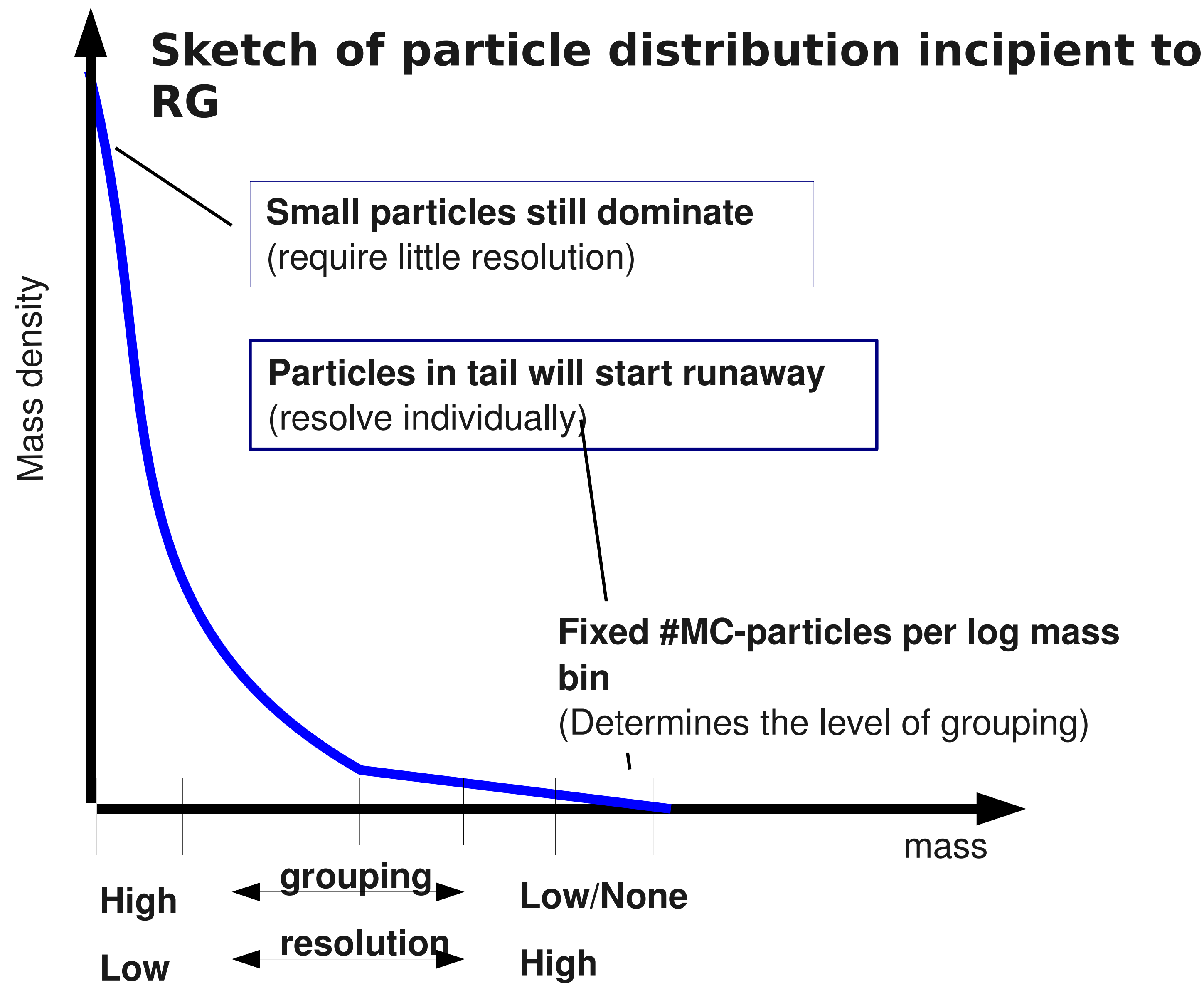
- But how to manage 10^X dust particles/planetesimals with MC?

- MC at high dynamic range: 2 tricks

Ingredient #1: grouping



Ingredient #2: flexible grouping



Strong runaway growth

- Test behavior of kernels $K \propto (m_i \; m_j)^{\nu}$

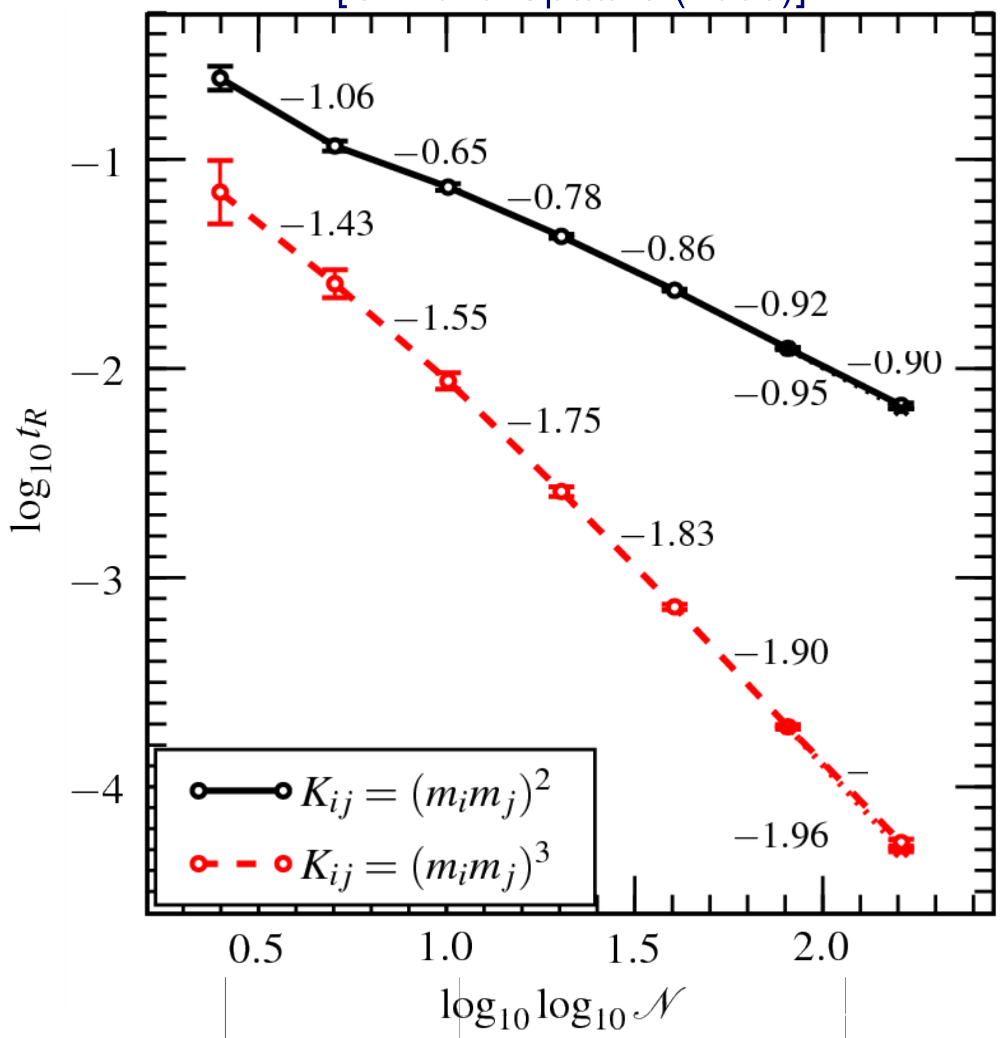
  with ν=2,3

  - Prediction: runaway time t_R scales as [Malyskin & Goodman (2001)]

$$t_R \propto (\log N)^{1-\nu}$$

slope -1, -2

[Ormel & Spaans (2008)]



Note: N (initial #particles) sets simulation size; number density same in all models!

N~300 10^{10} 10^{100}

Application: runaway growth in Kuiper Belt

- Model characteristics/approximations
  - Include velocity evolution; i.e., encounters can result in:
    - Collisions
    - Dynamical friction (energy equipartition)
    - Viscous stirring
  - Use order-of-magnitude expressions [Greenberg et al. (1991); Goldreich et al. (2004)]
  - Assume eccentricities~inclinations; calculate (random) velocity change.

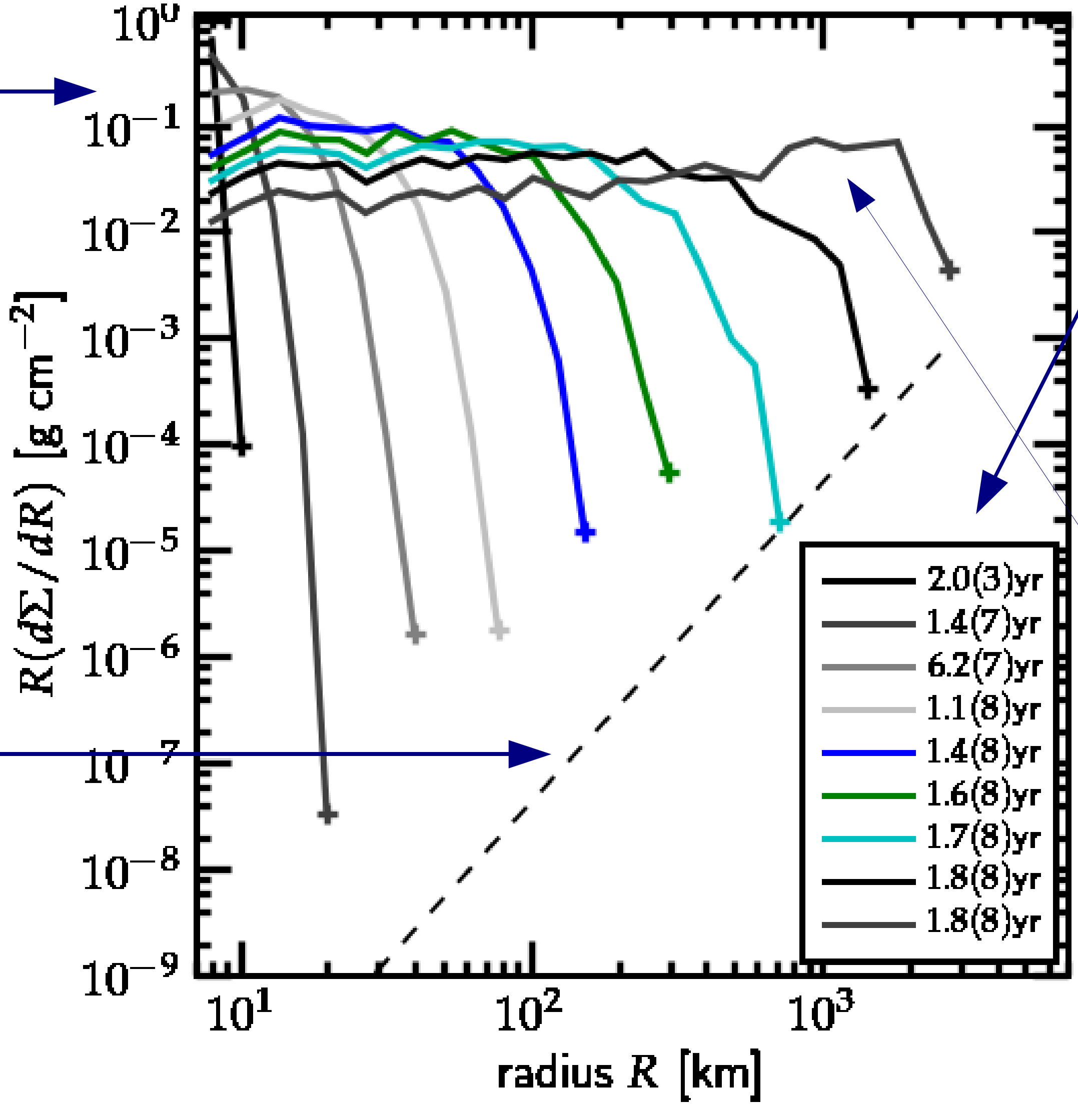
Approximations (ctd)

- .....
  - No fragmentation; all collisional collisions result in accretion; no gas drag
  - No spatial inhomogeneities
  - Simulate accretion @35 AU in a 6AU annulus (few Earth masses;
  - Start monodisperse planetesimal population of 8km and $\Delta v = 3$ m/s. (cf. Kenyon & Luu, 1999)

Results: w/o velocity evol.

-Y-axis indicates relative contribution by mass

1 particle/bin line

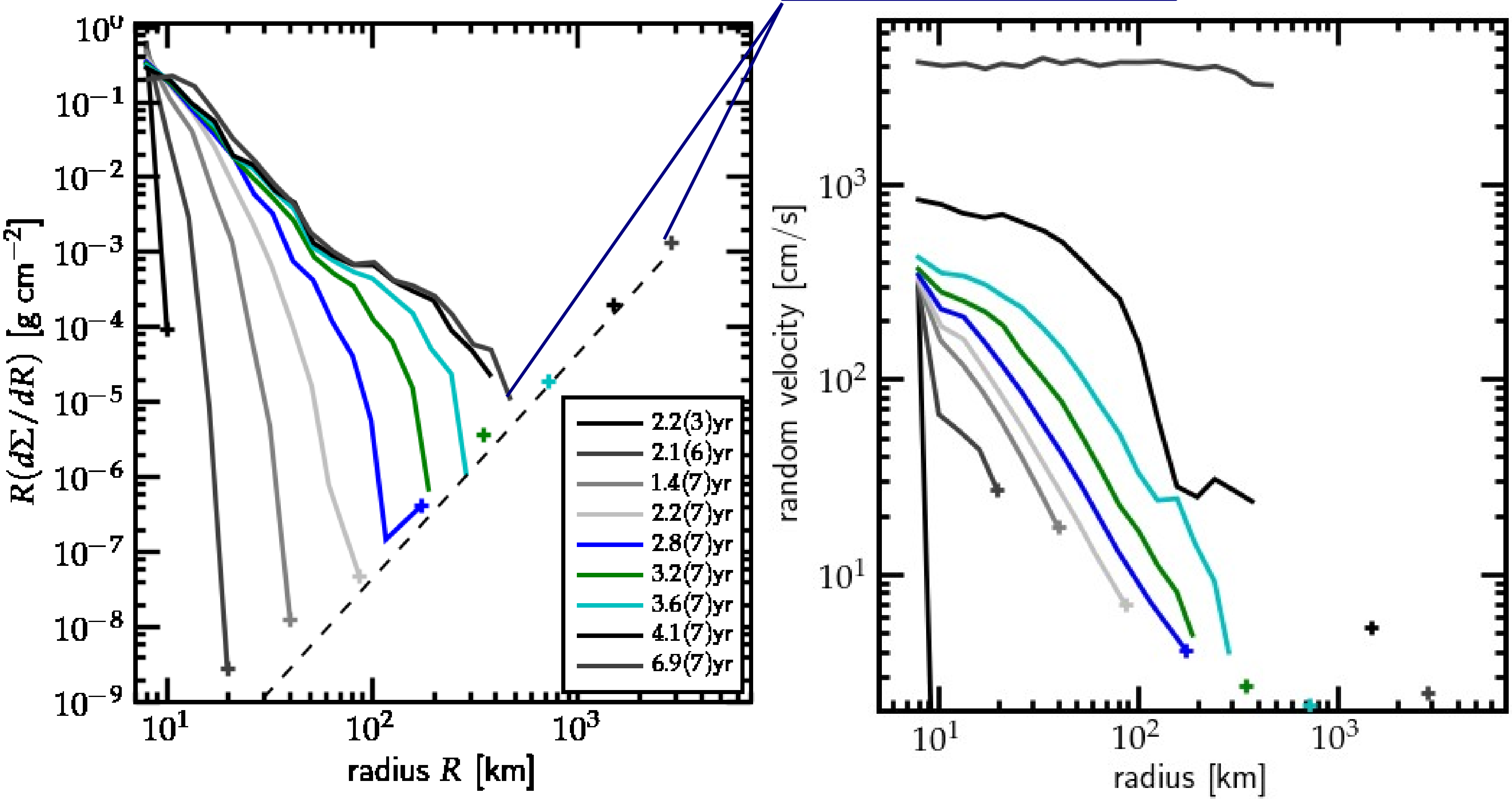


Distribution plotted after each factor of 2 increase in radius

3m/s < Hill velocity big bodies: runaway growth ceases

w/ velocity evolution

Large gap between m_1 and m_2



Conclusions/outlook

- Runaway accretion code tested
  - RG required for Kuiper Belt [cf. Kenyon & Luu, 1999]
  - Follow collisional evolution in more detail
  - E.g. model internal state of bodies (e.g., asteroids), binarity, etc.
- TBDs
  - Include fragmentation, gas drag, spatial inhomogeneities (oligarchy!), etc.
  - Compare/merge with N-body code