Formation of Multi-Planetary Systems in Turbulent Discs

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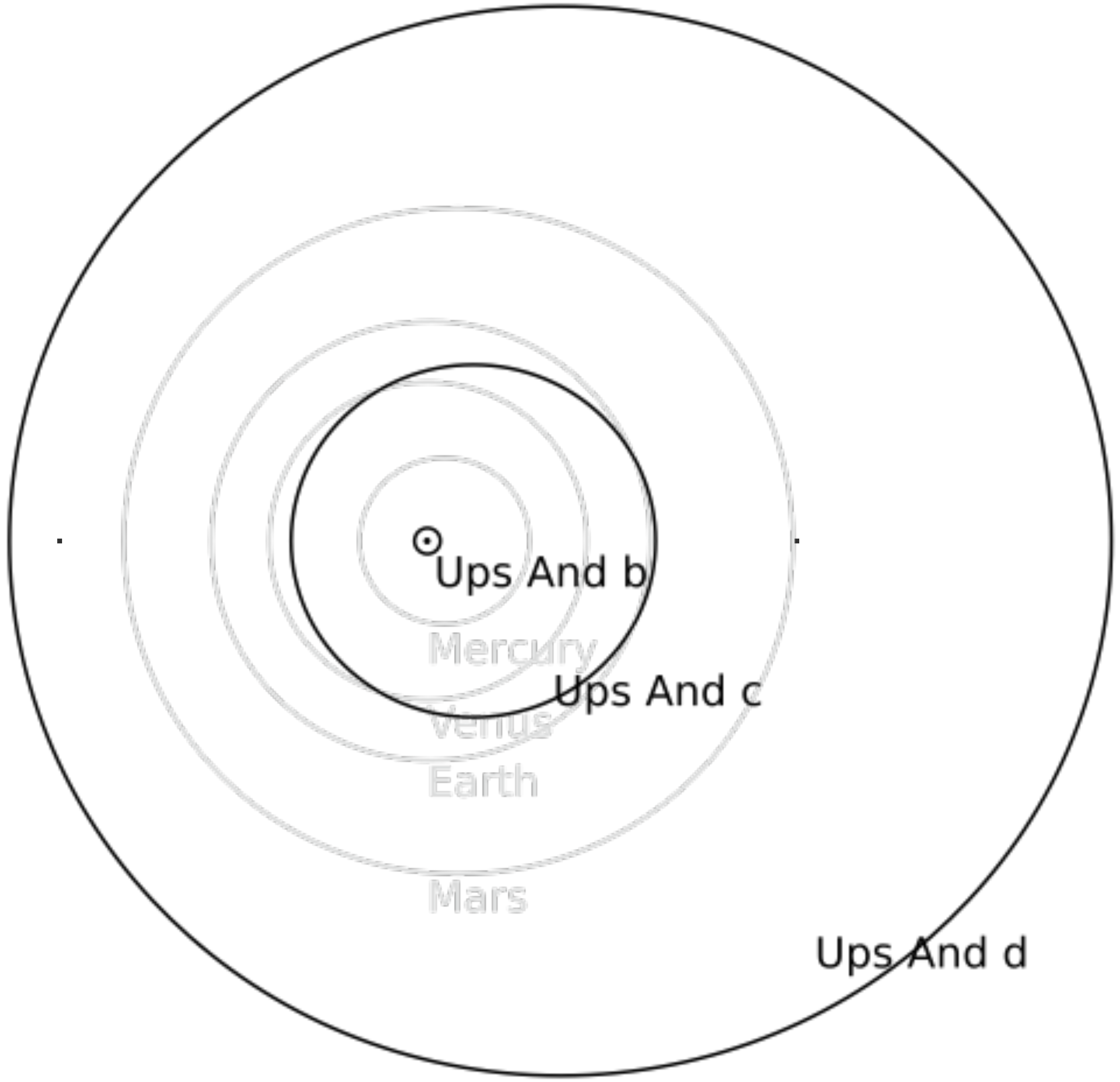
Rein & Papaloizou 2008 (A&A in press, arXiv:0811.1813)

Multi-Planetary Systems

Multi-planetary systems are more interesting than single planetary systems.

The complete History is encoded in the architecture.

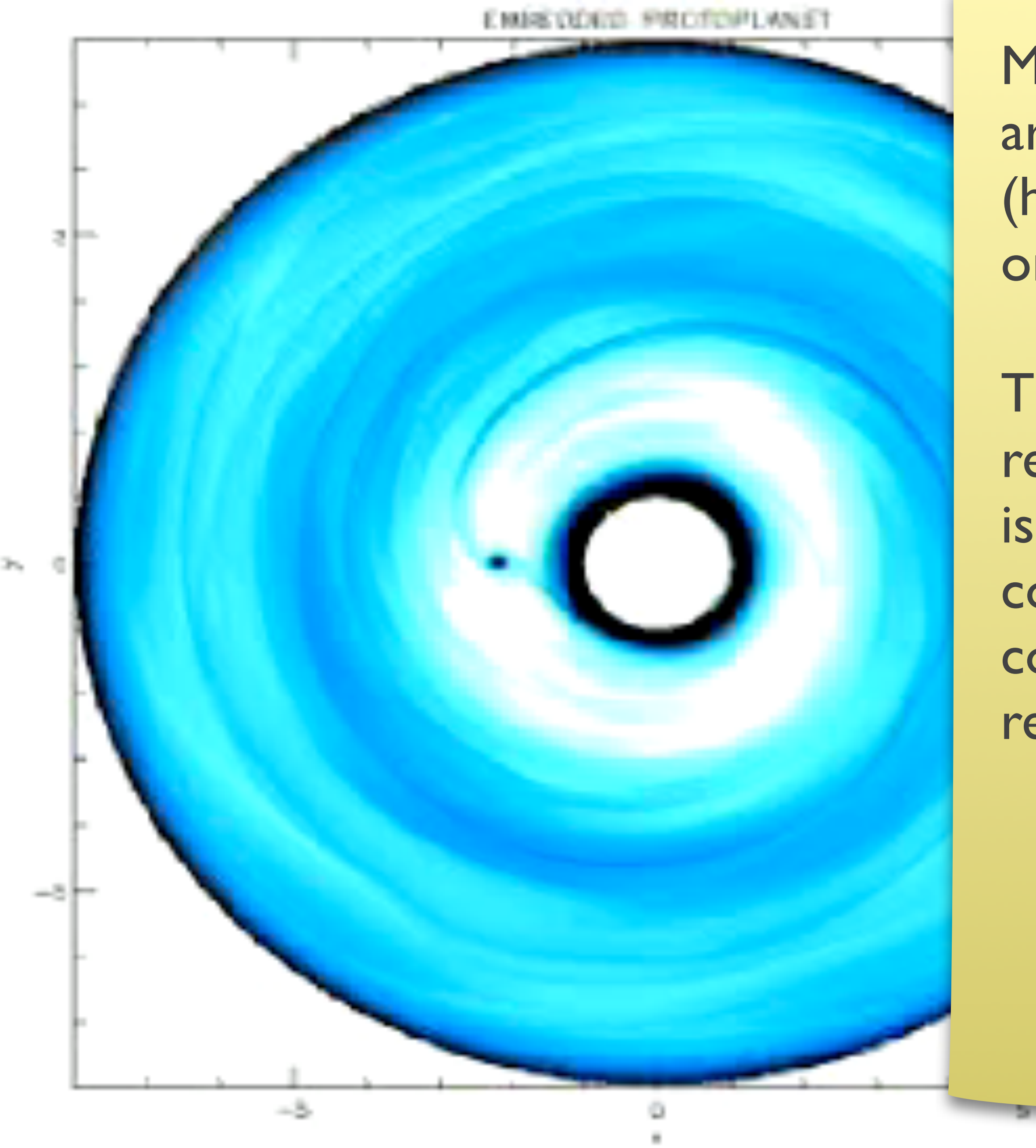


Orbital elements from exoplanet.eu

- Random forces
- An analytic description
- Stability of resonant systems
- Conclusions

Turbulent disc

- Angular momentum transport
- Magnetorotational instability
- Density perturbatior interact gravitationall with planets
- Random forces



MHD simulations are short (hundreds of orbits).

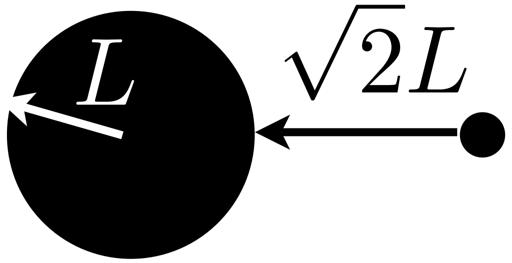
They have low resolution and the issue of convergence is not completely resolved.

Full MHD simulations by Nelson & Papaloizou (2004)
See also Laughlin et al. (2004) and Adams et al. (2008)

Scaling of MRI-forces

- Natural force scale

$$F_0(r) = M_{\rm pert} G/L^2 = \pi G \Sigma(r)/2$$



- Natural time scale (Correlation time)

$$\Omega^{-1}$$

- Reduction factors are crucial

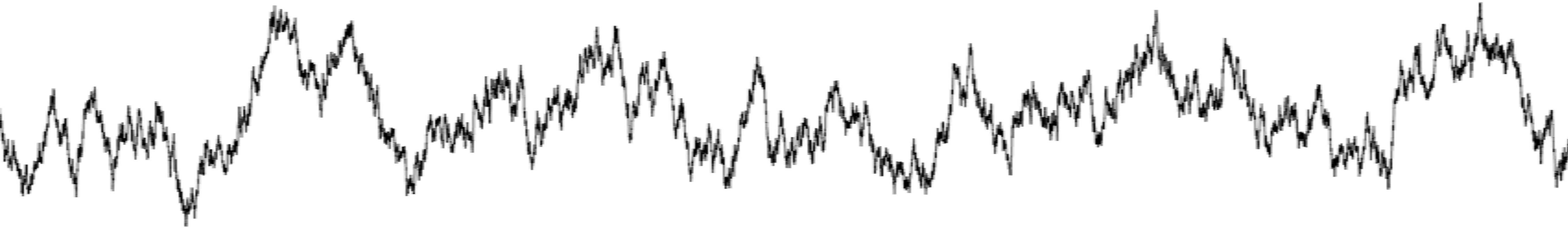
Density perturbation	0.1
Gap opening	0.1
...	...

Force scale doesn’t contain a preferred size scale.

Other factors which reduce the force scale are for example dead zones.

Estimates from MHD simulations by Oishi et al. (2007), Val-Borro et al. (2006), Nelson and Papaloizou (2004)

Stochastic forces



- Forces are stochastic and correlated

$$\langle F_i(t)F_i(t+\Delta t)\rangle_t = \langle F_i^2\rangle g(|\Delta t|)$$

- Auto correlation function

$$g(|\Delta t|) = \exp\left(-\frac{|\Delta t|}{\tau_c}\right)$$

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Autocorrelation function is similar to that of Oishi et al. (2007)
Implemented as a Markov process described by Kasdin (1995)

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Stochastic forces on a single planet

$$\begin{aligned}
\dot{L}_F &= m\left(\frac{\partial}{\partial\lambda}+\frac{\partial}{\partial\varpi}\right)(\mathbf{r}\cdot\mathbf{F}) \\
\dot{E}_F &= m\mathbf{v}\cdot\mathbf{F} \\
\dot{\varpi}_F &= \frac{\sqrt{(1-e^2)}}{n a\, e}\left[F_\theta\left(1+\frac{1}{1-e^2}\frac{r}{a}\right)\sin f - F_r\cos f\right] \\
\dot{\lambda}_F &= \left(1-\sqrt{1-e^2}\right)\dot{\varpi}_F+\frac{2an}{GM}\mathbf{r}\cdot\mathbf{F}
\end{aligned}$$

We add an additional term to the full Keplerian Hamiltonian and obtain the equations of motions.

This gives us the correct pre-factors.

Details described in Rein & Papaloizou (2008)

Growth of orbital parameters

$$
\begin{aligned}
(\Delta A)^2 &= \int_0^t \int_0^t F_i(t')F_i(t'')dt'dt'' \\
&= \int_0^t \int_0^t \langle F_i^2 \rangle \, g(|t'-t''|)dt'dt'' \\
&= \underbrace{2\langle F_i^2 \rangle \tau_c}_{D} \; t
\end{aligned}
$$

$$
\begin{aligned}
(\Delta a)^2 &= 4\frac{Dt}{n^2} \\
(\Delta e)^2 &= 2.5\frac{\gamma Dt}{n^2 a^2} \\
(\Delta \varpi)^2 &= \frac{2.5}{e^2}\frac{\gamma Dt}{n^2 a^2}
\end{aligned}
$$

We can now do the usual trick to obtain the growth rate of all orbital parameters.

All parameters are undergoing a random walk.

Note the different factors, especially the 1/e term.

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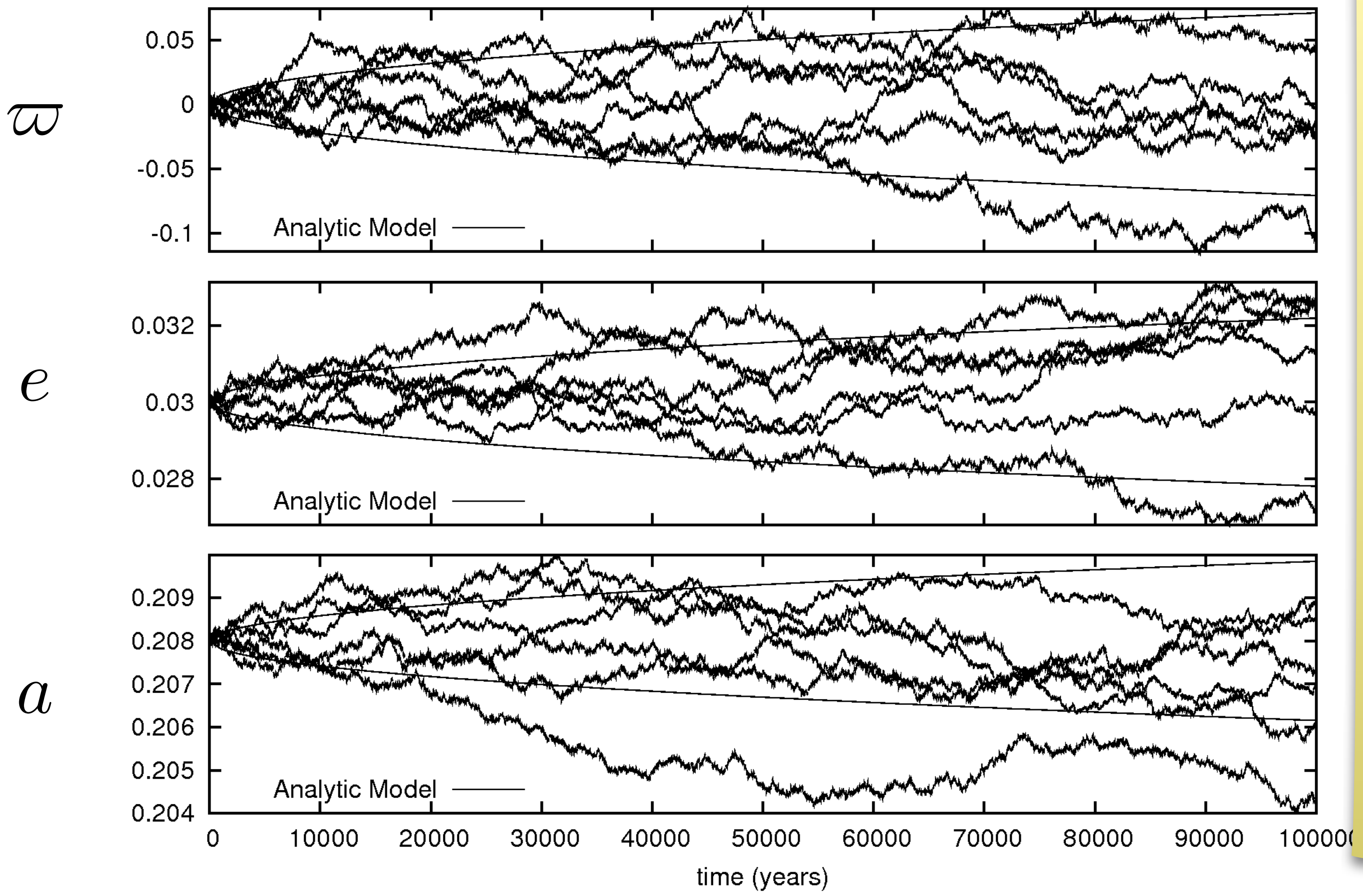
Growth of orbital parameters - single planet



6 realizations of the same initial conditions.

The mean growth is well characterized by the sqrt(t) laws from the previous slide.

Growth of orbital parameters - two planet case

- Same form as in single planet case
- Amplitude of harmonic oscillator

$$\frac{(\Delta\phi_1)^2}{(p+1)^2} = \frac{9\gamma_f}{a_1^2\omega_{lf}^2}\,D\,t$$

$$(\Delta(\Delta\varpi))^2 = \frac{5\gamma_s}{4a_1^2n_1^2e_1^2}\,D\,t$$

- Dependence on e

Don't get confused by the two \Deltas.

One is the name of the parameter, the difference in apsidal lines.

The other one describes the growth.

The 1/e dependence shows a coordinate singularity, not a physical instability.

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Details in Rein & Papaloizou (2008)

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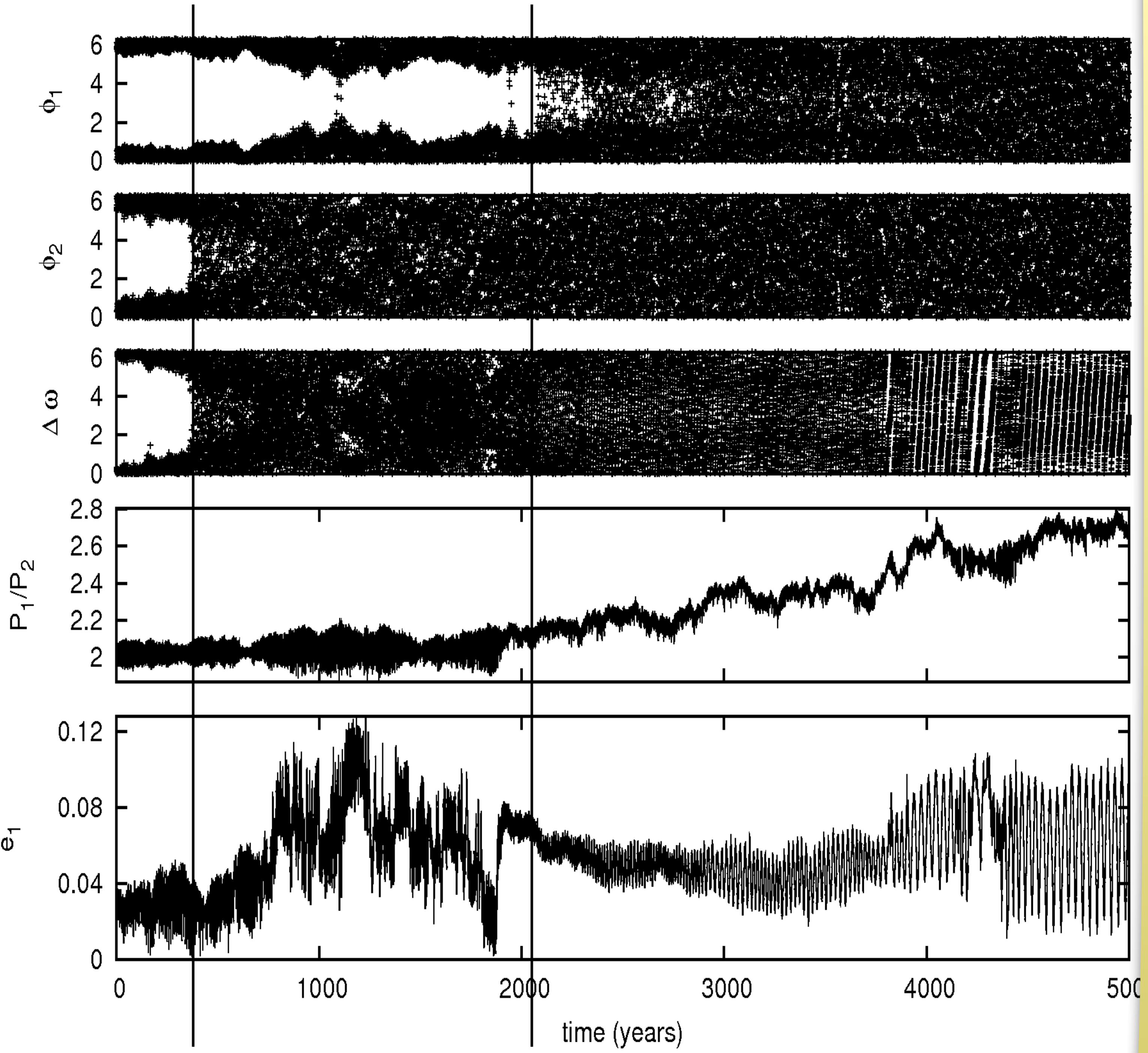
Breaking a mean motion resonance



The first thing to notice is that the difference in the apsidal lines will go out of libration first.

This is not due to the random walk in this parameter but note that the eccentricity obtains small values at exactly the same time.

The resonant angle phi1 is still in resonance. Nothing dramatic happens.

The amplitude of phi1 keeps on growing until it finally goes out of resonance.

At that point, the planets are basically undergoing two independent random walks.

Initial Conditions from Rivera et al. (2005)

Lifetime

We can now make use of the analytic description to get an estimate for the average lifetime of such a resonance.

All we have to do is solving this equation for t. We can also express this in terms of physical parameters of the system.

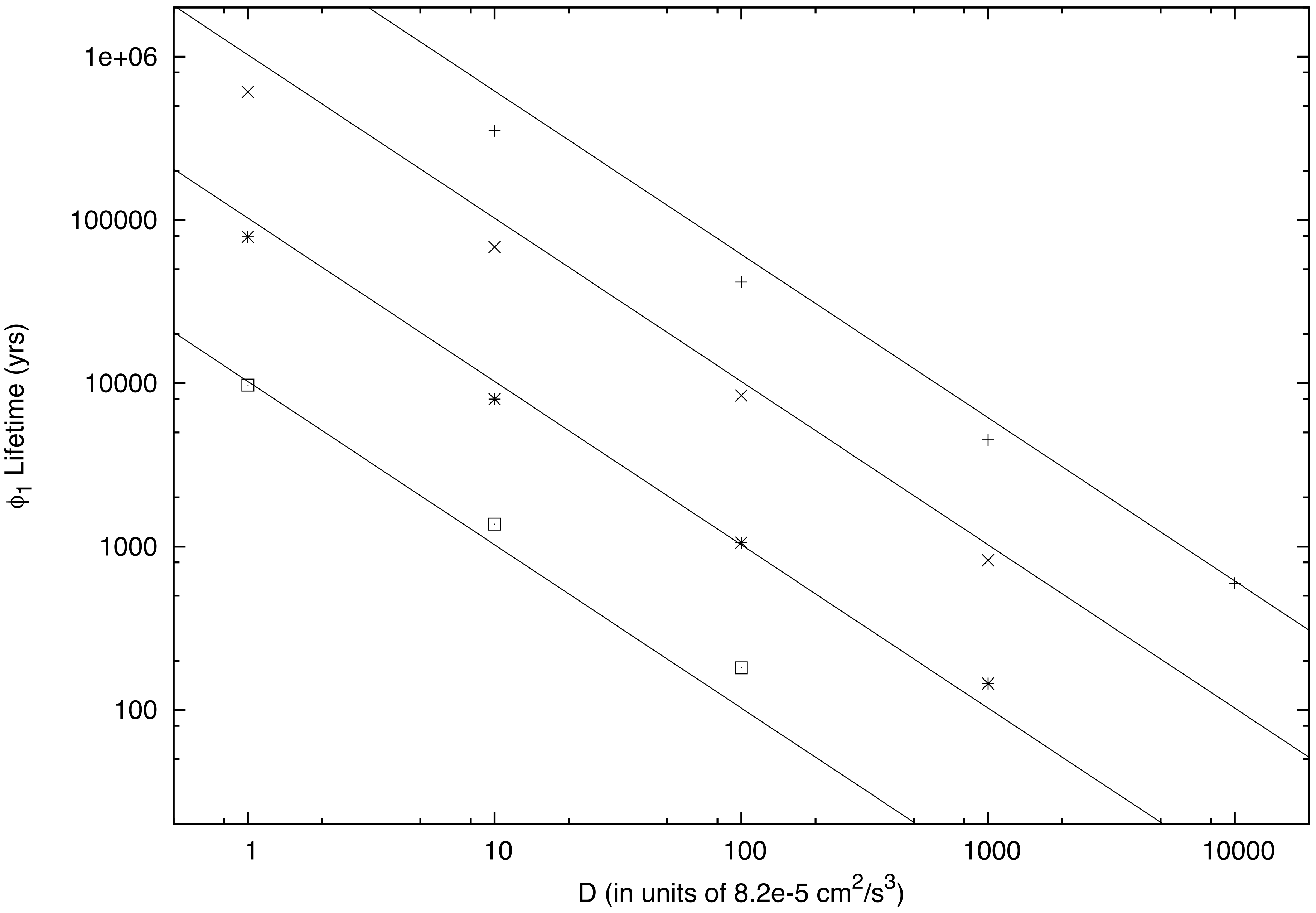
$$\frac{(\Delta\phi_1)^2}{(p+1)^2} = \frac{9\gamma_f}{a_1^2\omega_{lf}^2}\, D\, t \longrightarrow \tau \approx \frac{a_1^2\omega_{lf}^2}{9D}$$

$$\tau \approx 2.4\cdot 10^{-4}\left(\frac{a_1\, n_1^2}{\sqrt{\langle F_i^2\rangle}}\right)^2\left(\frac{1}{2n_1\tau_c}\right)\left(\frac{17\,\omega_{lf}\sqrt{q_{GJ}}}{2n_1\sqrt{q}}\right)^2\frac{q}{q_{GJ}}P_1$$

$$\frac{\text{central force}}{\text{turbulent force}}$$

$$\approx 1$$

Lifetime as a function of D



This plot shows a comparison between the lifetimes estimated from the analytic prescription and the numerical tests.

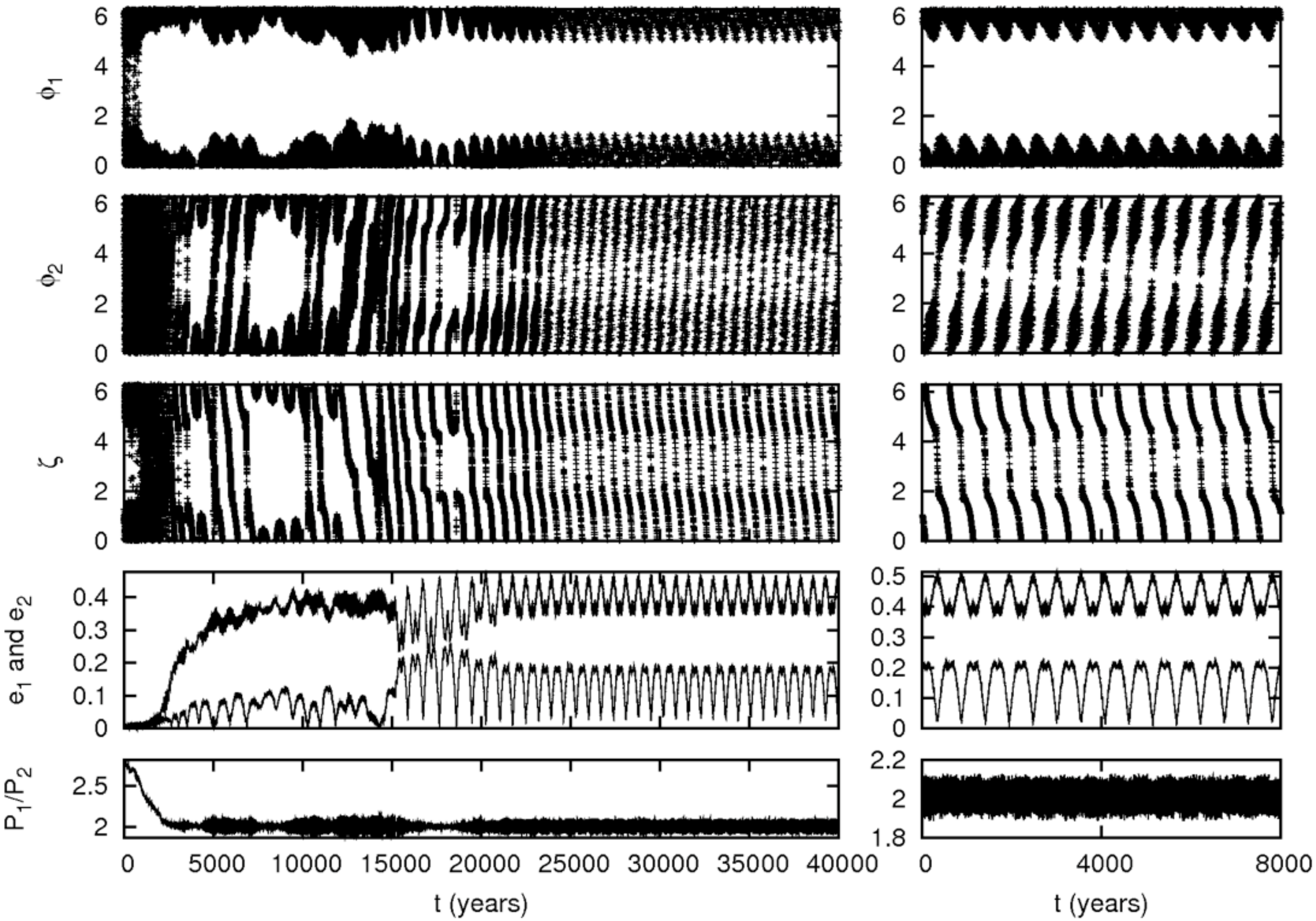
The lines correspond to the analytic lifetimes of different planet masses, ranging from Jupiter masses to terrestrial masses.

Two points are slightly off.

The one on the top is more a lower limit because I stop the simulations after a finite time.

The two points on the bottom appear to have a slightly longer lifetime then expected. This is due to the fact that the forces are so strong that the resonance is broken within one libration period.

Formation of HD128311



The most interesting thing to look at is the formation of the systems.

The right plot shows the observed system, the left one it's formation.

The HD128311 system has been studied before. It turns out that the system cannot reach it's current state with smooth migration only.

Turbulence might be the best explanation.

Observed orbital elements from Vogt et al. (2005)
Formation without stochastic forces by Sandor & Kley (2006)

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Conclusions

- Analytic description of stochastic forces from first principles

- Physical scaling laws allow us to cover large uncertainties

- The result is an analytic formula for the lifetime of resonances

$$\tau \approx \frac{a_1^2 \omega_{lf}^2}{9D}$$

- Turbulence naturally produces system with broken apsidal corotation and provides plausible formation scenarios for many system

- Future observations will allow us to constrain D and lead to a better understanding of turbulence

Thank you for your attention.

All details are described in
Rein & Papaloizou 2008
arXiv:0811.1813