Wilhelm und Else Heraeus Physics School: The Early Phase of Planet Formation
18-22Feb, 2008 Physikzentrum, Bad Honnef, Germany

# The Effect of Poloidal Magnetic Field on Type I Planetary Migration

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*We study the effect of poloidal magnetic field on type I planetary migration by linear perturbation analysis in the shearing-sheet approximation and the analytic results are compared with numerical calculations. In contrast to the unmagnetized case, the basic equations that describe the wake due to the planet in the disk allow magnetic resonances at which density perturbation diverges. In order to simplify the problem, we consider the case without magneto-rotational instability. We perform two sets of analyses: two-dimensional and three-dimensional. In two-dimensional analysis, we find the generalization of the torque formula previously known in unmagnetized case. In three-dimensional calculations, we focus on the disk with very strong magnetic field and derive a new analytic formula for the torque exerted on the planet. We find that when Alfven velocity is much larger than sound speed, two-dimensional torque is suppressed and three-dimensional modes dominate, in contrast to the unmagnetized case.*

Reference: Muto et al. arXiv:0712.1060, ApJ Accepted
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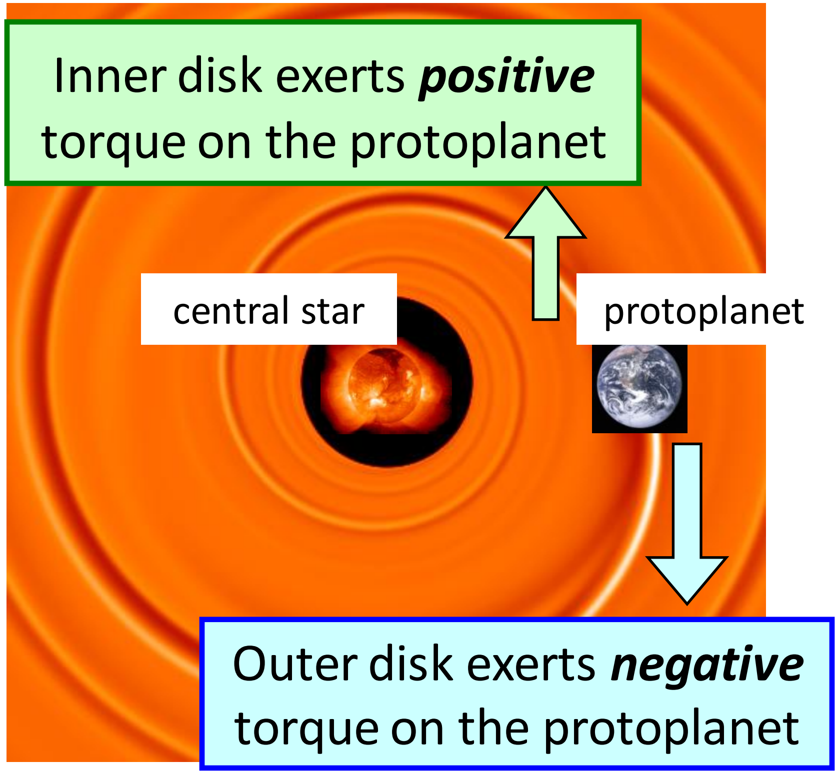
# (Type I) Planetary Migration

Density perturbation induced by a protoplanet




- Protoplanets generate ***density waves*** on protoplanetary disk
- Waves transport angular momenta
- Outer wave wins
- The planets ***fall towards the central star*** due to the ***back reaction*** of the waves

The background figure is adopted from F. Masset's web page: http://www.maths.qmul.ac.uk/~masset/moviesmpegs.html

Migration timescale: $\tau = (2.7 + 1.1\alpha)^{-1} \frac{M_c}{M_p} \frac{M_c}{\sigma_p r_p^2} \left( \frac{c}{r_p \Omega_p} \right)^2 \Omega_p^{-1}$

For Minimum Mass Solar Nebula: $\tau \sim 8 \times 10^5 \mathrm{yr} < \tau_{nebula} \sim 10^7 \mathrm{yr}$

Tanaka et al. (2002)

**Protoplanets Fall into the Central Star *BEFORE* Gas Dispersal**

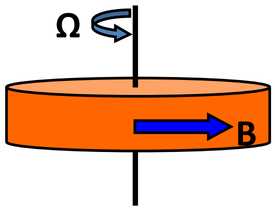
# Previous Work: Stopping Inward Planetary Migration by …

Terquem M.N.R.A.S. **341** 1157 (2003)

**Toroidal magnetic field** can strengthen the wave inside the planet orbit.

1. Density diverges at ***Magnetic Resonance***

2. Planets migrate ***outward*** when magnetic field decreases faster than $B \propto r^{-1}$



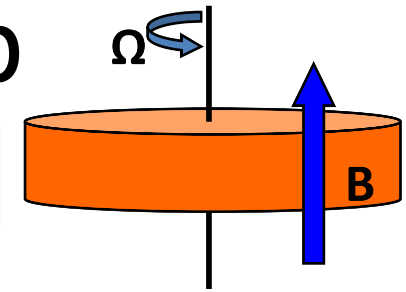

Magnetic Field may be Important when Considering Planetary Migration

Remaining questions in Terquem's analysis are…

- More detailed conditions of stopping migration?
- Is it possible to estimate the torque analytically?
- How about other configurations?

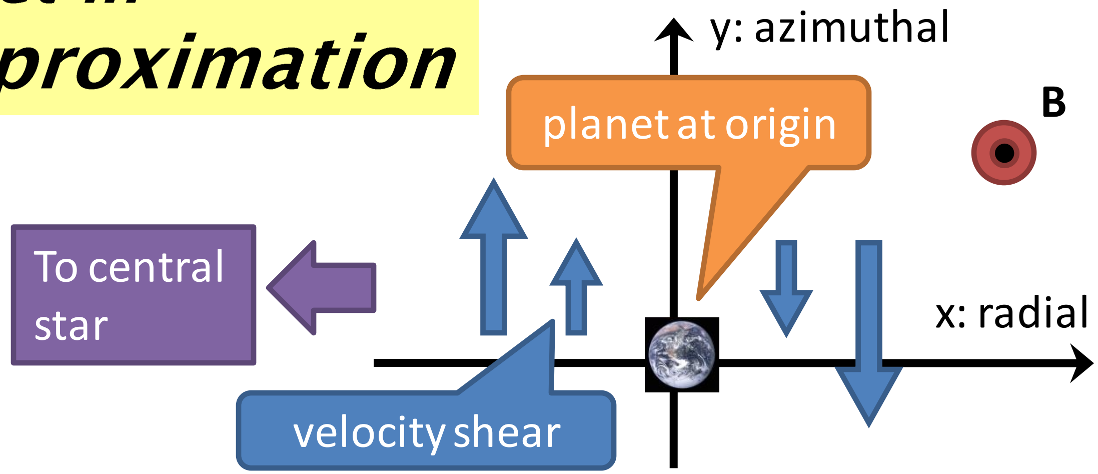
More *analytic treatment* is necessary

# 3 Ongoing Research: Setup




We consider when poloidal magnetic field is exerted on the disk

## *Ideal MHD+One Planet in shearing sheet approximation*




**Background disk:**

$$\mathbf{B}_0 = B_0 \mathbf{e}_z \quad \mathbf{v}_0 = -\frac{3}{2}\Omega_p x \mathbf{e}_y$$

EoC: $$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$

EoM: $$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} = -\frac{c^2}{\rho}\nabla\rho - \nabla\psi_{eff} - 2\Omega_p \mathbf{e}_z \times \mathbf{v} - \frac{1}{4\pi\rho}\mathbf{B} \times (\nabla \times \mathbf{B})$$

$$\psi_{eff} = -\frac{3}{2}\Omega_p^2 x^2 + \psi_p$$

Coriolis force

Lorentz force

Tidal force

Planet's gravitation, treated as perturbation

For simplicity ... No disk self-gravity, Isothermal EoS, No background structure in z direction

# Formulation

Linear perturbation analysis: $\rho = \rho_0 + \delta\rho$

Fourier transform for y- and z-directions: $\delta\rho = \sum_{k_y, k_z} \delta\rho(x) e^{-i(\omega t - k_y y - k_z z)}$

Stationary perturbation (with small viscosity when necessary)

$$\omega = 0 + i\gamma$$

➔obtain an ODE describing wave excitation/propagation in radial direction

➔boundary condition: wave propagates away from the planet

$$\frac{d^2 f}{dr^2} + \mathcal{A}_1 \frac{df}{dr} + \mathcal{A}_0 f = \mathcal{S}$$

LHS describes **wave propagation**

RHS describes **wave excitation** by the planet

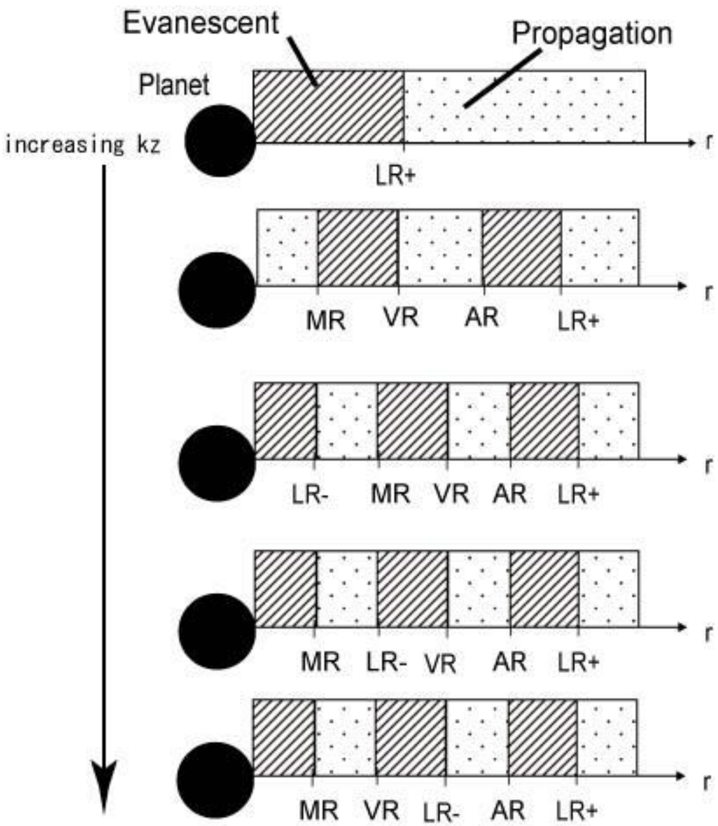
$$f(x) \equiv \frac{1}{\sigma^2}\left[\left\{(c^2 + v_A^2)\sigma^2 - c^2 v_A^2 k_z^2\right\} \frac{\delta\rho}{\rho_0} + (\sigma^2 - v_A^2 k_z^2)\psi_p\right]$$

$$\mathcal{A}_1 = \frac{d}{dx} \ln \frac{\sigma^2 - v_A^2 k_z^2}{D}, \quad \mathcal{A}_2 = \frac{(\sigma^2 - c^2 k_z^2) D}{\left\{(c^2 + v_A^2)\sigma^2 - c^2 v_A^2 k_z^2\right\}(\sigma^2 - v_A^2 k_z^2)} + \frac{2\Omega_p \sigma k_y}{\sigma^2 - v_A^2 k_z^2} \frac{d}{dx}(\ln D) - k_y^2,$$

$$\mathcal{S} = \frac{\sigma^2 D}{\left\{(c^2 + v_A^2)\sigma^2 - c^2 v_A^2 k_z^2\right\}(\sigma^2 - v_A^2 k_z^2)} \psi_p.$$

$$D = (\sigma^2 - v_A^2 k_z^2)(\sigma^2 - v_A^2 k_z^2 + 3\Omega_p^2) - 4\sigma^2 \Omega_p^2. \qquad \sigma(x) \equiv \omega + \frac{3}{2}\Omega_p k_y x = \frac{3}{2}\Omega_p k_y x.$$

# 5 Wave Propagation Property of the Disk




**Resonances in WKB regime:**

a. Infinite radial wavelength

Lindblad resonance: $\sigma^2 = v_A^2 k_z^2 + \frac{\kappa^2 \pm \sqrt{\kappa^4 + 16\Omega^2 v_A^2 k_z^2}}{2}$

Vertical resonance: $\sigma^2 = c^2 k_z^2$

b. Zero radial wavelength (and viscosity is effective)

Alfven resonance: $\sigma^2 = v_A^2 k_z^2$

Magnetic resonance: $\sigma^2 = c^2 v_A^2 k_z^2 / (c^2 + v_A^2)$

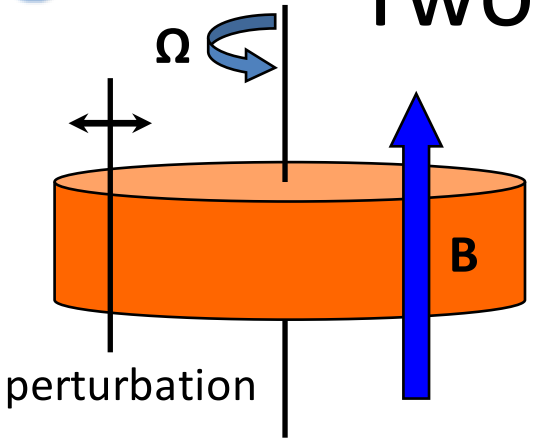
**Wave propagation property is totally different from unmagnetized case**

However, the problem simplifies in some cases:

1. two-dimensional disk ( $k_z = 0$ )
2. strong magnetic field case ($\beta \to 0$)

Since MRI does not occur in these cases, we can compare the analytic results directly with numerical calculation

# Two-dimensional mode $k_z = 0$




No magnetic tension in 2-d mode

The ***only*** effect of magnetic field is to **change effective sound speed** by magnetic pressure

$$c^2 \rightarrow c^2 + v_A^2$$

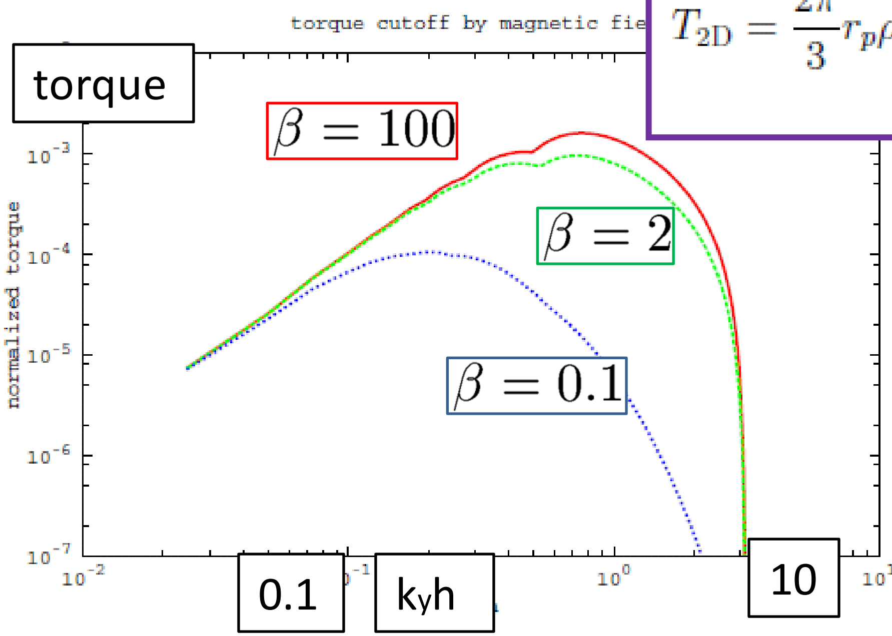
Using the same methods with Artymowicz (1993), it is easy to obtain the torque formula, ***without employing WKB approximation***

$$T_{\rm 2D} = \frac{2\pi}{3} r_p \rho_0 L_y L_z \frac{\Omega_p}{\Omega_p^2 + 4c^2 k_y^2 (1+\beta^{-1})} \frac{1}{\sqrt{\Omega_p^2 + c^2 k_y^2 (1+\beta^{-1})}} \Psi_{\rm eff}^2,$$

$$\Psi_{\rm eff} = \frac{d\psi_p}{dx}(x_{\rm eff}) - 2k_y \frac{\sqrt{\Omega_p^2 + c^2 k_y^2 (1+\beta^{-1})}}{\Omega_p} \psi_p(x_{\rm eff}).$$

$$\frac{x_{\rm eff}}{h} = \frac{2}{3k_y h} \sqrt{1 + k_y^2 h^2 (1+\beta^{-1})}$$

$$h = c/\Omega_p$$




Magnetic field ***suppresses*** the 2D torque

# 3D mode with strong field

Consider $\beta = 0$

Wave equation: $\frac{d^2 f}{dx^2} - k_z^2 f = S(x), \quad S(x) = -\frac{\sigma^2}{\sigma^2 - c^2 k_z^2}\psi_p.$

Waves are ***evanescent*** around the corotation, since MR and VR coincide and all the other resonances go infinite away

Yet ***the source term is divergent.***
This comes from the divergence at MR

Boundary conditions: perturbation vanishes at infinity

We have derive a ***new analytic torque formula***

$$T_{\rm MR} = \frac{2\pi}{3} L_y L_z \frac{\rho_0 r_p k_z}{\Omega_p c} \psi^2_{p,\rm MR}.$$

Prediction: In the strong field limit, the torque does not depend on the magnetic field strength

# Numerical Calculation

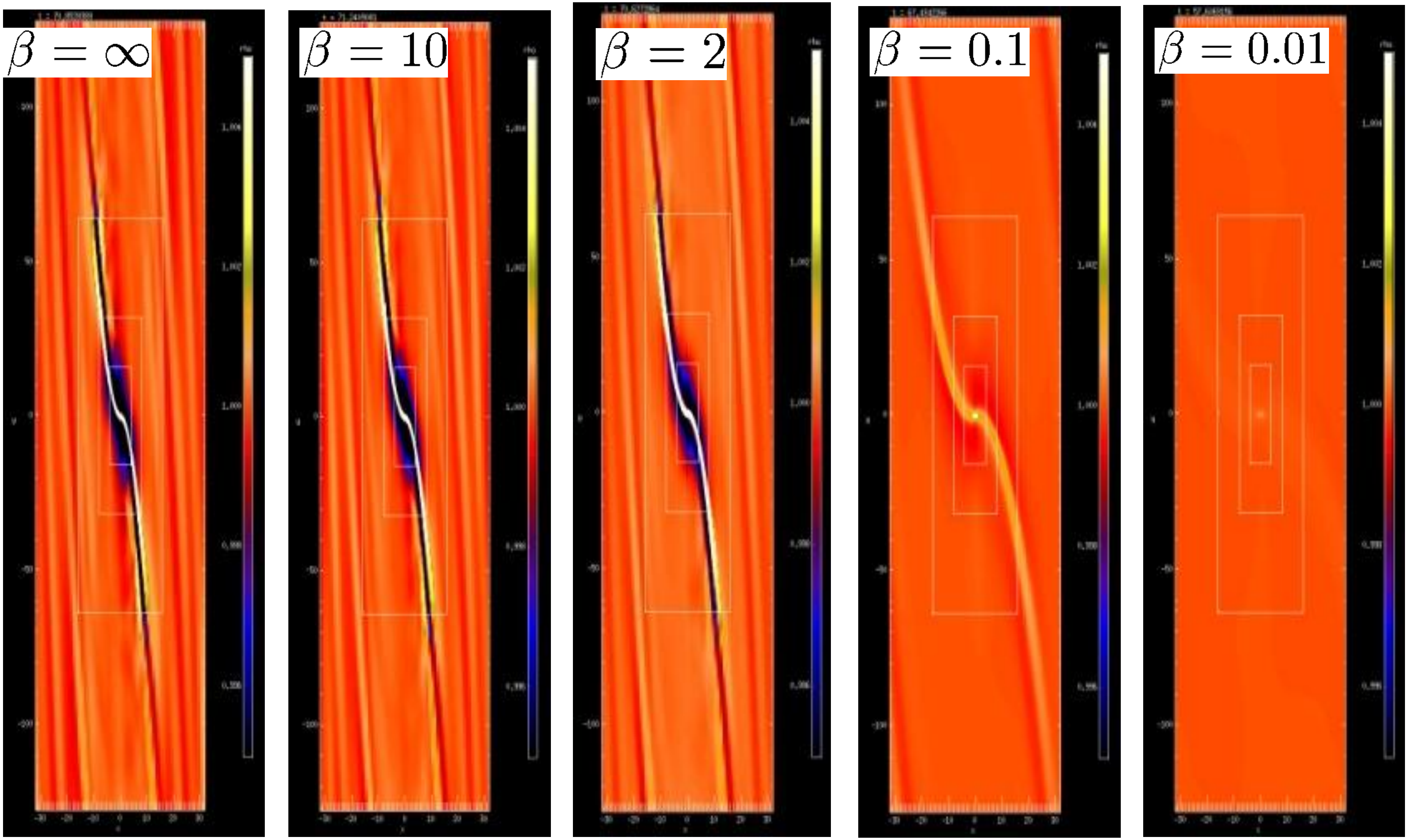
- Code: nested grid
  - see e.g., Matsumoto and Hanawa 2003
- Notes on 2D calculation
  - No z-dependence of planet potential
  - Calculate the mode without z-dependence

$$\psi_p = \frac{GM_p}{(x^2 + y^2)^{1/2}}$$

- Parameters for 2D calculation
  - Hill radius =0.3 times scale height ($3M_{Earth}$ at 1AU, MSSN)
  - 4 grid levels
  - Mesh size of finest grid: 0.125H at finest, 1H at coarsest
  - Box size: $-32H<x<32H$, $-128H<y<128H$
  - 8 parameters for beta
    - infinity, 100, 10, 2, 0.1, 0.3, 0.01, 0.001
- Parameters for 3D calculation
  - Hill radius =0.3 times scale height ($3M_{Earth}$ at 1AU, MSSN)
  - 5 grid levels
  - Mesh size of finest grid: 0.0625H at finest, 1H at coarsest
  - Box size: $-32H<x<32H$, $-128H<y<128H$ and $0<z<16H$
  - 8 parameters for beta, but only model with 0.01 and 0.001 are investigated, since we observe the MRI in other calculations

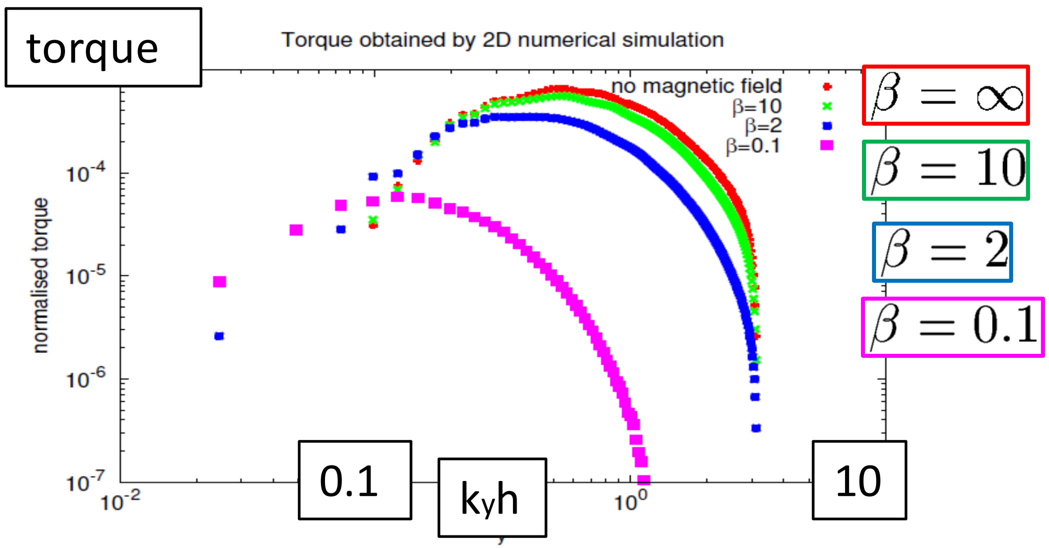
# Results of 2D calculation

**The stronger the field, the smaller the amplitude**

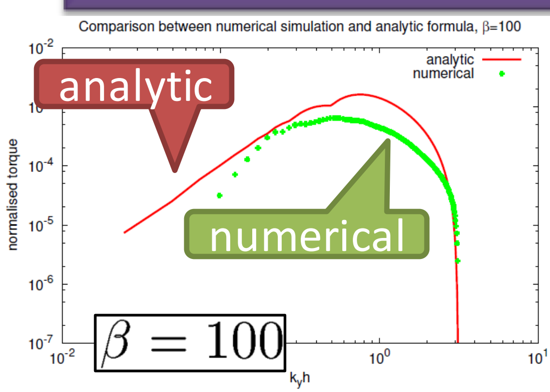


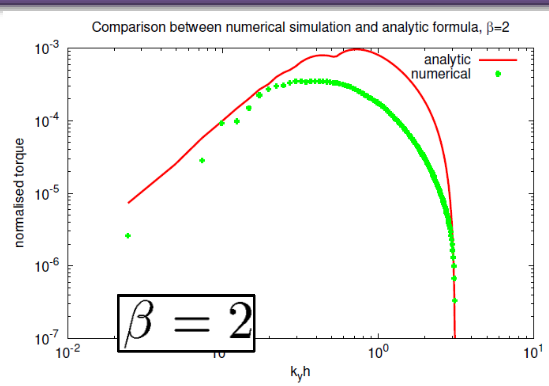
# 2D Torque Cutoff by Magnetic Field

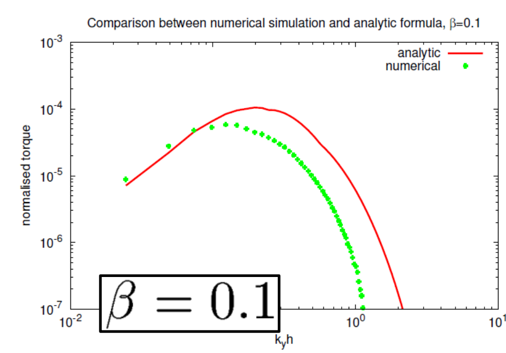
## Torque for each Fourier mode obtained by 2D calculation



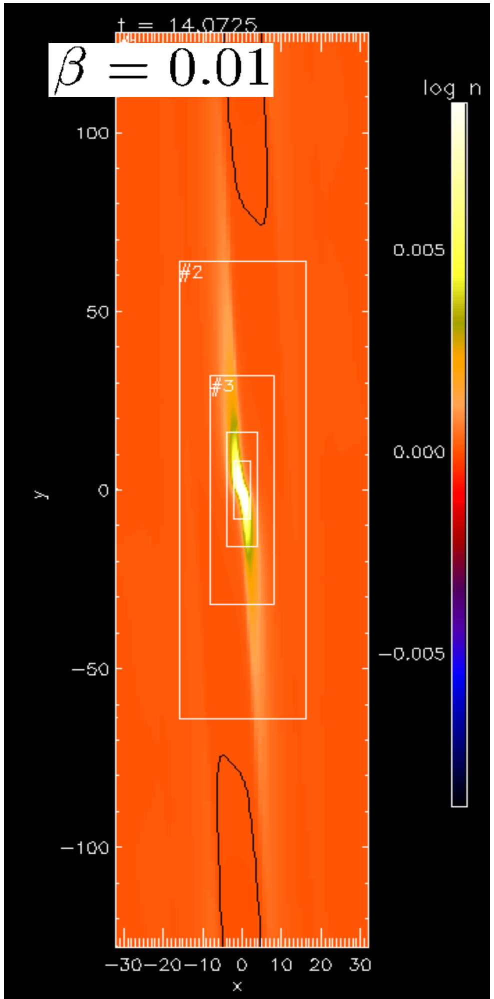

*we have an order-of magnitude agreement between analytic estimate and numerical calculation for all beta*

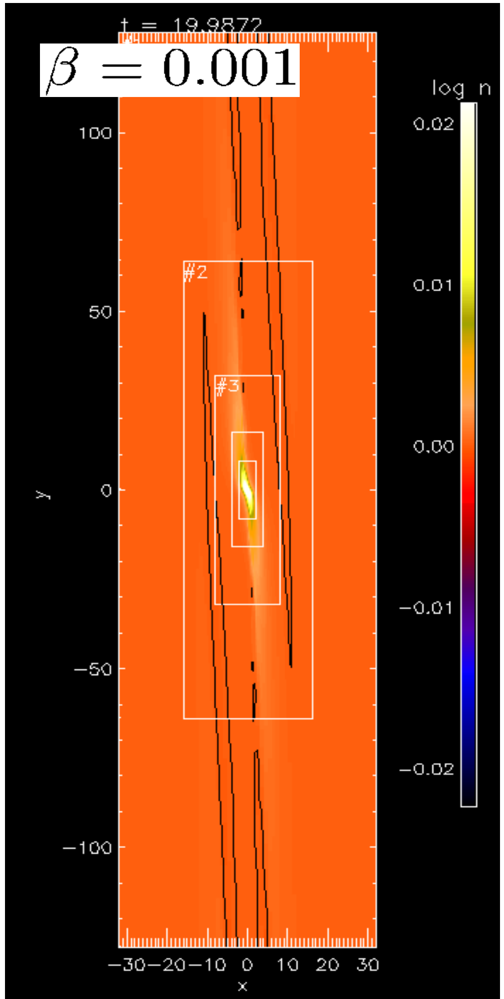


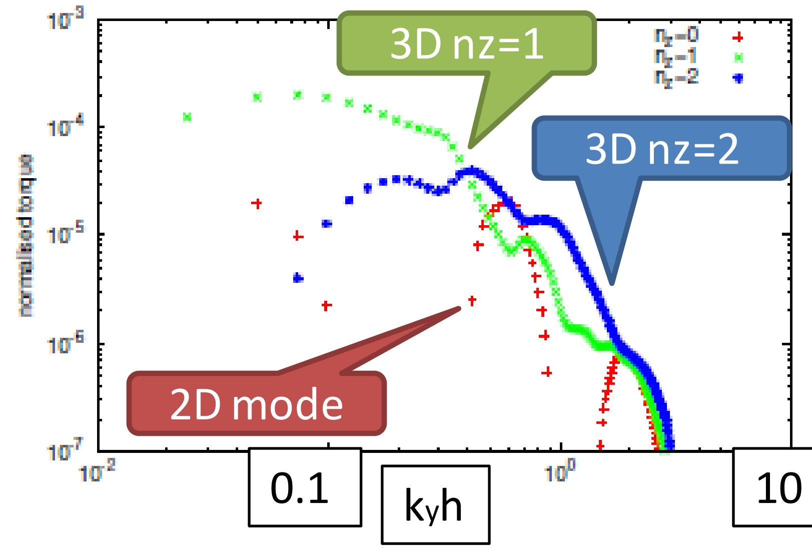



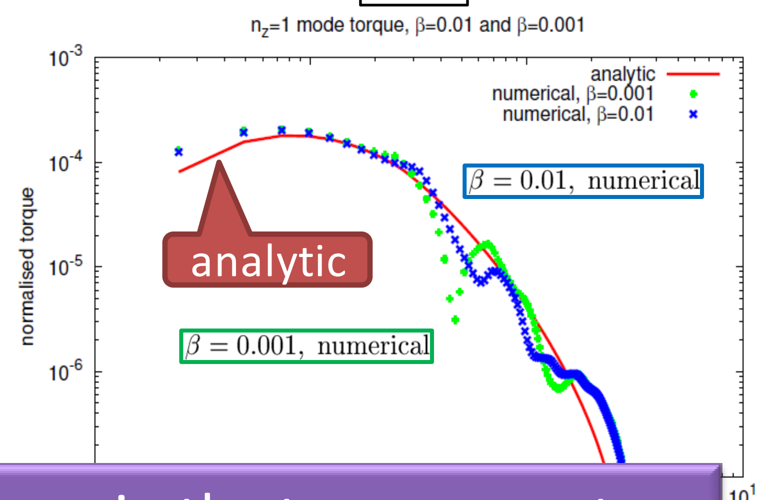
# Results of 3D calculation




Torque for beta=0.01



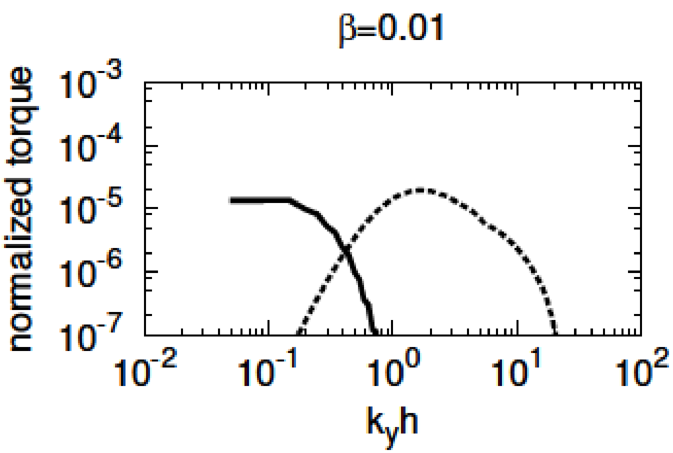



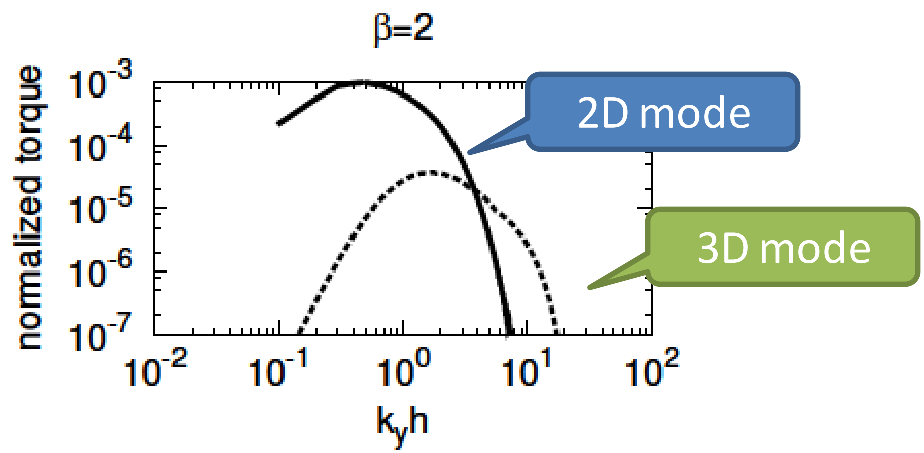

- The wave pattern appears to be the *same* in the two parameters
- ***3D mode torque is stronger*** than the 2D mode
- ***Excellent agreement between numerical and analytic results***

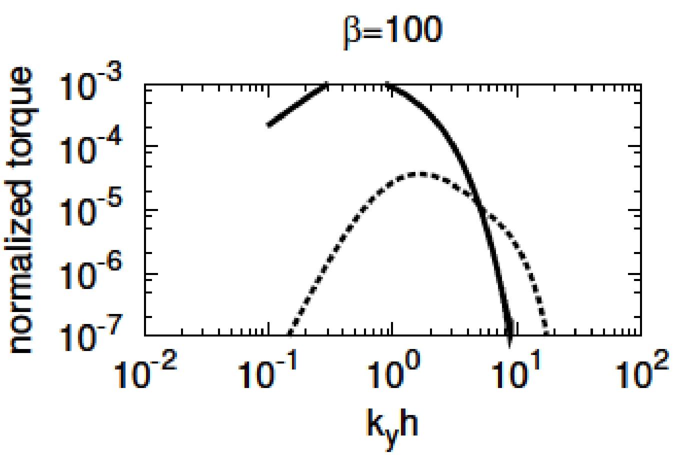
# Migration timescale estimate

Estimate by analytic formula for thin disk (0<z<2h)

Note: numerical calculation assumed 0<z<16H




- Total torque may be ***smaller by two orders of magnitude*** if magnetic field is very strong
- ***Migration may be slow*** if magnetic field is exerted on the disk
  - Note: cannot define the differential torque with shearing sheet approximation, future work

# Brief Summary

- We investigate the ***torque exerted on a planet*** embedded in a ***disk with poloidal magnetic field*** in a local shearing sheet approximation when magneto-rotational instability does not occur.
- We derive the ***new torque formula*** exerted on the planet, which can be used to estimate the migration timescale.
- ***2D torque is suppressed*** by the strong magnetic field, while ***3D torque seems to be enhanced***, and in the strong magnetic field limit, 3D torque converges to a single value.
- ***Migration rate may be reduced*** by the strong magnetic field: possible way to 'save the Earth'