Planetary Resonances and Structure Formation in Debris Disks

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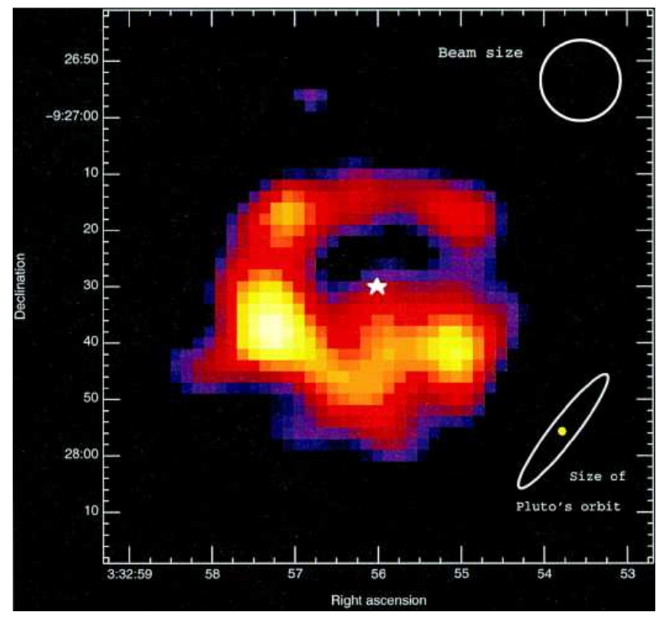
Abstract



Many structures such as inner gaps, rings, arcs and clumps observed in debris disks around other stars can be attributed to resonances with unseen planets. Here we compare two conceivable scenarios for the origin of resonant structures: one in which dust from non-resonant parent bodies is captured into resonance and another one in which dust is originally produced by resonant bodies and stays locked in resonance.
We show that, in all debris disks resolved so far, the first scenario may yield axisymmetric features (rings, gaps), whereas formation of azimuthal structure (clumps, arcs) is precluded by dust-dust collisions. In contrast, the second scenario can naturally explain clumpy structure seen in several debris disks. In fainter disks, expected to become detectable with future instruments, both scenarios are able to efficiently generate a variety of structures.
The examination of scenario I also includes an exploration of the resonance capture.

Resonant Dynamics in Brief

We consider external **mean motion resonances** (MMR), i.e. $(p+q) : p$ commensurabilities of the planet's and the disk particles' mean motion. Resonance capture of the particles occurs with a probability p_{res} when they are nearing

$$a_{res} = a_p(1-\beta)^{1/3}\left(\frac{p+q}{p}\right)^{2/3} ,$$

where a_p is the planet's semimajor axis, β the radiation pressure coefficient. The resonance is characterised by libration of the **resonant argument** Φ,

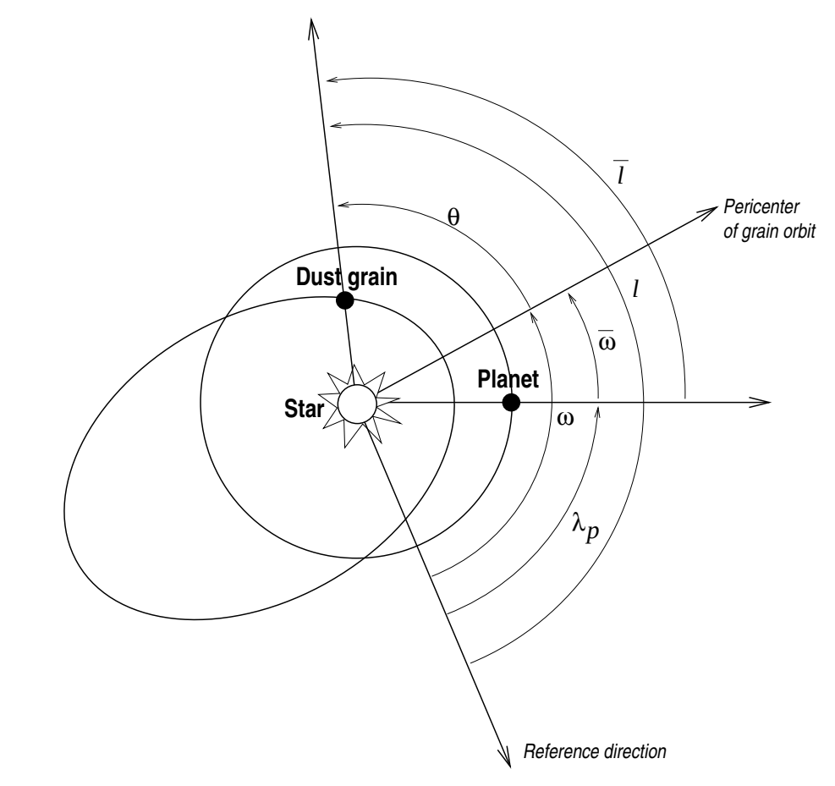


Figure 1: Geometry for resonant particles.

$$\Phi = (p+q)\lambda - p\lambda_P - q\tilde{\omega}$$

around $\Phi \equiv \Phi_0$ with the given amplitude A.
Resonance trapping results in

- preservation of **semimajor axis**, $a = a_{res}$
- pumping up of **eccentricity**,

$$e^2(t) = \frac{q}{3p}\left[1 - exp\left(-\frac{3p}{p+q}Bt\right)\right]$$

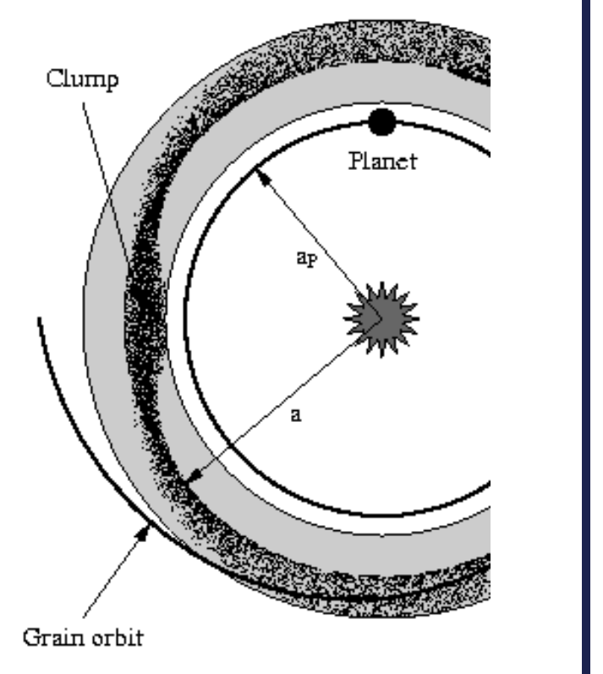
with $B = \frac{2GM_*\beta}{ca^2}$, $\beta = \beta(s) \sim 0$ for planetesimals

- decrease of inclination i

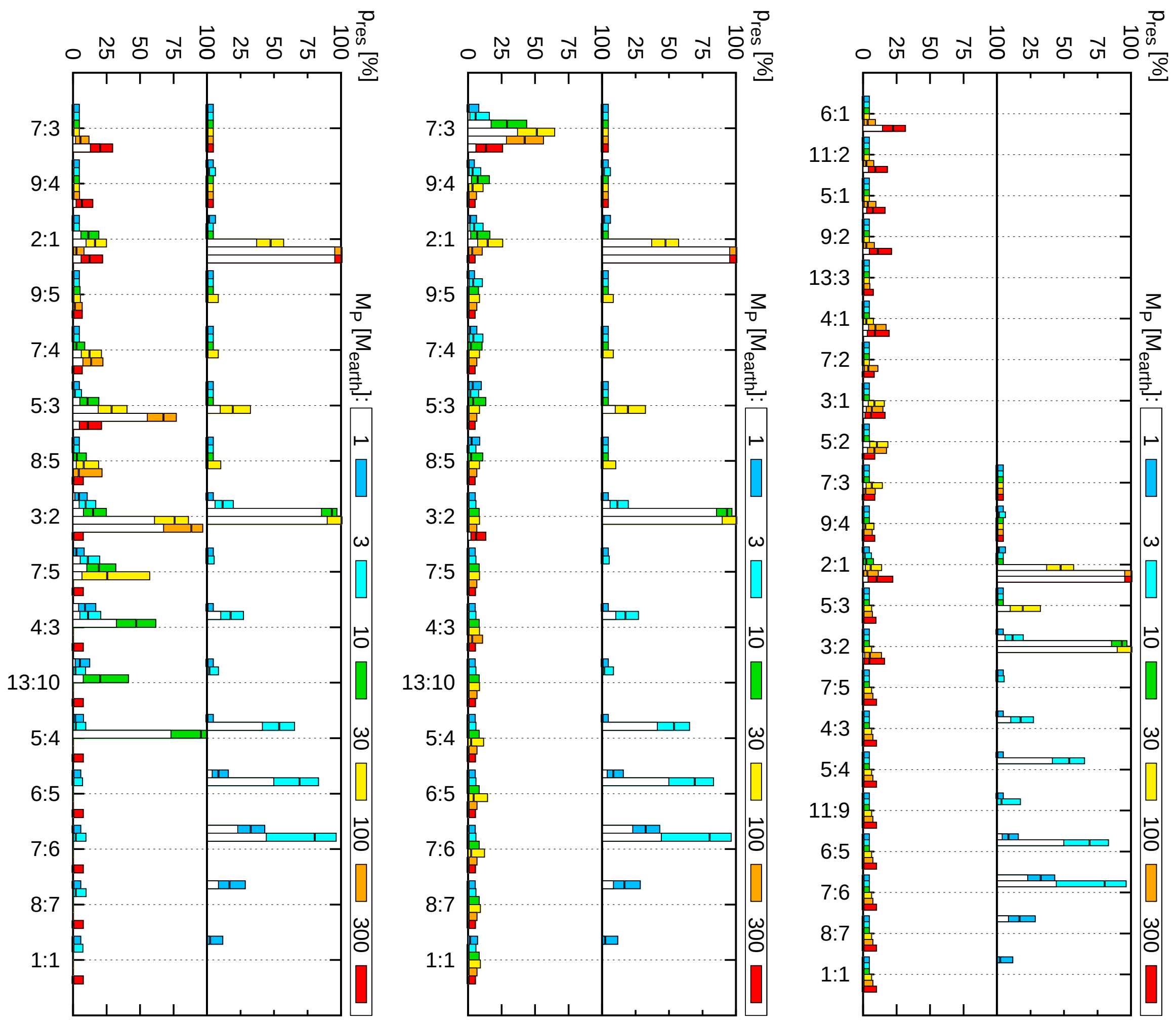
Before e reaches $e_{res} = \sqrt{1/(3p)}$, the **particle** becomes a **planet crosser** and is ejected from the resonance by gravitational scattering.

Scenario I – The Classical

- there is a **reservoir of dust** grains
- due to **Poynting-Robertson** and **stellar wind drag** they are drifting inwards, such that $\dot{n}^+$ grains cross $r = a_{res}$ per unit time ($\dot{n}^+ = B\,\frac{a^2}{s^2}\,\tau_0$)
- **grains** get **captured** in resonance with probability p_{res}
- **captured grains** form oberservable **structures**
- meanwhile eccentricity is pumped up, when $e > e_{res}$ grains are ejected from resonance



Capture into Resonance



$e_p = 0.1$ vs. 0.0 $e_{belt} = 0.3$ vs. 0.0 $\beta = 0.3$ vs. 0.03

Probability of resonant capture as function of external MMRs in order of spatial appearance (top: further out, bottom: further in). Left columns are deviations from standard case ($M_* = M_{sun},\ i = e_p = e_{belt} = 0,\ \beta = 0.03$).
Inclination variation was also tested, but did not have significant influence.

Timescales

There are 4 natural timescales in this scenario:
(1) time to drift through the annulus due to drag forces T_{drag},
(2) time of residence in resonance T_{res}, until $e = e_{res}$
(3) time of collisions with background grains T^0_{coll} and
(4) time of collisions with resonant grains T_{coll}.
$\Rightarrow$ actual **lifetime** T of resonant grains is

$$\boxed{\frac{1}{T} \equiv \frac{1}{T_{res}} + \frac{1}{T^0_{coll}} + \frac{1}{T_{coll}} .}$$

Limited lifetime T also limits the maximal eccentricity the grains can reach before they are destroyed.

$$e_{max} = e(T) \approx \sqrt{\frac{q}{p+q}B\,T} .$$

Dense disks ($\tau_0 \simeq 10^{-4}$): collisions of large grains destroy the grains faster than drag can resplenish them, for small grains drag is efficient enough. Dilute disks: drag efficient for all grain sizes.

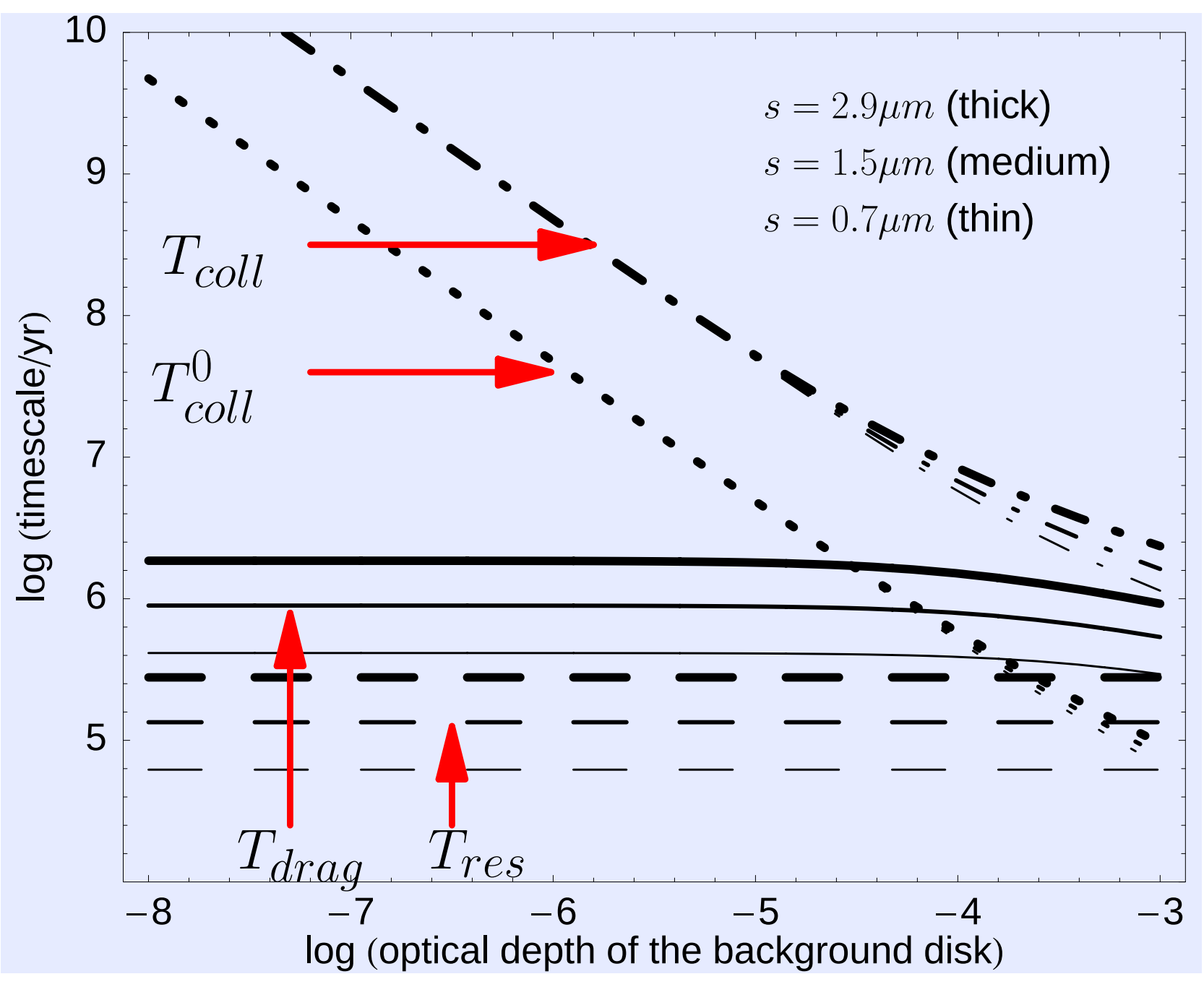


Figure 2: Typical timescales as functions of the normal optical depth τ_0 of the non-resonant disk.

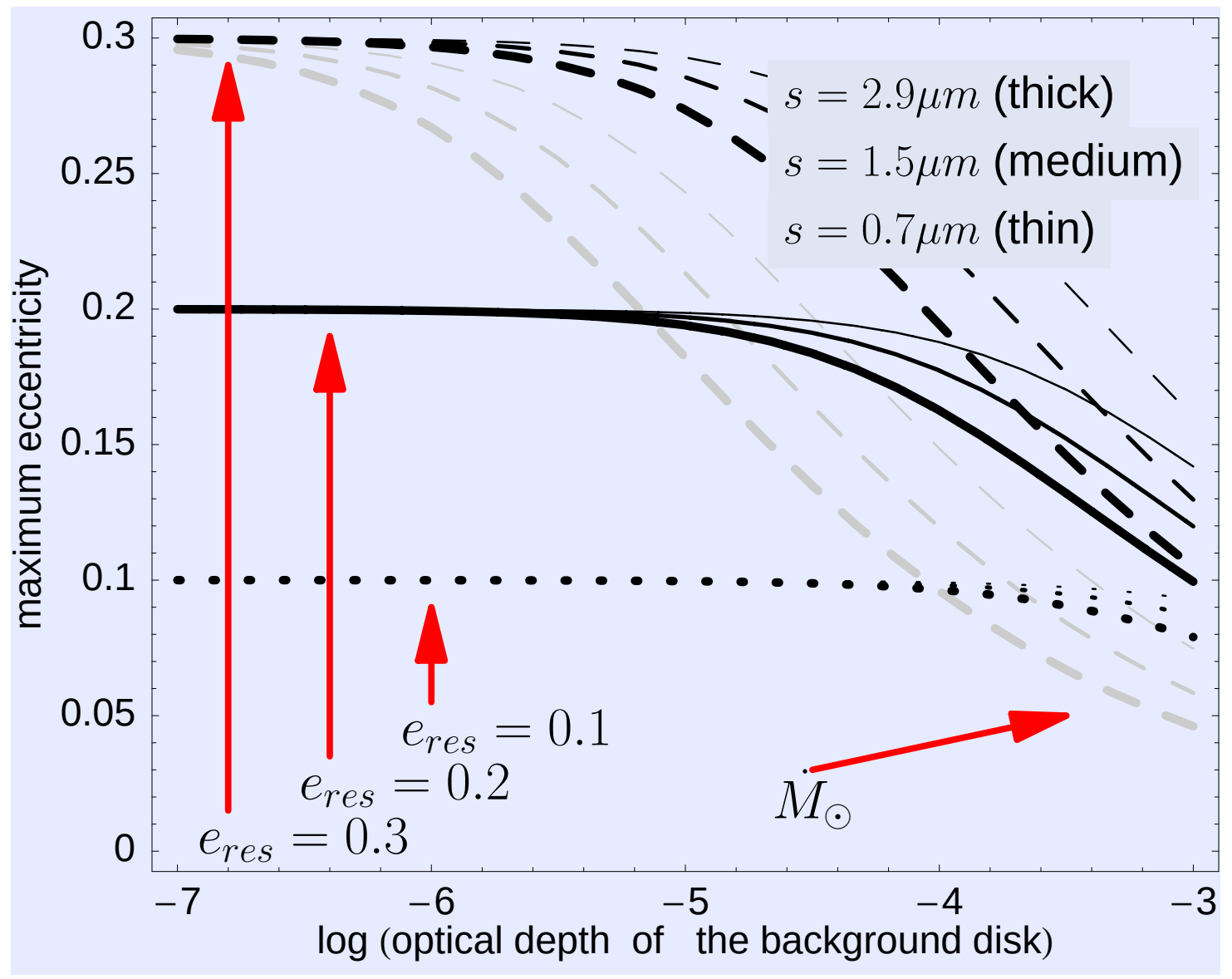


Figure 3: Maximum orbital eccentricity of the clump grains. In dilute disks $e_{max} \sim e_{res}$, but drops in denser disks, i.e. there are no clumps, but bright rings only.

Optical depth

With $n(t)$ the total number of grains with radius s contained in the resonant population at time t, the **balance equation** can be written as

$$\boxed{\frac{dn}{dt} = \dot{n}^+ p_{res} - \frac{n}{T_{res}} - \frac{n}{T^0_{coll}} - \frac{n}{T_{coll}}.}$$

With $\hat{S} \equiv S/S_0$ the fractional area occupied by the clumps, their optical depth is

$$\tau = n\frac{s^2}{4\hat{S}a^2e_{max}}$$

$$= \tau\left(\tau_0\right) .$$

Only for dilute disks ($\tau_0 \leq 10^{-6}$) is $\tau \sim \tau_0$, in denser ones optical depth of clumps stays $\tilde{1}0\%$ below that of the backround.

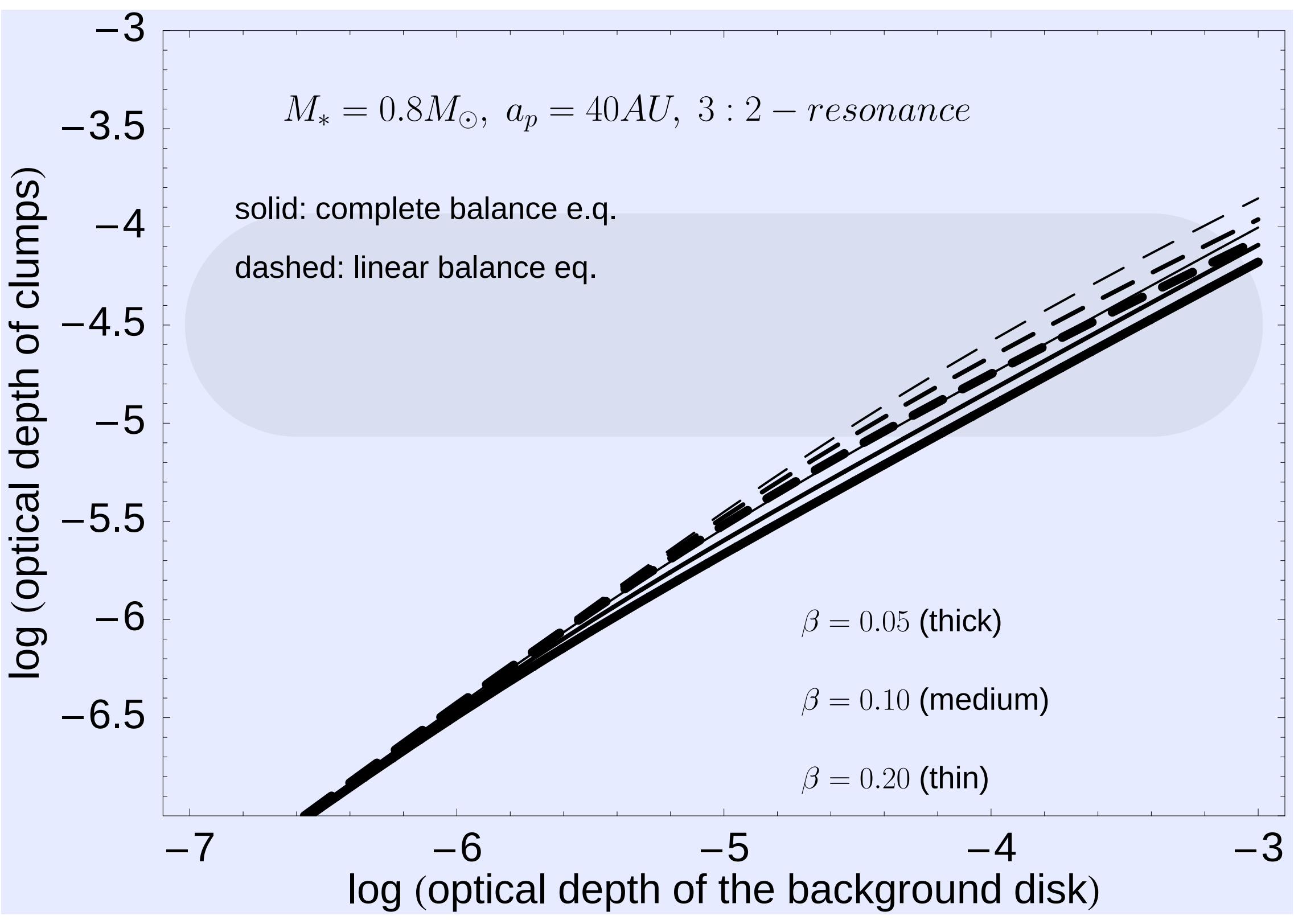
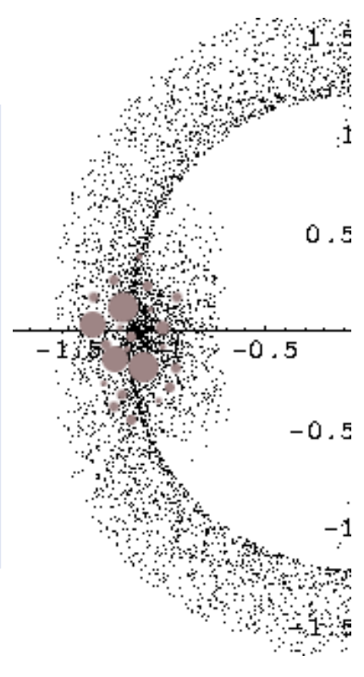
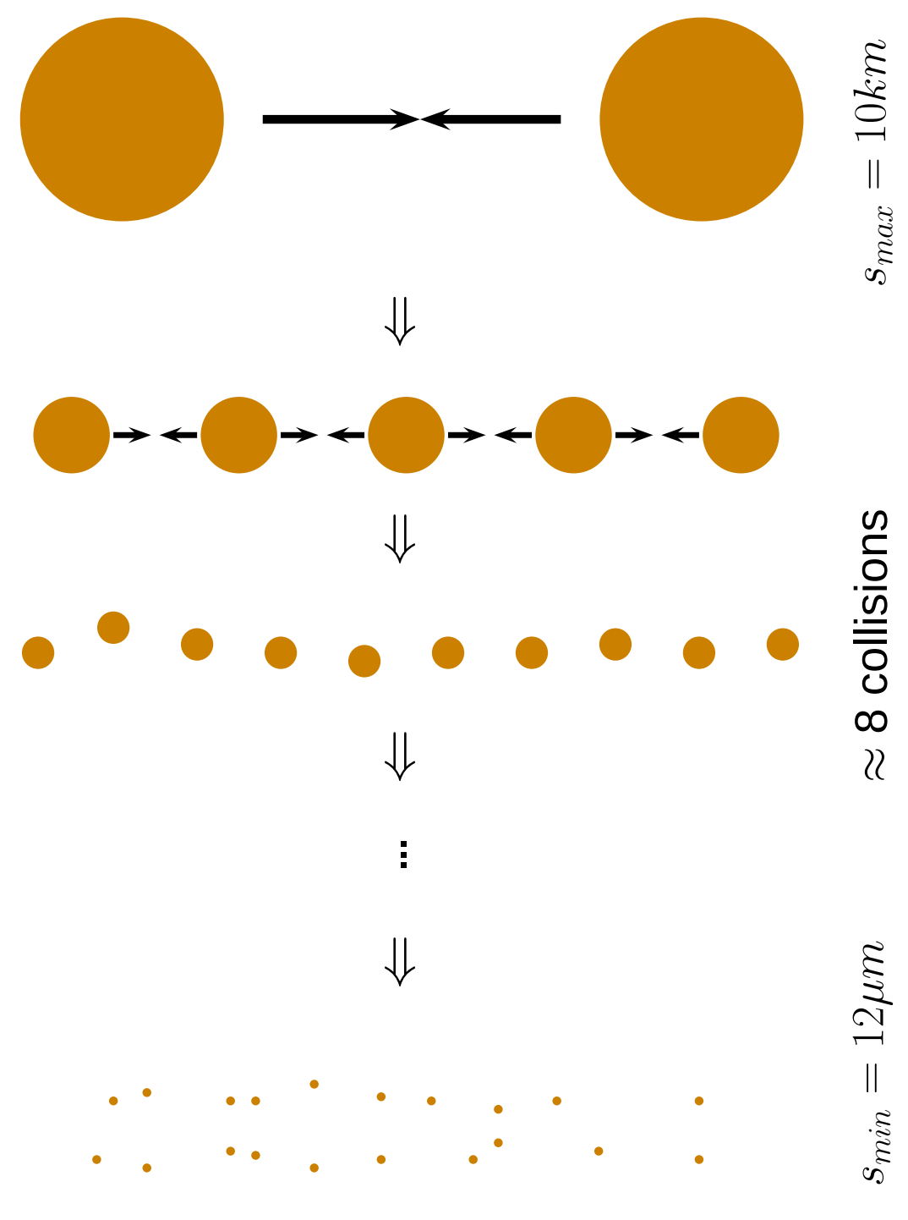


Figure 4: Normal optical depth of the resonant clumps τ versus the optical depth of the "regular" disk τ_0. Darker background shows the minimum optical depth observable today (ϵ Eridani has an optical depth of about 10^{-4}.)

Scenario II – The Toddler

- **planetesimals** locked in resonance
- **mutual collisions** among them create clouds of fragments and dust
- **fragments** have small relative velocities, therefore stay locked in resonance and create the **observed clumps**
- dust smaller than s_{min} is blown from resonance





A **collisional cascade** grinds the particles in the cloud from km-sized planetesimals to μm-sized dust. Part of the collisional outcome will be **ejected** from the resonance, due to too high relative velocity, $u \geq u_{crit}$.

$$\frac{u_{crit}}{v_{kepler}} = 0.007\left(\frac{M_p/M_{Jup}}{M_*/M_\odot}\right)^{0.28} ,$$

$$u_{crit} = 16\ldots30\frac{m}{s}$$

Fraction of material ejected after collision

$$1-\Psi \approx \left(\frac{u_0}{u_{crit}}\right)^\gamma \simeq 3\%\ldots10\% ,$$

for $\gamma = 2$ ($\Rightarrow$ solid bodies), $M_* = 0.8M_\odot$, $M_p = 0.1\ldots1.0M_{Jup}$.

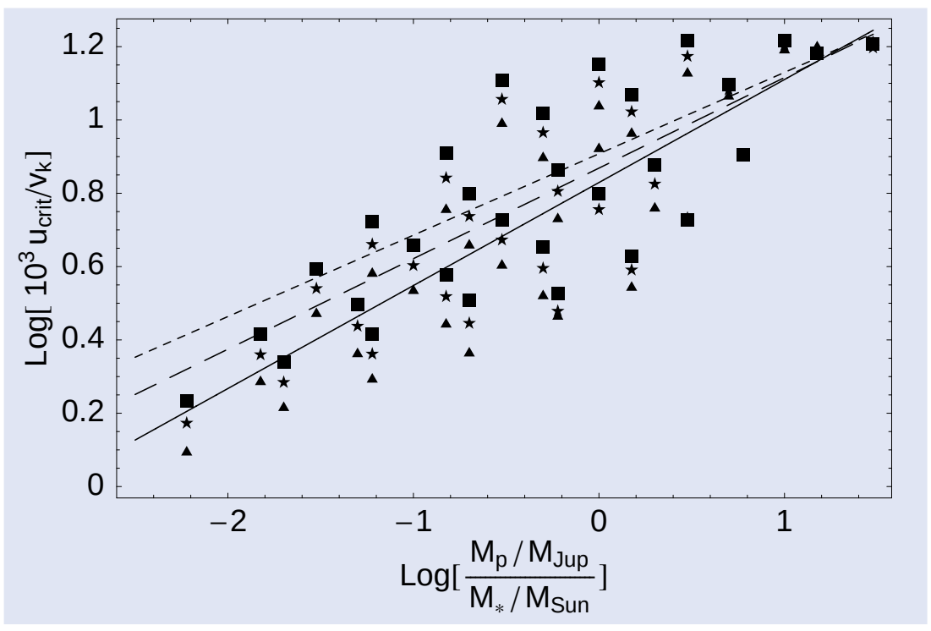


Figure 5: **Critical relative velocity** of fragments to stay in the resonance. Symbols: results of individual runs; lines: power law fits. Different maximum A of particles were adopted as a criterion of staying in the resonance: $A = 30°$ (triangles and solid line), $A = 40°$ (squares and dashed line), $A = 50°$ (stars and dotted line).

$\Rightarrow$ Cascade represents a **closed system** to which the **Dohnanyi theory** applies,

$$n(m) \sim m^{-1-\alpha} \quad , \quad \alpha = 0.83 \text{ (Dohnanyi '69)}$$
$$\alpha = 0.87 \text{ (Durda\&Dermott '97)} .$$

Limit for **blow out** from the resonant swarm,

$$\beta_{crit} = 0.034\left(\frac{M_p/M_{Jupiter}}{M_*/M_\odot}\right)^{1/2} = 0.012$$

$$\Leftrightarrow s_{min} = 12\mu m .$$

Optical depth

With ρ being the material density of the objects, M the cloud's total mass, m_{min} the minimum mass (corresponding to s_{min}) and m_{max} the maximum mass (that of the largest planetesimal) in the resonant swarm, the total cross section of the steady-state population is given by

$$\Sigma = \pi^{1/3}\left(\frac{3}{4\rho}\right)^{2/3}\frac{M}{m_{min}^{1/6}m_{max}^{1/6}}\,\Psi^8 .$$

Dividing this by the clumps area we get the resonant population's optical depth,

$$\tau = \frac{\Sigma}{\hat{S}4\pi a^2 e_{max}}$$

$$= \tau\left(M_{planetesimals}\right) .$$

A planetesimal mass of only $1\ldots10\% M_\oplus$ will already create clumps obervable today.

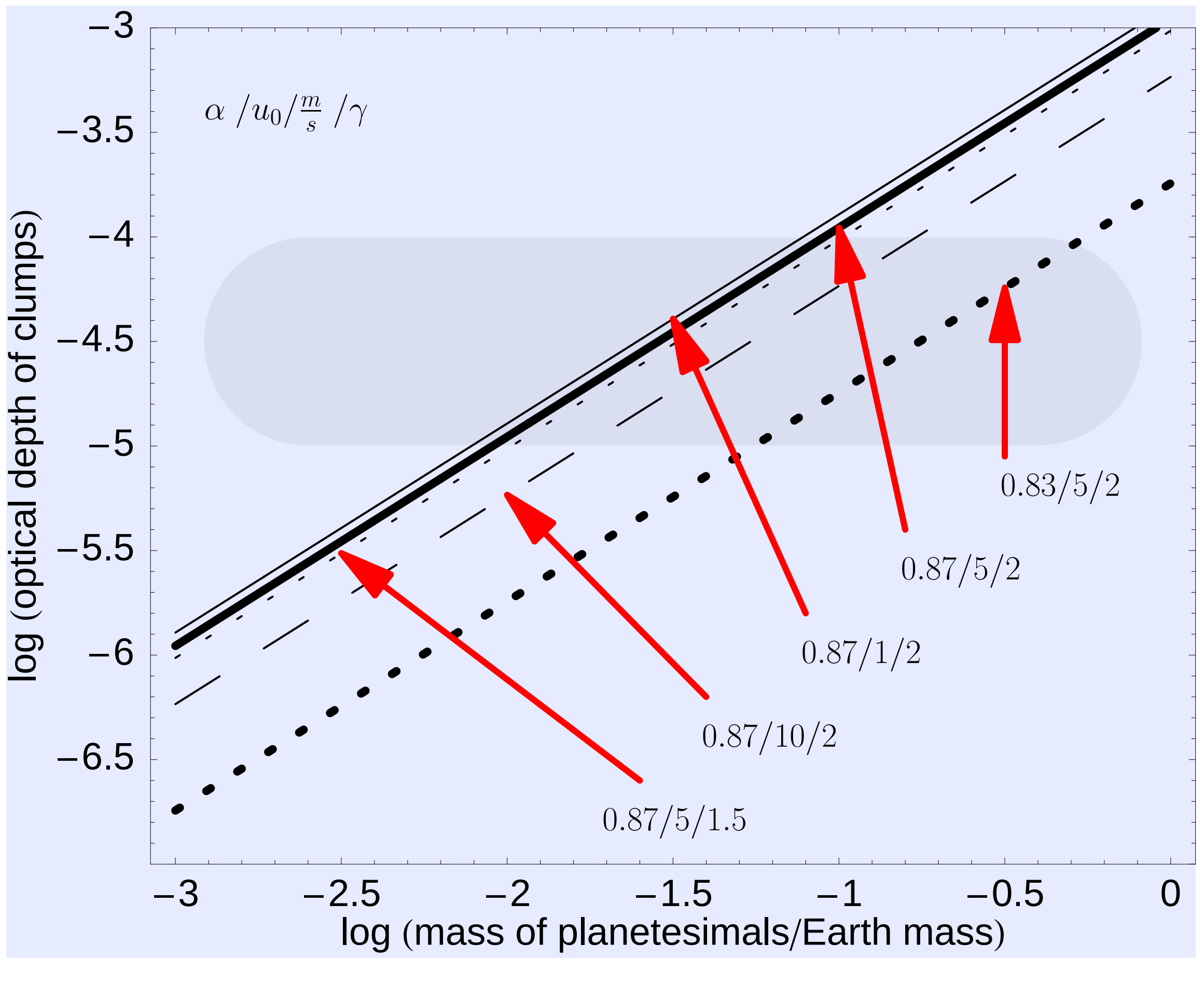


Figure 6: Normal optical depth τ of the resonant clumps versus mass M of parent bodies. Darker background shows the minimum optical depth observable today. (ϵ Eri has an optical depth of about 10^{-4}.) Dependency on impact velocity u_0 and material properties is weak, correct slope α is most important.

Conclusions

- **first scenario**
  - for giant planets **capture into** 2:1 **resonance** highly efficient, for less massive ones more resonances populated (closer to 1:1)
  - high e_p makes only quantitative differences, high β, e_{belt} prevent resonance capture
  - **efficiency** of scenario dependent on τ_0, $\dot{M}_*$, clump brightness $\sim\ p_{res}$

    $\Rightarrow$ in **very dilute disks** clumps brighter than background possible
    in denser disks collisions kill clumps and make rings
- **second scenario** determined by total planetesimal mass in resonance

  $\Rightarrow$ for **debris disks resolved so far** already $M = 0.01\ldots0.1M_\oplus$ creates $\tau \sim 10^{-5}$
- first scenario sensitive to mass loss rate, second scenario very sensitive to details of collisional cascade — determination of both cumbersome

Ref.: KRIVOV, A. V.; QUECK, M.; LÖHNE, T.; SREMČEVIĆ, M.: On the nature of clumps in debris disks *Astronomy & Astrophysics* **462**, *199*; REIDEMEISTER, M.: Statistik des Resonanzeinfangs von Staubteilchen in Trümmerscheiben, *Diplomarbeit*