

EVOLUTION OF PLANETESIMALS

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Introduction

The goal is to develop a method capable of following explicitly the collisional growth of planetesimals over a range extending from planetesimals of a few tens of kilometers to planetary embryos several thousand kilometers in size. For this, we have developed an orbit-averaging Monte Carlo approach that scales with the number of bodies like $N \log N$ and therefore allows to follow the history of several tens of millions of bodies over the disk lifetime [3]. We present here a few tests of the method and illustrate its capability by analyzing the collisional history of a sample calculation.

Method

- Time evolution of orbital elements of individual particles.
- Symmetry around vertical axis of rotation, eliminates Ω (longitude of ascending node).
- Random choice of orbital positions each timestep. Taking into account the nonuniform distribution of position probabilities on orbits with $e > 0$.
- A new interaction partner is determined at each timestep and every time an interaction was applied.
- The interaction partner is needed as a representative to sample local disk properties.
- Pairwise interactions in the case of collisions and close encounters. Or, in the case of relaxation, background sampling by representative bodies.

Leading to an effective pairwise encounter. The direction of deflection of pairs' relative velocities \vec{v}_{rel} is randomly chosen.

- Cloning of bodies possible if the number of particles gets low due to sticking collisions. Allows to keep good resolution and statistics.

Particles & Processes

Timestep Constrained by

$$t_{\text{dynamic}} \ll \Delta t \ll t_{\text{relax}}$$

required by random choice of orbital position and to ensure sampling of many interactions in different regions of the disk before considerable change of orbital elements has occurred.

Planetesimal Described by orbital elements, material density and mass. A new interaction partner is refferd every time an interaction was applied.

Matching Pairs are determined in the (r, z) -plane, therefore space is recursively subdivided using a quadtree datastructure.

Relaxation Chandrasekar theory is applied. Central mass is neglected for short duration of encounter. Position and velocity of center of mass is conserved. The numerous encounters that would occur during Δt are replaced by a single representative encounter. The direction into which \vec{v}_{rel} is tiled is randomly chosen by sphere point picking. The deflection angle is given by

$$\Delta\vartheta = \frac{\pi}{2} \frac{\Delta t}{t_{\text{relax}}}$$

For each pair of bodies a particular t_{relax} is computed.

Collisions In contrast to N-body simulations, positions of bodies are not unique due to the assumed symmetry around the vertical axis of rotation. One has to switch to statistics. Each pair (i, j) holds its collision probability

$$P_{\text{col}} = \Delta t n v_{\text{rel}} A_{\text{col}}$$

with the collision cross section $A_{\text{col}} = \pi b_{\text{max}}^2$ and local number density n . If gravitational focusing is taken into account

$$b_{\text{max}} = \left((R_i + R_j)^2 + \frac{2G(m_i + m_j)(R_i + R_j)}{v_{\text{rel}}^2} \right)^{1/2}$$

Close encounters The impact parameter is

$$b_{\text{max}} = b_0 = \frac{G(m_i + m_j)}{v_{\text{rel}}^2}$$

Encounters with $b < b_0$ led to deflection of \vec{v}_{rel} larger than $\frac{\pi}{2}$. Every matched pair is assigned a close encounter probability the same way as P_{col} above. A_{col} is replaced by $A_{\text{close}} = \pi b_{\text{max}}^2$.

Gasdrag The alteration of orbital elements is computed following equations derived in [2]. Gas density and temperature are, for the moment, specified by means of power laws but could include a fully consistent disk evolution model.

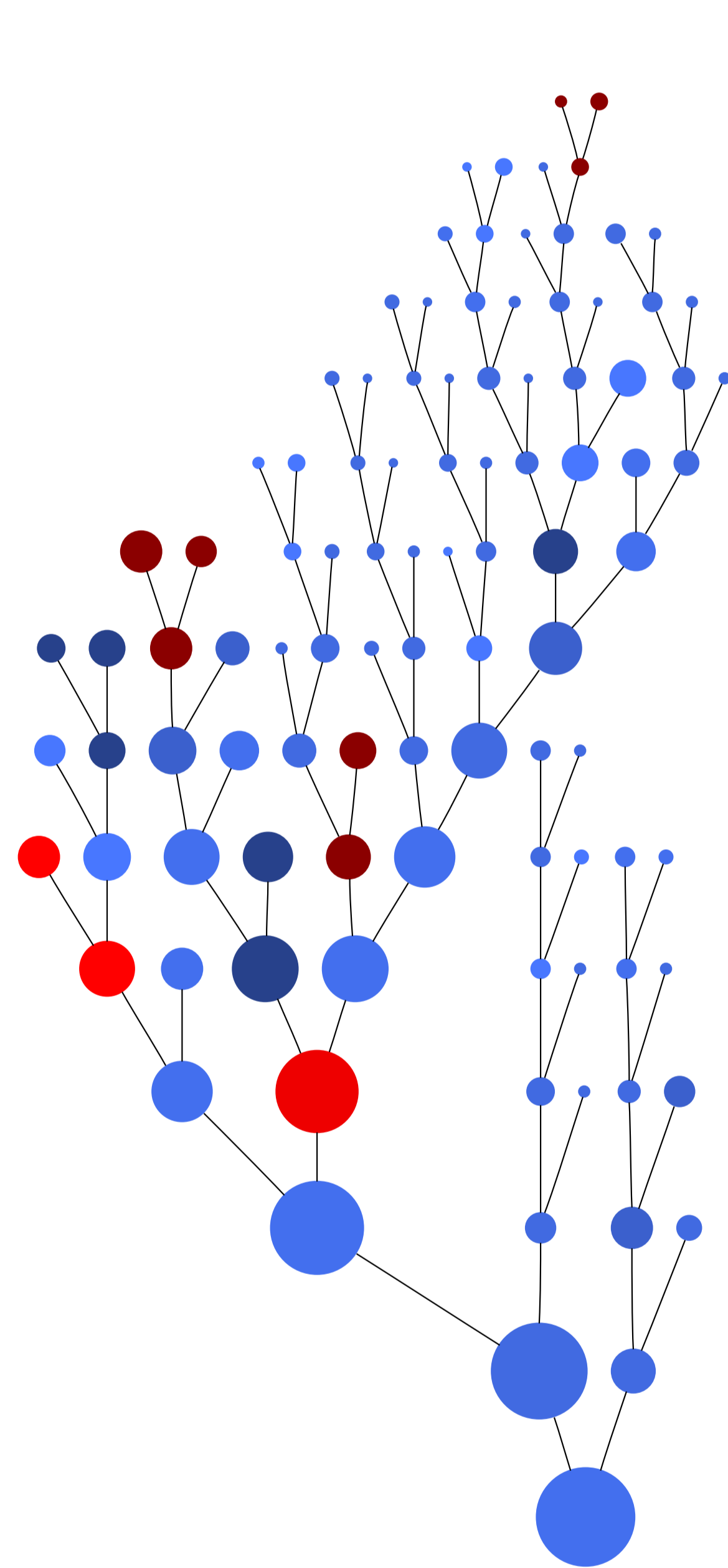


FIGURE 3 The most massive impacts on the particle that gained the maximum mass at the end. Size of nodes represents merger mass. Color indicates the ratio $q = v_{\text{col}}/v_{\text{esc}}$. For the most energetic impact $q = 1.506$, corresponding to the red node at the bottom. Light blue corresponds to the lowest q . Same initial conditions as in figure 2.

Summary & Outlook

The comparison with N-body results shows good agreement. Since the computation time for the Monte Carlo approach is shorter the number of particles can be increased. Further it is possible to follow the collisional growth of individual particles over disk lifetime. Future work will include:

- Detailed studies of collisions
- Fragmentation

- Temperature history of individual bodies
- Composition and water content of bodies
- Advanced gas disk evolution model
- N-body follow up, when the number of particles gets low towards the end of runs
- Individual timesteps for different regions of the disk, allowing large computational domains

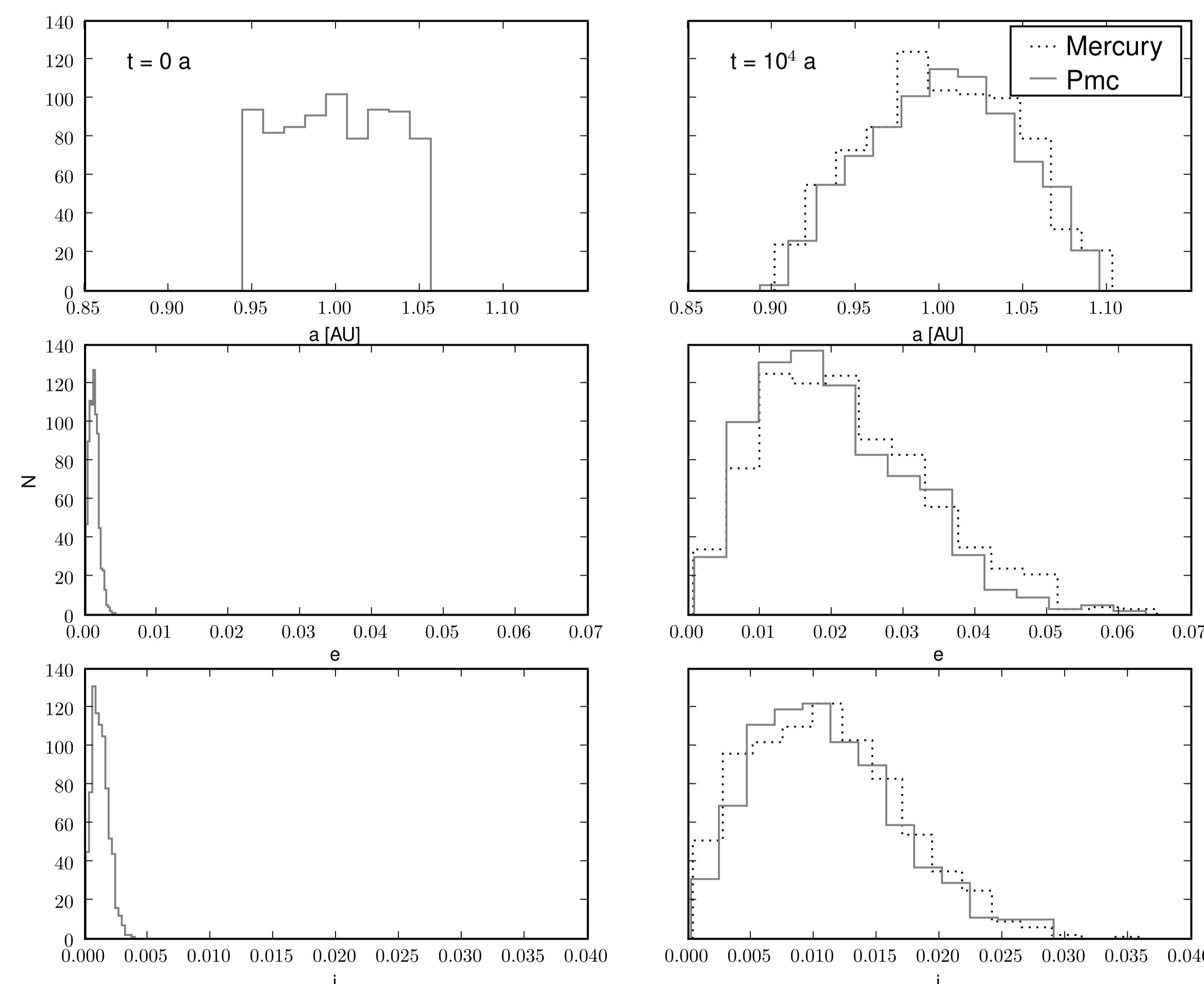


FIGURE 1 Comparison with Mercury [1]. Simultaneous evolution of two populations of bodies, 400 with $m = 2 \cdot 10^{21}$ and 400 with $m = 2 \cdot 10^{22}$ kg. The Pmc code included relaxation and close encounters. Computation time for Mercury was of the order of days. The Pmc run took less than 5 seconds running on a single CPU.

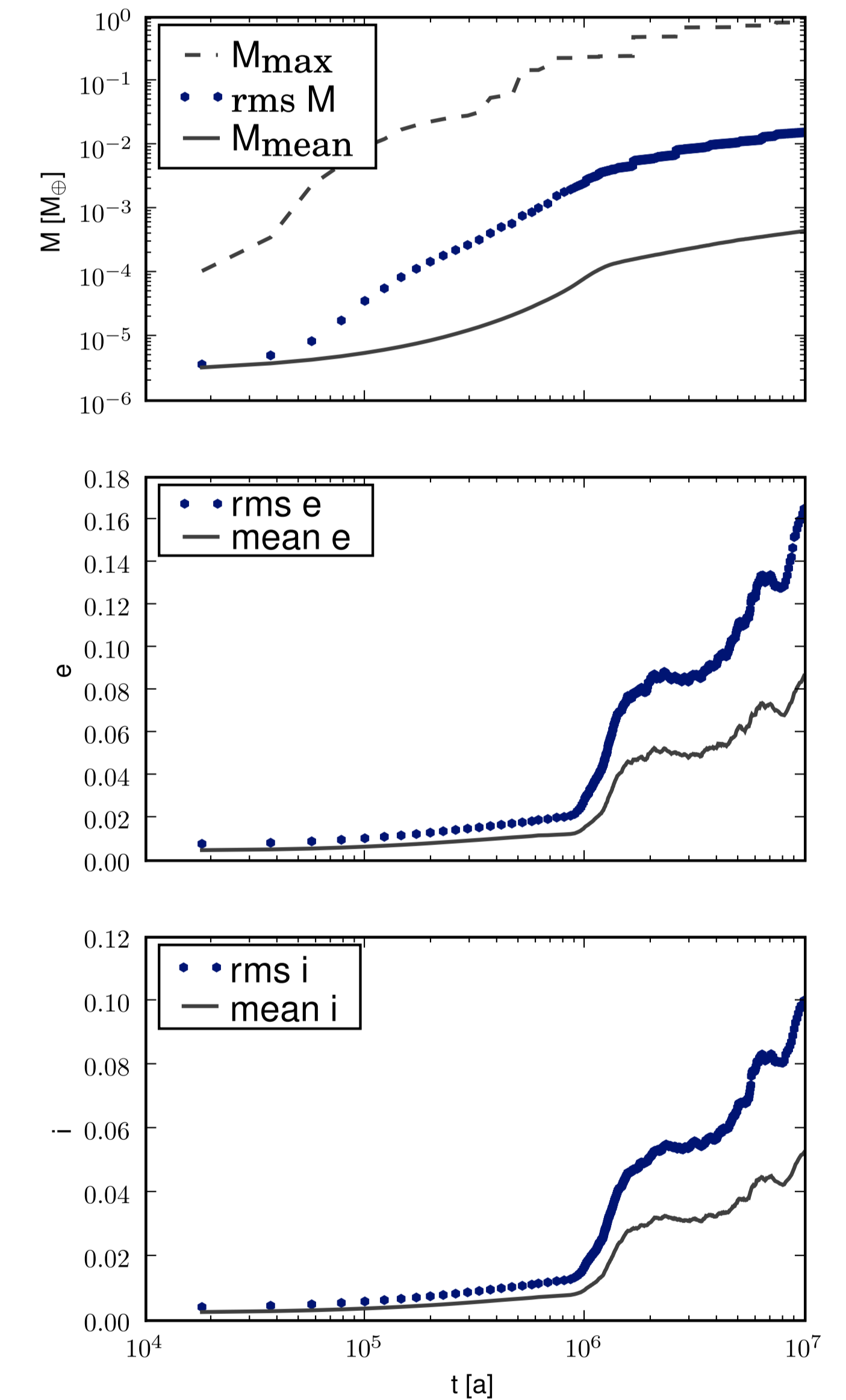


FIGURE 2 Evolution of a system containing of $8 \cdot 10^5$ particles distributed over $a = (0.6, 1.2)$ AU. The initial particle size was 100 km. With a body density of $4 \cdot 10^3$ kg/m³ the initial surface density at 1 AU was 150 kg/m². At the end $m_{\text{max}} \approx 0.4 m_{\text{tot}} (t = 0)$.

References

- [1] J. E. Chambers. A hybrid symplectic integrator that permits close encounters between massive bodies. *MNRAS*, 304:793–799, April 1999.
- [2] S. Inaba, H. Tanaka, K. Nakazawa, G. W. Wetherill, and E. Kokubo. High-Accuracy Statistical Simulation of Planetary Accretion: II. Comparison with N-Body Simulation. *Icarus*, 149:235–250, January 2001.
- [3] Oliver Nyffenegger. *Modelling Protoplanetary Discs Using an Orbit-averaged Monte Carlo Technique*. PhD thesis, Universität Bern, 2005.

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