Measuring the Whirling of Spacetime

Special Lectures on Experimental Gravity

Newtonian vs General Relativistic gravity

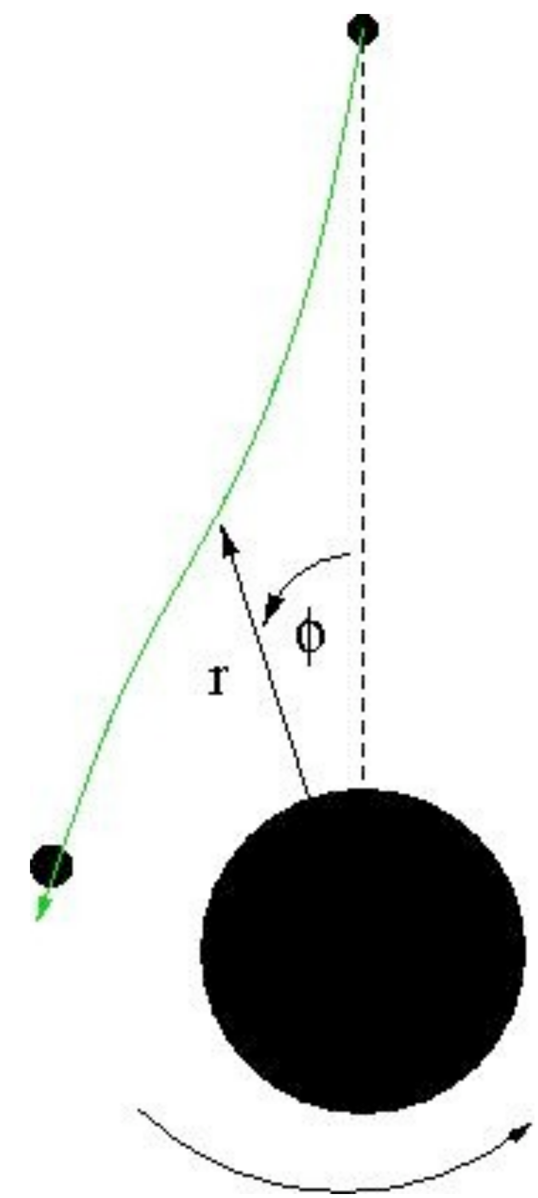
Newtonian field equations	**GR field equations**
$\nabla^2\Phi = 4\pi G\rho$	$G^{ab} = \frac{8\pi G}{c^4} T^{ab}$
Source: mass density Gravitational field: scalar Φ	Source: energy-momentum tensor (includes mass densities/currents) Gravitational field: metric tensor g_{ab}

- Inspection of the field equations hints at the fact that in GR "spin gravitates"

Frame dragging

- The freely falling test body (could be a photon) has zero angular momentum
- The gravitating source has spin J (pointing towards you)
- The test body acquires an angular velocity:

$$\omega = \frac{d\phi}{dt} = -\frac{g_{t\phi}}{g_{\phi\phi}} \approx \frac{2GJ}{c^2 r^3}$$



- Test body is “dragged” along the direction of the source’s rotation (also known as the **Lense-Thirring effect**)
- Frame dragging is intimately related to $g_{t\phi}$

Gravitomagnetism

- Consider the limit of **weak gravity**: $GM/rc^2 \ll 1$
- At leading order we have the usual Newtonian potential: $\Phi = -\frac{GM}{r}$
- The leading order potential due to the spin of the source is

$$g_{it} = \frac{2}{c} A^i, \quad \vec{A} = \frac{G}{cr^3} (\vec{J} \times \vec{r})$$

- The "**gravitomagnetic**" field is: $\vec{B} = \nabla \times \vec{A} = \frac{GJ}{cr^3} \left[3(\hat{J} \cdot \hat{r})\hat{r} - \hat{J} \right]$

- Gravitomagnetic force: $\vec{F}_{gm} = -\frac{2m}{c} \vec{v} \times \vec{B}$

Gyroscopes and gravity I

- The gravitomagnetic character of GR also means that the **spin S** of a test-body (the "gyroscope") will itself couple to a given "background" gravitational field

✓ Spin is forced to precess: $\partial_t \vec{S} = \vec{S} \times \vec{\Omega}_{\rm p}$

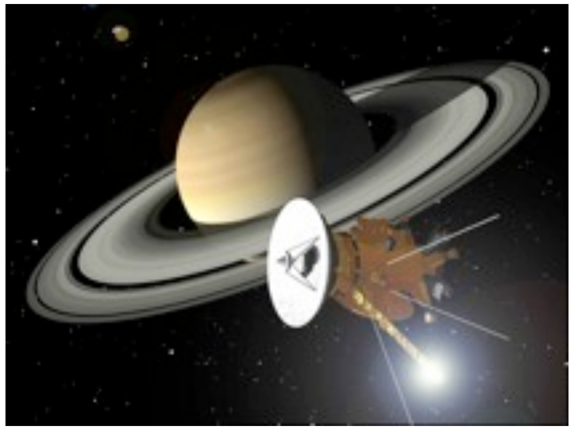
✓ Precession frequency: $\vec{\Omega}_{\rm p} = \frac{3GM}{2c^2 r^3}(\vec{r} \times \vec{v}) + \frac{GJ}{c^2 r^3}\left[3(\hat{J} \cdot \hat{r}) - \hat{J}\right]$

Mass-spin coupling   Spin-spin coupling

✓ This result was first derived by Schiff (1960)

✓ The spin-spin contribution is just the torque: $\vec{\tau} = -\frac{1}{c}\vec{S} \times \vec{B}$

Measuring the whirling spacetime



- Existing solar-system measurements:

✓ Frame-dragging at ~ a 10% accuracy, using the ultra-precise laser tracking system of the LAGEOS satellite constellation

✓ The geodetic effect has been measured at the level of ~ 1% accuracy, using lunar laser ranging data and navigation data from the Cassini probe

- Need for higher accuracy (< 1%) measurements

- This is a challenging task since $\Omega_{\rm p} \sim 10^{-10}\Omega_{\oplus}$

- Hi-tech gyros could be suitable, but they need to be in a noise-free environment. Free fall (i.e. in orbit) is the best option

The Gravity Probe B experiment

- The world's most advanced drag-free gyroscopes in polar orbit (~650 km), completing over 5000 revolutions per year

- Gyro spin-axis precession measured with respect to some fixed star

- **GP-B launched in 2004**, cost ~ $ 700 million, (and 90 related PhDs completed!)

- **Mission objective**:
  measure the gyro spin-axis precession induced by the coupling with Earth's mass and spin, to unprecedented accuracy.

✓ The measured frequency is $\Omega_{\rm p}$

More on the Gravity Probe B

- The GP-B gyros are sensitive to both contributions to the precession frequency $\Omega_{\rm p}$

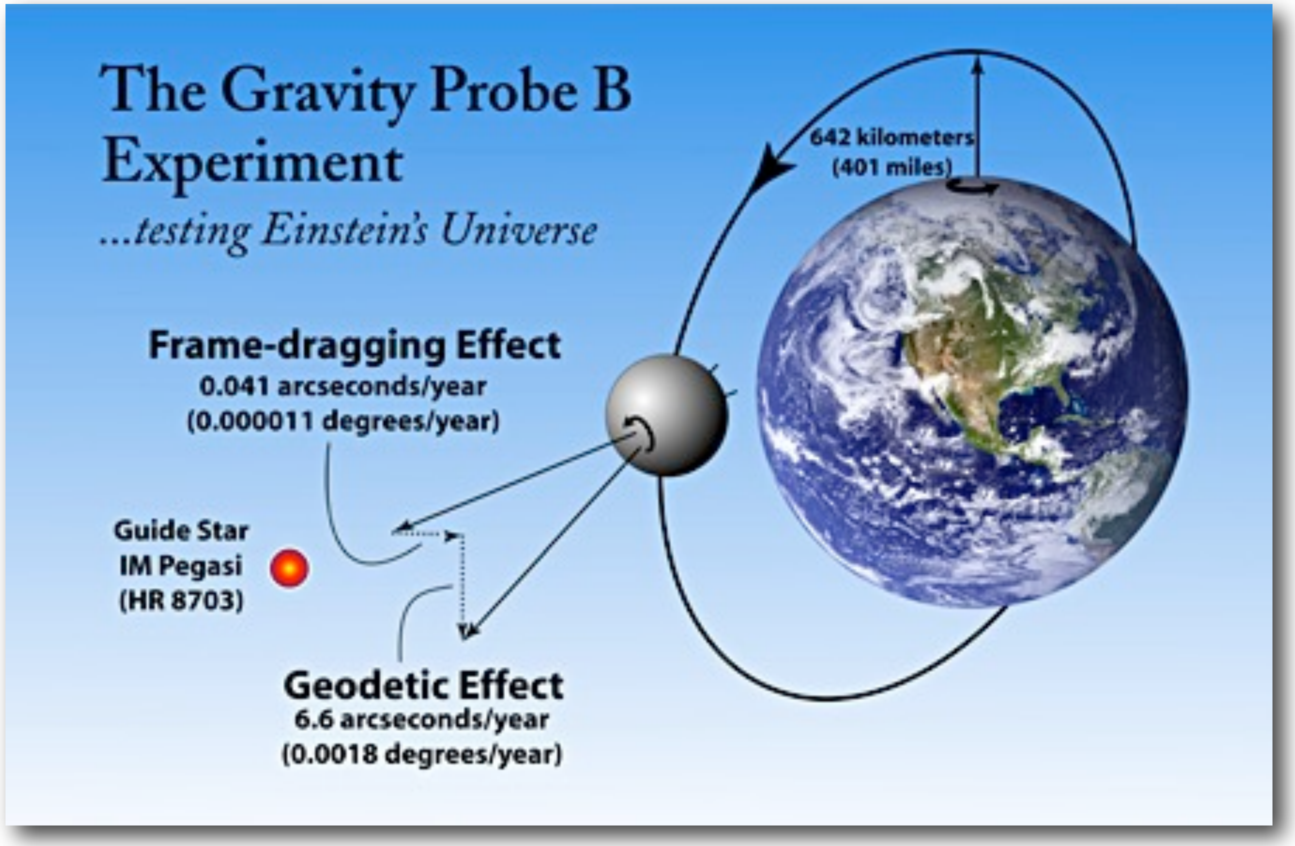
- The mass-spin term is the "**geodetic effect**":

$$\vec{\Omega}_{\rm geod} = \frac{3GM}{2c^2r^3}\,(\,\vec{r}\times\vec{v}\,)$$

- The spin-spin term is the f**rame dragging:**

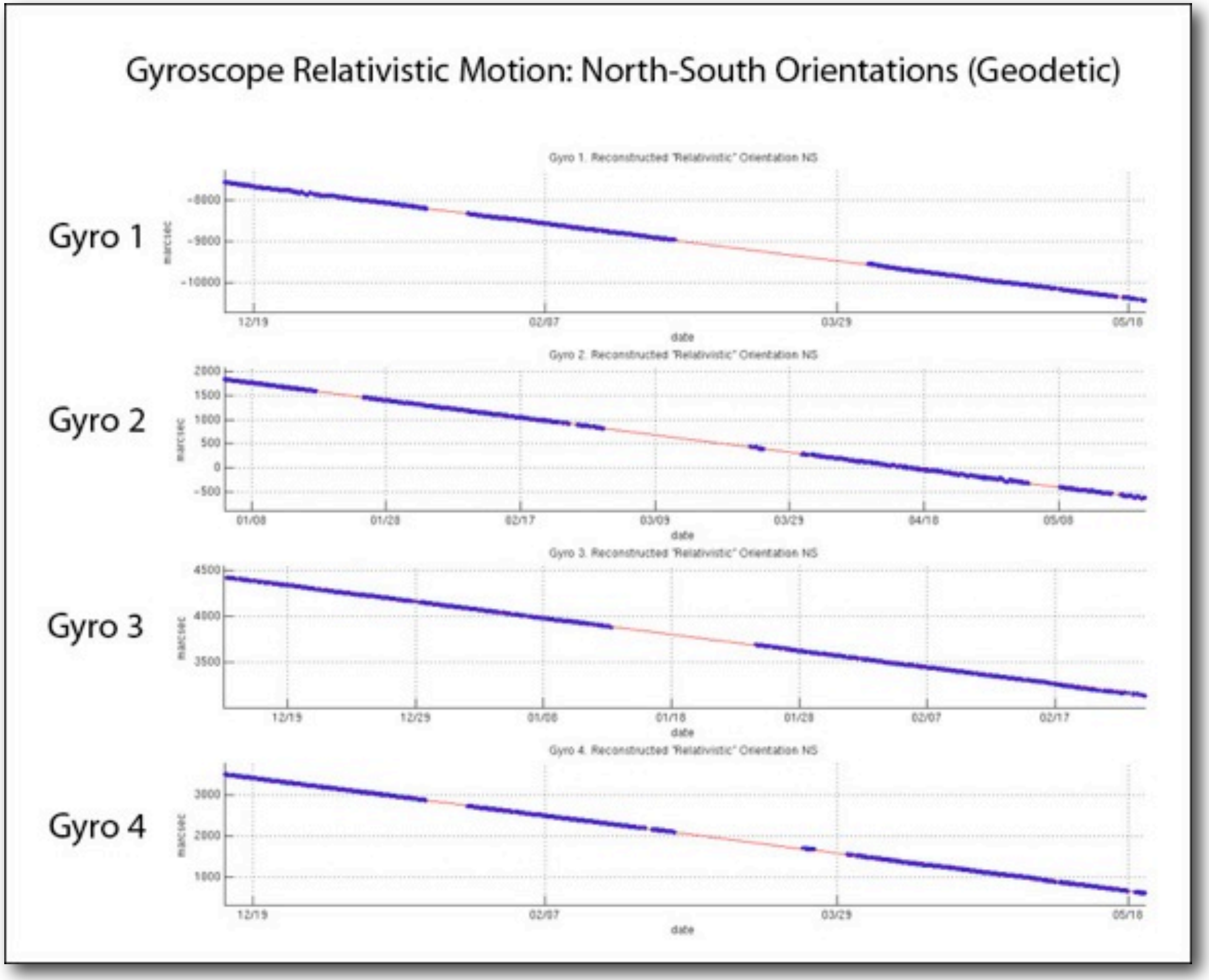
$$\vec{\Omega}_{\rm fd} = \frac{GJ}{c^2r^3}\left[3(\hat{J}\cdot\hat{r})\hat{r} - \hat{J}\right]$$

✓ For a polar orbit: $\vec{\Omega}_{\rm fd} \perp \vec{\Omega}_{\rm geod}$



GP-B results on the geodetic effect

GP-B data	GR prediction
Geodetic **-0.001818 (26) deg/yr**	Geodetic **-0.0018 deg/yr**

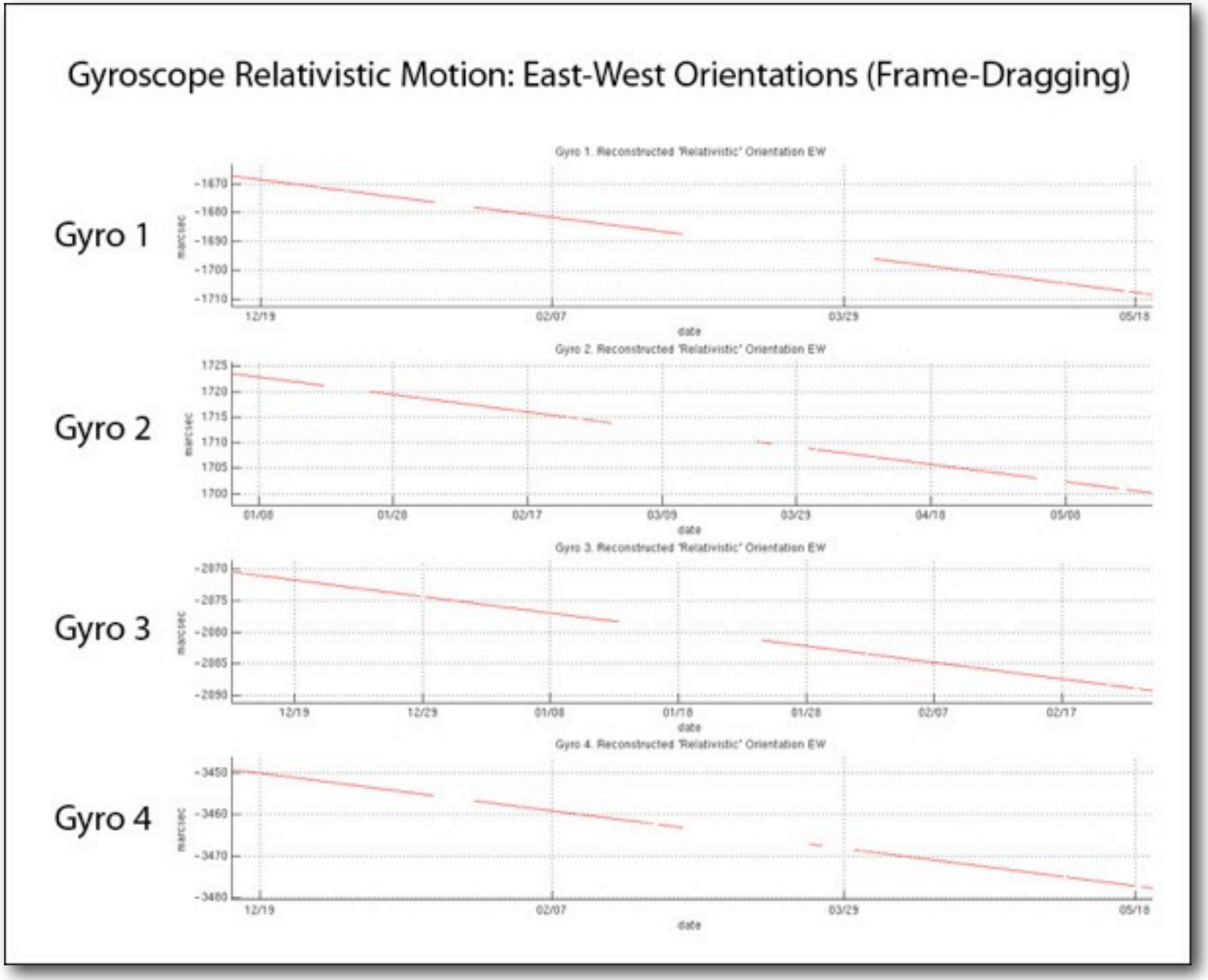


GP-B results on frame-dragging

- After some teething problems with the data analysis and noise removal, all four gyros have detected frame dragging

- Analysis still incomplete, latest data has a ~15 % accuracy, this will improve with time ...

- ... so stay tuned at: **http://einstein.stanford.edu/**



Whirling to the extreme: Kerr black holes

- Black holes have become part of everyday astrophysics. But more hard evidence is needed ...

✓ Supermassive BHs ($M \sim 10^6 - 10^9 M_\odot$) reside in most galactic nuclei (including our own Galaxy)

✓ Less massive BHs ($M \sim 10 M_\odot$) can be found in Galactic accreting systems

- The famous **Kerr solution** of the GR equations provides a precise description of astrophysical spinning BHs.

✓ It has only two parameters : mass M and spin J

$$a \equiv J/M \leq M$$

✓ Existing astrophysical measurements suggest rapidly spinning BHs

Welcome to the ergosphere !

- Matter and radiation in the vicinity of a Kerr hole experience strong frame dragging

✓ Remaining at a fixed position is <u>not</u> possible within the radius (given in G=c=1 units)

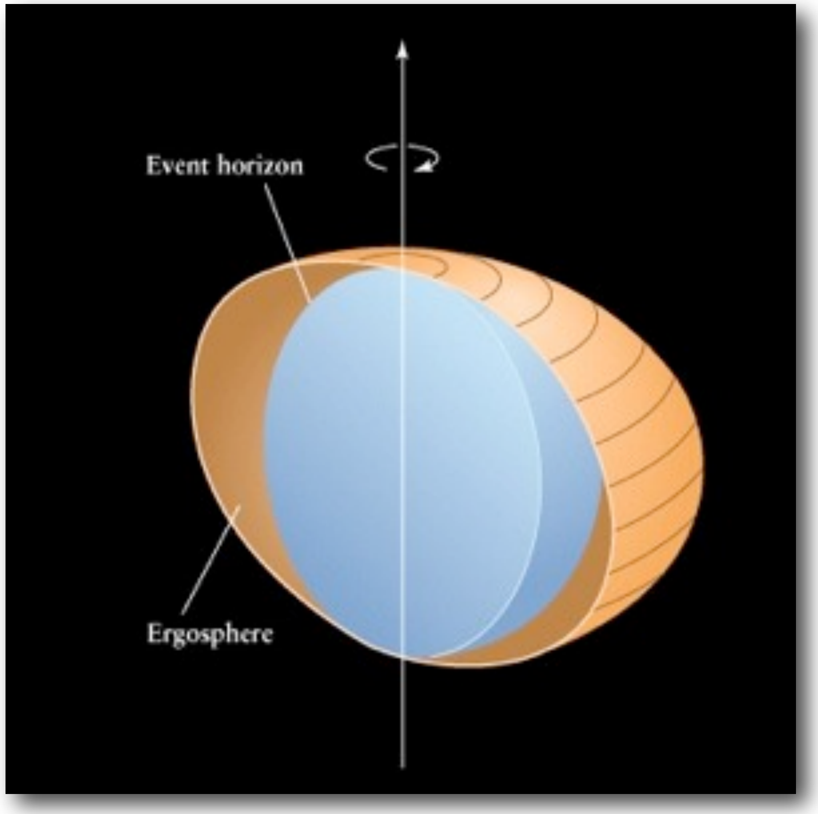
$$r_{\mathrm{erg}}(\theta) = M + \sqrt{M^2 - a^2\cos^2\theta}$$

- The event horizon is at:

$$r_+(\theta) = M + \sqrt{M^2 - a^2}$$

- In the **ergosphere** $r_+ < r < r_{\mathrm{erg}}$ everything has to rotate with the black hole !

- Future observation of gravitational waves from small BHs orbiting around supermassive ones should provide strong evidence for the presence of ergospheres



Summer assignments !

- Search the literature for the equations of motion of a spinning test-body
- Assuming a uniform gravitational field, does a spinning particle fall at the same rate as a non-spinning one ?
- Do rotating compact stars have an ergosphere ?
- The Kerr black hole ergosphere is related to a phenomenon called “superradiance” by which the BH rotational energy can be extracted. Try to learn more about it !
- Derive the force due to the spin-spin coupling in weak gravity