Classical Field Theory

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Field Theory : Introduction

- A **classical field theory** is a physical theory that describes the study of how one or more physical fields interact with matter. The word 'classical' is used in contrast to those field theories that incorporate quantum mechanics (quantum field theories).
- The term 'classical field theory' is commonly reserved for describing those physical theories that describe electromagnetism and gravitation, two of the fundamental forces of nature.
- Classical field theories are usually categorised as non-relativistic and relativistic.

Field Theories : Non-relativistic

• **Newtonial Gravitation** : classical field theory describing gravity. The gravitational force as a mutual interaction between two masses (Newton's law of gravitation)

$$\vec{F} = -\frac{Gm_1 m_2}{r^2}\vec{r}$$

The **gravitational field strength** due to a mass m is

$$\vec{g} = -\frac{Gm}{r^2}\vec{r}$$

Due to the experimental observation that the inertial mass and the gravitational mass are equal (equivalence principle) the gravitational field strength is identical to the acceleration experienced by a particle.

• **Electrostatics**: The force $\vec{F}$ acting on a charged test particle depends on its charge. The **electric field** $\vec{E}$, due to a single charged particle, can be written as $\vec{F} = q\vec{E}$ and thanks to Coulomb's law we get :

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\vec{r}\,.$$

Field Theories : Relativistic

- **Principles**

Lorentz Covariance is the key to formulate a classical relativistic theory, and it is a fundamental aspect of nature.

- **Tools**

Lagrangians are the keys for the mathematical formulation of field theories. The Lagrangian of a system describes its dynamics and when subjected to an action principle gives rise to field equations and a conservation law for the theory.

Electrostatics - Magnetostatics

- Electrostatics
- Green Functions
- Laplace Equation in Spherical Coordinates
- Legendre Equation and Legendre Polynomials
- Associated Legendre Functions and Spherical Harmonics
- Spherical Harmonics $Y_{lm}(\theta, \phi)$
- Multipole Expansion
- Energy of a Charge Distribution in an External Field
- Magnetostatics
- Biot - Savart Law
- ...

Classical Field Theory: Maxwell Equations

- Faraday's Law of Induction
- Energy in the Magnetic Field
- Maxwell Equations
- Vector and Scalar Potentials
- Gauge Transformations
- Green Functions for the Wave Equations
- Poynting's Theorem
- ...

Electromagnetic Waves

- Plane Electromagnetic Waves
- Linear and Circular Polarization of EM Waves
- Stokes Parameters
- Elliptically Polarized EM Waves
- ...

Simple Radiating Systems

- Fields and Radiation of a Localized Oscillating Source
- Electric Monopole Fields
- Electric Dipole Fields and Radiation
- Electric Dipole : Power Radiated
- Magnetic Dipole Fields
- Electric Quadrupole Fields
- ...

Radiation by Moving Charges

- Liénard - Wiechert Potentials
- Power radiated by an accelerated charge
- Angular Distribution of Radiation Emitted by an Accelerated Charge
- Radiation from a charge in arbitrary motion
- What Is Synchrotron Light?
- Thomson Scattering of Radiation
- ...

Special Theory of Relativity

- Tensors
- Covariance in Electrodynamics
- ...

Dynamics of Relativistic Particles and EM Fields

- Lagrangian & Hamiltonian for a Relativistic Charged Particle
- Relativistic Lagrangian
- Relativistic Hamiltonian
- Motion in a Uniform, Static Magnetic Field
- Motion in Combined, Uniform, Static E- and B- Field
- Lowest Order Relativistic Corrections to the Lagrangian...
- Lagrangian for the Electromagnetic Field
- Conservation Laws : Canonical Stress Tensor
- Conservation Laws for EM fields interacting with Charged Particles
- ...

A Short Introduction to Tensor Analysis

- Scalars, Vectors & Tensors
- Tensor Algebra
- Covariant Differentiation
- Parallel Transport
- Curvature Tensor
- Geodesics
- Metric Tensor
- Euler-Lagrange Equations
- Riemann, Ricci and Einstein Tensors
- ...

Physics on Curved Spaces

- Electromagnetism in arbitrary coordinates
- Equations of motion for charged particles
- Energy-Momentum Tensor for EM fields
- Energy-Momentum Tensor for matter
- The Energy-Momentum tensor of a perfect fluid
- Linear Field Equations for Gravitation
- ...

Einstein's Equations

- Equivalence Principle
- Einstein's Equations
- Newtonian Limit
- Schwarzschild Solution
- Motion of massive particles
- Trajectories of photons
- The Classical Tests:
  - Perihelion Advance
  - Deflection of Light Rays
  - Gravitational Redshift
  - Radar Delay
- ...

Solutions of Einstein's Equations & Black Holes

- Schwarzschild Solution: Black Holes
- Wormholes
- Kerr Solution
- Schwarzschild Solution
- Black-Hole: Mechanics & Thermodynamics
- Relativistic Stars
- ...

Grading

- 70% : Final exams
- 40% : Exercises