# Field Theory

## 4th Set of Problems

### RADIATION BY MOVING CHARGES

1. Using the Liénard-Wiechert fields, discuss the time-averaged power radiated per unit solid angle in nonrelativistic motion of a particle with charge $e$, moving
(a) along the $z$ axis with instantaneous position $z(t) = a\cos\omega_0 t$,
(b) in a circle of radius $R$ in the $x - y$ plane with constant angular frequency $\omega_0$.
Sketch the angular distribution of the radiation and determine the total power radiated in each case.

2. A relativistically moving particle of charge $ze$ and mass $m$ passes a fixed point charge $Ze$ in an approximately straight-line path at impact parameter $b$ and nearly constant speed $v$. Show that the total energy radiated in the encounter is
$$\Delta W = \frac{\pi z^4 Z^2 e^6}{4m^2c^4\beta}\left(\gamma^2 + \frac{1}{3}\right)\frac{1}{b^3}.$$

3. A particle of charge $ze$ and mass $m$ moves in external electric and magnetic fields $\vec{E}$ and $\vec{B}$.
(a)Show that the classical relativistic result for the instantaneous energy radiated per unit time can be written
$$P = \frac{2}{3}\frac{z^4e^4}{m^2c^3}\gamma^2\left[\left(\vec{E} + \vec{\beta}\times\vec{B}\right)^2 - \left(\vec{\beta}\cdot\vec{E}\right)^2\right],$$
where $\vec{E}$ and $\vec{B}$ are evaluated at the position on the particle and $\gamma$ is the particle's instantaneous Lorentz factor.
(b)Show that the expression in part (a) can be put into the manifestly Lorentz-invariant form,
$$P = \frac{2z^4r_0^2}{3m^2c}\cdot F^{\mu\nu}p_\nu p^\lambda F_{\lambda\mu},$$
where $r_0 = e^2/mc^2$ is the classical charged particle radius.