

6th Set of Problems

1. Derive step by step the form of $h_{\mu\nu}$ in the TT gauge.
2. Prove that in the T-T gauge the for a GW propagating in the z -direction the only non-vanishing components of the GW stress-energy tensor are:

$$t_{00}^{GW} = \frac{t_{zz}^{GW}}{c^2} = -\frac{t_{0z}^{GW}}{c} = \frac{1}{32\pi G} \frac{c^2}{\omega^2} (h_+^2 + h_\times^2) \quad (1)$$

3. Estimate the amplitude of the GWs from the binary pulsar PSR 1913+16 on earth.
4. Does a single particle feels any acceleration by a passing by gravitational wave? If instead of single particle we consider two connected by a massless spring and the wavelength of a passing by gravitational wave is considerably larger than the distance between the two particles, will the system (particles+spring) feel the passage of the wave?
5. Does the center of mass of the previous system (two equal masses + spring) feels the passage of a gravitational wave?
6. Calculate the quadrupole tensor of Q_{ij} of a triaxial ellipsoid (a, b, c) with uniform density ρ rotating around the z -axis with angular velocity Ω .
7. Calculate the components of $h_{\mu\nu}$ in the TT gauge and estimate the luminosity in gravitational waves of the triaxial ellipsoid if $a = 10km$, $b = 11km$ and $c = 9km$ and the average density is $5 \times 10^{14} \text{gr/cm}^3$.