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GRAVITATIONAL WAVE PHYSICS

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Abstract: Gravitational waves are propagating fluctuations of gravitational fields, that is, "ripples" in spacetime, generated mainly by moving massive bodies. These distortions of spacetime travel with the speed of light. Every body in the path of such a wave feels a tidal gravitational force that acts perpendicular to the waves direction of propagation; these forces change the distance between points, and the size of the changes is proportional to the distance between the points. Gravitational waves can be detected by devices which measure the induced length changes. The frequencies and the amplitudes of the waves are related to the motion of the masses involved. Thus, the analysis of gravitational waveforms allows us to learn about their source and, if there are more than two detectors involved in observation, to estimate the distance and position of their source on the sky.

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1. INTRODUCTION

Einstein first postulated the existence of gravitational waves in 1916 as a consequence of his theory of General Relativity, but no direct detection of such waves has been made yet. The best evidence thus far for their existence is due to the work of 1993 Nobel laureates Joseph Taylor and Russell Hulse. They observed, in 1974, two neutron stars orbiting faster and faster around each other, exactly what would be expected if the binary neutron star was losing energy in the form of emitted gravitational waves. The predicted rate of orbital acceleration caused by gravitational radiation emission according to general relativity was verified observationally, with high precision.

Cosmic gravitational waves, upon arriving on earth, are much weaker than the corresponding electromagnetic waves. The reason is that strong gravitational waves are emitted by *very* massive compact sources undergoing *very* violent dynamics. These kinds of sources are not very common and so the corresponding gravitational waves come from large astronomical distances. On the other hand, the waves thus produced propagate essentially unscathed through space, without being scattered or absorbed from intervening matter.

1.1 Why are gravitational waves interesting?

Detection of gravitational waves is important for two reasons: First, their detection is expected to open up a new window for observational astronomy since the information carried by gravitational waves is very different from that carried by electromagnetic waves. This new window onto universe will complement our view of the cosmos and will help us unveil the fabric of spacetime around black-holes, observe directly the formation of black holes or the merging of binary systems consisting of black holes or neutron stars, search for rapidly spinning neutron stars, dig deep into the very early moments of the origin of the universe, and look at the very center of the galaxies where supermassive black holes weighting millions of solar masses are hidden. These are only a few of the great scientific discoveries that scientists will witness during the first decade of the 21st century. Second, detecting gravitational waves is important for our understanding of the fundamental laws of physics; the proof that gravitational waves exist will verify a fundamental 85-year-old prediction of general relativity. Also, by comparing the arrival times of light and gravitational waves, from, e.g., supernovae, Einstein's prediction that light and gravitational waves travel at the same speed could be checked. Finally, we could verify that they have the polarization predicted by general relativity.

1.2 How we will detect them?

Up to now, the only indication of the existence of gravitational waves is the indirect evidence that the orbital energy in the Hulse-Taylor binary pulsar is drained away at a rate consistent with the prediction of general relativity. The gravitational wave is a signal, the shape of which depends upon the changes in the gravitational field of its source. As it has been mentioned earlier, any body in the path of the wave will feel an oscillating tidal gravitational force that acts in a plane perpendicular to the waves direction of propagation. This means that a group of freely moving masses, placed on a plane perpendicular to the direction of propagation of the wave, will oscillate as long as the wave passes through them, and the distance between them will vary as a function of time as in Figure 1. Thus, the detection of gravitational waves can be accomplished by monitoring the tiny changes in the distance between freely moving test masses. These changes are extremely small; for example, when the Hulse-Taylor binary system finally merges, the strong gravitational wave signal that will be emitted will induce changes in the distance of two particles on earth, that are 1 km apart much smaller than the diameter of the atomic nucleus! To measure such motions of macroscopic objects is a tremendous challenge for experimentalists. As early as the mid-1960s, Joseph Weber designed and constructed heavy metal bars, seismically isolated, to which a set of piezoelectric strain transducers were bonded in such a way that they could detect vibrations of the bar if it had been excited by a gravitational wave. Today, there are a number of such apparatuses operating around the world which have achieved unprecedented sensitivities, but they still are not sensitive enough to detect gravitational waves. Another form of gravitational wave detector that is more promising uses laser beams to measure the distance between two well-separated masses. Such devices are basically kilometer sized laser interferometers consisting of three masses placed in an L-shaped configuration. The laser beams are reflected back and forth between the mirrors attached to the three masses, the mirrors lying several kilometers away from each other. A gravitational wave passing by will cause the lengths of the two arms to oscillate with time. When one arm contracts, the other expands, and this pattern alternates. The result is that the interference pattern of the two laser beams changes with time. With this technique, higher sensitivities could be achieved than are possible with the bar detectors. It is expected that laser interferometric detectors are the ones that will provide us with the first direct detection of gravitational waves.

2. THEORY OF GRAVITATIONAL WAVES

Newton's theory of gravity has enjoyed great success in describing many aspects of our every-day life and additionally explains most of the motions of celestial bodies in the universe. General relativity corrected Newton's theory and is recognized as one of the most ingenious creations of the human mind. The laws of general relativity, though, in the case of slowly moving bodies and weak gravitational fields reduce to the standard laws of Newtonian theory. Nevertheless, general relativity is conceptually different from Newton's theory as it introduces the notion of spacetime and its geometry. One of the basic differences of the two theories concerns the speed of propagation of any change in a gravitational field. As the apple falls from the tree, we have a rearrangement of the distribution of mass of the earth, the gravitational field changes, and a distant observer with a high-precision instrument will detect this change. According to Newton, the changes of the field are instantaneous, i.e., they propagate with infinite speed; if this were true, however the principle of causality would break down. No information can travel faster than the speed of light. In Einstein's theory there is no such ambiguity; the information of the varying gravitational field propagates with finite speed, the speed of light, as a ripple in the

Figure 1: The effects of a gravitational wave travelling perpendicular the plane of a circular ring of particles, is sketched as a series of snapshots. The deformations due the two polarizations are shown.

fabric of spacetime. These are the gravitational waves. The existence of gravitational waves is an immediate consequence of any relativistic theory of gravity. However, the strength and the form of the waves depend on the details of the gravitational theory. This means that the detection of gravitational waves will also serve as a test of basic gravitational theory.

The fundamental geometrical framework of relativistic metric theories of gravity is spacetime, which mathematically can be described as a four-dimensional manifold whose points are called *events*. Every event is labeled by four coordinates x^μ ($\mu = 0, 1, 2, 3$); the three coordinates x^i ($i = 1, 2, 3$) give the spatial position of the event, while x^0 is related to the coordinate time t ($x^0 = ct$, where c is the speed of light, which unless otherwise stated will be set equal to 1). The choice of the coordinate system is quite arbitrary and coordinate transformations of the form $\tilde{x}^\mu = f^\mu(x^\lambda)$ are allowed. The motion of a test particle is described by a curve in spacetime. The distance ds between two neighboring events, one with coordinates x^μ and the other with coordinates $x^\mu + dx^\mu$, can be expressed as a function of the coordinates via a symmetric tensor $g_{\mu\nu}(x^\lambda) = g_{\nu\mu}(x^\lambda)$, i.e.,

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu \tag{2.1}$$

This is a generalization of the standard measure of distance between two points in Euclidian space. For the Minkowski spacetime (the spacetime of special relativity),

$g_{\mu\nu} \equiv \eta_{\mu\nu} = \mathrm{diag}(-1, 1, 1, 1)$. The symmetric tensor is called the *metric tensor* or simply the metric of the spacetime. In general relativity the gravitational field is described by the metric tensor alone, but in many other theories one or more supplementary fields may be needed as well. In what follows, we will consider only the general relativistic description of gravitational fields, since most of the alternative theories fail to pass the experimental tests.

The information about the degree of curvature (i.e., the deviation from flatness) of a spacetime is encoded in the metric of the spacetime. According to general relativity, any distribution of mass bends the spacetime fabric and the Riemann tensor $R_{\kappa\lambda\mu\nu}$ (that is a function of the metric tensor $g_{\mu\nu}$ and of its first and second derivatives) is a measure of the spacetime curvature. The Riemann tensor has 20 independent components. When it vanishes the corresponding spacetime is flat.

In the following presentation, we will consider mass distributions, which we will describe by the stress-energy tensor $T^{\mu\nu}(x^\lambda)$. For a perfect fluid (a fluid or gas with isotropic pressure but without viscosity or shear stresses) the stress-energy tensor is given by the following expression

$$T^{\mu\nu}(x^\lambda) = (\rho + p)u^\mu u^\nu + pg^{\mu\nu}, \tag{2.2}$$

where $p(x^\lambda)$ is the local pressure, $\rho(x^\lambda)$ is the local energy density and $u^\mu(x^\lambda)$ is the four velocity of the infinitesimal fluid element characterized by the event x^λ.

Einstein's gravitational field equations connect the curvature tensor (see below) and the stress-energy tensor through the fundamental relation

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = kT_{\mu\nu} \tag{2.3}$$

This means that the gravitational field, which is directly connected to the geometry of spacetime, is related to the distribution of matter and radiation in the universe. By solving the field equations, both the gravitational field (the $g_{\mu\nu}$) and the motion of matter is determined. $R_{\mu\nu}$ is the so- called Ricci tensor and comes from a contraction of the Riemann tensor ($R_{\mu\nu} = g^{\alpha\sigma}R_{\sigma\mu\rho\nu}$), R is the scalar curvature ($R = g^{\rho\sigma}R_{\rho\sigma}$), while $G_{\mu\nu}$ is the so-called Einstein tensor, $k = 8\pi G/c^4$ is the coupling constant of the theory and is the gravitational constant, which, unless otherwise stated will be considered equal to 1. The vanishing of the Ricci tensor corresponds to a spacetime free of any matter distribution. However, this does not imply that the Riemann tensor is zero. As a consequence, in the empty space far from any matter distribution, the Ricci tensor will vanish while the Riemann tensor can be nonzero; this means that the effects of a propagating gravitational wave in an empty spacetime will be described via the Riemann tensor.

2.1 Linearized theory

Now lets assume that an observer is far away from a given static matter distribution, and the spacetime in which he or she lives is described by a metric $g_{\mu\nu}$. Any change in the matter distribution, i.e., in $T_{\mu\nu}$, will induce a change in the gravitational field, which will be recorded as a change in metric. The new metric will be

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} + h_{\mu\nu} \tag{2.4}$$

where $h_{\mu\nu}$ is a tensor describing the variations induced in the spacetime metric. As we will describe analytically later, this new tensor describes the propagation of ripples in spacetime curvature, i.e., *the gravitational waves*. In order to calculate the new tensor we have to solve Einsteins equations for the varying matter distribution. This is not an easy task in general. However, there is a convenient, yet powerful, way to proceed, namely to assume that $h_{\mu\nu}$ is small ($|h_{\mu\nu}| \ll 1$), so that we need only keep terms linear in $h_{\mu\nu}$ in our calculations. In making this approximation we are effectively assuming that the disturbances produced in spacetime are not huge. This *linearization approach* has proved extremely useful for calculations, and for weak fields at least, gives accurate results for the generation of the waves and for their propagation.

The first attempt to prove that in general relativity gravitational perturbations propagate as waves with the speed of light is due to Einstein himself. Shortly after the formulation of his theory - the year after - he proved that by assuming linearized perturbations around a flat metric, i.e., $g_{\mu\nu} = \eta_{\mu\nu}$, then the tensor

$$\tilde{h}_{\mu\nu} \equiv h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} h^{\alpha}_{\alpha} \tag{2.5}$$

is governed by a wave equation, which admits plane wave solutions similar to the ones of electromagnetism; here $h_{\mu\nu}$ is the metric perturbation and $\tilde{h}_{\mu\nu}$ is the gravitational field (or the trace reverse of $h_{\mu\nu}$). Then the linear field equations in vacuum have the form

$$\left(-\frac{\partial^2}{\partial t^2} + \nabla^2\right)\tilde{h}^{\mu\nu} \equiv \partial_\lambda \partial^\lambda \tilde{h}^{\mu\nu} = 0 \tag{2.6}$$

($\partial_\mu k^\alpha \equiv \partial k^\alpha / \partial x^\mu$), which is the three-dimensional wave equation. To obtain the above simplified form, the condition $\partial_\mu \tilde{h}^{\mu\nu} = 0$, known as *Hilbert's gauge condition* (equivalent to the Lorentz gauge condition of electromagnetism), has been assumed. A gauge transformation is a suitable change of coordinates defined by

$$x'^{\mu} \equiv x^\mu + \xi^\mu \tag{2.7}$$

which induces a redefinition of the gravitational field tensor

$$\tilde{h}'_{\mu\nu} \equiv \tilde{h}_{\mu\nu} - \partial_\mu \xi_\nu - \partial_\nu \xi_\mu + \eta_{\mu\nu}\partial_\lambda \xi^\lambda. \tag{2.8}$$

It can be easily proved that ξ^μ must satisfy the condition

$$\partial_\nu \partial^\nu \xi^\mu = 0, \tag{2.9}$$

so that the new gravitational field is in agreement with the Hilberts gauge condition. The solution of equation (2.9) defines the four components of ξ^μ so that the new tensor $\tilde{h}'_{\mu\nu}$ is also a solution of the wave equation (2.6), and thus, have the same physical meaning as $\tilde{h}_{\mu\nu}$. In general, gauge transformations correspond to symmetries of the field equations, which means that the field equations are invariant under such transformations. This implies that the field equations do not determine the field uniquely; however, this ambiguity in determining the field is devoid of any physical meaning.

The simplest solution to the wave equation (2.6) is a plane wave solution of the form

$$\tilde{h}^{\mu\nu} = A^{\mu\nu} e^{ik_\alpha x^\alpha}, \tag{2.10}$$

where $A^{\mu\nu}$ is a constant symmetric tensor, the *polarization tensor*, in which information about the amplitude and the polarization of the waves is encoded, while k_α is a constant vector, the *wave vector*, that determines the propagation direction of the wave and its frequency. In physical applications we will use only the real part of the above wave solution. By applying the Hilbert gauge condition on the plane wave solution we obtain the relation $A^{\mu\nu} k_\mu = 0$, the geometrical meaning of which is that $A^{\mu\nu}$ and k_μ are orthogonal. This relation can be written as four equations that impose four conditions on $A^{\mu\nu}$, and this is the first step in reducing the number of its independent components. As a consequence, $A^{\mu\nu}$, instead of having 10 independent components (as has every symmetric second rank tensor in a four dimensional space), has only 6 independent ones. Further substitution of the plane wave solution in the wave equation leads to the important equation $k_\mu k^\mu = k_0^2 - (k_x^2 + k_y^2 + k_z^2) = 0$, which means that k_μ is a *lightlike* or *null vector*, i.e., the wave propagates on the light-cone. This means that the speed of the wave is 1, i.e., equal to the speed of light. The frequency of the wave is $\omega = k^0$.

Up to this point it has been proven that $A^{\mu\nu}$ has six arbitrary components, but due to the gauge freedom, i.e., the freedom in choosing the four components of the vector ξ^μ, the actual number of its independent components can be reduced to **two**, in a suitable chosen gauge. The *transverse–traceless* or *TT gauge* is an example of such a gauge. In this gauge, only the spatial components of $\tilde{h}_{\mu\nu}$ are non-zero (hence $\tilde{h}_{\mu 0} = 0$), which means that the wave is *transverse* to its own direction of propagation, and, additionally, the sum of the diagonal components is zero ($\tilde{h}^\mu_\mu = h^0_0 + h^1_1 + h^2_2 + h^3_3 \equiv \tilde{h} = h = 0$) (*traceless*). Due to this last property and equation (5), in this gauge there is no difference between $h_{\mu\nu}$(the perturbation of the metric) and $\tilde{h}_{\mu\nu}$ (the gravitational field). It is customary to write the gravitational wave solution in the TT gauge as $h^{TT}_{\mu\nu}$. That $A_{\mu\nu}$ has only two independent components means that a gravitational wave is completely described by two dimensionless amplitudes, h_+ and $h_\times$, say. If, for example, we assume a wave propagating along the z-direction, then the amplitude $A^{\mu\nu}$ can be written as

$$A^{\mu\nu} = h_+ \epsilon_+^{\mu\nu} + h_\times \epsilon_\times^{\mu\nu} \tag{2.11}$$

where $\epsilon_+^{\mu\nu}$ and $\epsilon_\times^{\mu\nu}$ are the so-called *unit polarization tensors* defined by

$$\epsilon_+^{\mu\nu} \equiv \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \qquad \epsilon_\times^{\mu\nu} \equiv \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{2.12}$$

As mentioned earlier, the Riemann tensor is a measure of the curvature of spacetime. A gravitational wave, propagating in a flat spacetime, generates periodic distortions, which can be described in terms of the Riemann tensor. In linearized theory the Riemann tensor takes the following gauge-independent form:

$$R_{\kappa\lambda\mu\nu} = \frac{1}{2}\left(\partial_{\nu\kappa}h_{\lambda\mu} + \partial_{\lambda\mu}h_{\kappa\nu} - \partial_{\kappa\mu}h_{\lambda\nu} - \partial_{\lambda\nu}h_{\kappa\mu}\right), \tag{2.13}$$

which is considerably simplified by choosing the TT gauge:

$$R_{j0k0}^{TT} = -\frac{1}{2}\frac{\partial^2}{\partial t^2}h_{jk}^{TT}, \quad j,k = 1,2,3. \tag{2.14}$$

Furthermore, in the Newtonian limit

$$R_{j0k0}^{TT} \approx \frac{\partial^2 \Phi}{\partial x^j \partial x^k}, \tag{2.15}$$

where Φ describes the gravitational potential in Newtonian theory. Earlier we defined the Riemann tensor as a geometrical object, but this tensor has a simple physical interpretation: it is the tidal force field and describes the relative acceleration between two particles in free fall. If we assume two particles moving freely along geodesics of a curved spacetime with coordinates $x^\mu(\tau)$ and $x^\mu(\tau) + \xi^\mu(\tau)$ (for a given value of the proper time τ, $\xi^\mu(\tau)$ is the displacement vector connecting the two events) it can be shown that, in the case of slowly moving particles,

$$\frac{d^2\xi^k}{dt^2} \approx -R^{k\ \ TT}_{0j0}\xi^j. \tag{2.16}$$

This is a simplified form of the equation of *geodesic deviation*. Hence, the tidal force acting on a particle is:

$$f^k \approx -mR^k_{0j0}\xi^j, \tag{2.17}$$

where m is the mass of the particle. Equation (2.17) corresponds to the standard Newtonian relation for the tidal force acting on a particle in a field Φ.

Keeping this in mind, we will try to visualize the effect of a gravitational wave. Let us first consider two freely falling particles hit by a gravitational wave travelling along the z-direction, with the (+) polarization present only, i.e.,

$$h^{\mu\nu} = h_+ \epsilon^{\mu\nu} \cos[\omega(t - z)]. \tag{2.18}$$

Then, the measured distance ξ^x between the two particles, originally at a distance ξ^x_0, along the x direction, will be

$$\frac{\xi^x}{\xi^x_0} = 1 - \frac{1}{2} h_+ \cos[\omega(t - z)] \quad \text{or} \quad \delta\xi^x = \xi^x - \xi^x_0 = -\frac{1}{2} h_+ \cos[\omega(t - z)]\xi^x_0 \tag{2.19}$$

which implies that the relative distance $\delta\xi^x$ between the two particles will oscillate with frequency ω. This does not mean that the particles coordinate positions change; instead they remain at rest relative to the coordinates, but the coordinate distance oscillates. If the particles were placed originally along the y-direction the coordinate distance would oscillate according to

$$\frac{\xi^y}{\xi^y_0} = 1 + \frac{1}{2} h_+ \cos[\omega(t - z)] \quad \text{or} \quad \delta\xi^y = \xi^y - \xi^y_0 = \frac{1}{2} h_+ \cos[\omega(t - z)]\xi^y_0 \tag{2.20}$$

In other words, the coordinate distances along the two axes oscillate out of phase, that is, when the distance between two particles along the x direction is maximum the distance of two other particles along the y direction is minimum, and after half a period, it is the other way around. The effects are similar for the other polarization, where the axes along which the oscillations are out of phase are at an angle of 45^o with respect to the first ones. This can be visualized in Figure 1, where the effect of a passing gravitational wave on a ring of particles is shown as a series of snapshots closely separated in time.

Another way of understanding the effects of gravitational waves is to study the tidal force field lines. In the TT gauge the equation of the geodesic deviation (2.16) takes the simple form

$$\frac{d^2\xi^k}{dt^2} \approx \frac{1}{2} \frac{d^2 h^{TT}_{jk}}{dt^2} \xi^j \tag{2.21}$$

and the corresponding tidal force is

$$f^k \approx \frac{m}{2} \frac{d^2 h^{TT}_{jk}}{dt^2} \xi^j \tag{2.22}$$

For the wave given by equation (2.18) the two nonzero components of the tidal force are

$$f^x \approx \frac{m}{2} h_+ \omega^2 \cos[\omega(t - z)]\xi^x_0, \quad \text{and} \quad f^y \approx -\frac{m}{2} h_+ \omega^2 \cos[\omega(t - z)]\xi^y_0. \tag{2.23}$$

It can be easily proved that the divergence of the tidal force is zero ($\partial f^x/\partial\xi^x_0 + \partial f^y/\partial\xi^y_0 = 0$). It can therefore be represented graphically by field lines as in Figure 2.

Figure 2: The tidal field lines of force for a gravitational wave with polarization (+) (left panel) and (×) (right panel). The orientation of the field lines changes every half period producing the deformations as seen in Figure 1. Any point accelerates in the directions of the arrows, and the denser are the lines, the strongest is the acceleration. Since the acceleration is proportional to the distance from the center of mass, the force lines get denser as one moves away from the origin. For the polarization (×) the force lines undergo a 45^0 rotation.

Let us now return back to the two polarization states represented by the two matrices $\epsilon_+^{\mu\nu}$ and $\epsilon_\times^{\mu\nu}$. It is impossible to construct the (+) pattern from the (×) pattern and vice versa; they are orthogonal polarization states. By analogy with electromagnetic waves, the two polarizations could be added with phase difference $(\pm\pi/2)$ to obtain circularly polarized waves. The effect of circularly polarized waves on a ring of particles is to deform the ring into a rotating ellipse with either positive or negative helicity. The particles themselves do not rotate; they only oscillate in and out around their initial positions. These circularly polarized waves carry angular momentum the amount of which is $(2/\omega)$ times the energy carried by the wave. If we consider the gravitational field, then, according to quantum field theory, the waves are associated with fundamental particles responsible for the gravitational interaction, and a quantum of the field will have energy $\hbar\omega$ and consequently spin $2\hbar$. This means that the quanta of the gravitational field, the *gravitons*, are spin-2, massless particles (since they travel with the speed of light). Another way of explaining why a graviton should be a spin-2 particle comes from observing Figure 1. One can see that a gravitational wave is invariant under rotations of 180^0 about its direction of propagation; the pattern repeats itself after half period. For comparison, we mention that electromagnetic waves are invariant under rotations of 360^0. In the quantum mechanical description of massless particles, the wavefunction of a particle is invariant under rotations of $360^0/s$, where s is the spin of the particle. Thus the photon is a spin-1 particle and the graviton is a spin-2 particle. In other relativistic theories of gravity, the wave field has other symmetries and therefore they attribute different spins to the gravitons.

2.2 Properties of gravitational waves

Gravitational waves, once they are generated, propagate almost unimpeded. Indeed, it has been proven that they are even harder to stop than neutrinos! The only significant change they suffer as they propagate is the decrease in amplitude while they travel away from their source, and the *redshift* they feel (cosmological, gravitational or Doppler), as is the case for electromagnetic waves.

There are other effects that marginally influence the gravitational waveforms, for instance, *absorption* by interstellar or intergalactic matter intervening between the observer and the source, which is extremely weak (actually, the extremely weak coupling of gravitational waves with matter is the main reason that gravitational waves have not been observed). *Scattering* and *dispersion* of gravitational waves are also practically unimportant, although they may have been important during the early phases of the universe (this is also true for the absorption). Gravitational waves can be *focused* by strong gravitational fields and also can be *diffracted*, exactly as it happens with the electromagnetic waves.

There are also a number of "exotic" effects that gravitational waves can experience, that are due to the nonlinear nature of Einsteins equations (purely general-relativistic effects), such as: *scattering* by the background curvature, the existence of *tails* of the waves that interact with the waves themselves, *parametric amplification* by the background curvature, *nonlinear coupling of the waves with themselves* (creation of *geons*, that is, bundles of gravitational waves held together by their own self- generated curvature) and even *formation of singularities* by colliding waves (for such exotic phenomena see the extensive review by Thorne[4]). These aspects of nonlinearity affect the majority of the gravitational wave sources and from this point of view our understanding of gravitational-wave generation is based on approximations. However, it is expected that the error in these approximations for most of the processes that generate gravitational waves is quite small. Powerful numerical codes, using state-of-the-art computer software and hardware, have been developed (and continue to be developed) for minimizing all possible sources of error in order to have as accurate as possible an understanding of the processes that generate gravitational waves and of the waveforms produced.

For most of the properties mentioned above there is a correspondence with electromagnetic waves. Gravitational waves are fundamentally different, however, even though they share similar wave properties away from the source. Gravitational waves are emitted by coherent bulk motions of matter (for example, by the implosion of the core of a star during a supernova explosion) or by coherent oscillations of spacetime curvature, and thus they serve as a probe of such phenomena. By contrast, cosmic electromagnetic waves are mainly the result of incoherent radiation by individual atoms or charged particles. As a consequence, from the cosmic electromagnetic radiation we mainly learn about the form of matter in various regions of the universe, especially about its temperature and density, or about the existence of magnetic fields. Strong gravitational waves, are emitted from regions of spacetime where gravity is very strong and the velocities of the bulk motions of matter are near the speed of light. Since most of the time these areas are either surrounded by thick layers of matter that absorb electromagnetic radiation or they do not emit any electromagnetic radiation at all (black holes), the only way to study these regions of the universe is via gravitational waves.

2.3 Energy flux carried by gravitational waves

Gravitational waves carry energy and cause a deformation of spacetime. The stress-energy carried by gravitational waves cannot be localized within a wavelength. Instead, one can say that a certain amount of stress-energy is contained in a region of the space which extends over several wavelengths. It can be proven that in the TT gauge of linearized theory the stress-energy tensor of a gravitational wave (in analogy with the stress-energy tensor of a perfect fluid that we have defined earlier) is given by

$$t_{\mu\nu}^{GW} = \frac{1}{32\pi}\langle\left(\partial_\mu h_{ij}^{TT}\right)\left(\partial_\nu h_{ij}^{TT}\right)\rangle. \tag{2.24}$$

where the angular brackets are used to indicate averaging over several wavelengths. For the special case of a plane wave propagating in the z direction, which we considered earlier, the stress-energy tensor has only three non-zero components, which take the simple form

$$t_{00}^{GW} = \frac{t_{zz}^{GW}}{c^2} = -\frac{t_{0z}^{GW}}{c} = \frac{1}{32\pi}\frac{c^2}{G}\omega^2\left(h_+^2 + h_\times^2\right), \tag{2.25}$$

where t_{00}^{GW} is the energy density, t_{zz}^{GW} is the momentum flux and t_{0z}^{GW} the energy flow along the z direction per unit area and unit time (for practical reasons we have restored the normal units). The energy flux has all the properties one would anticipate by analogy with electromagnetic waves: (a) it is conserved (the amplitude dies out as $1/r$, the flux as $1/r^2$), (b) it can be absorbed by detectors, and (c) it can generate curvature like any other energy source in Einstein's formulation of relativity. As an example, by using the above relation, we will estimate the energy flux in gravitational waves from the collapse of the core of a supernova to create a 10 $M_\odot$ black hole at a distance of 50-million-light-years ($\sim$15 Mpc) from the earth (at the distance of the Virgo cluster of galaxies). A conservative estimate of the amplitude of the waves on earth (as we will show later) is of the order of 10^{-22} (at a frequency of about 1kHz). This corresponds to a flux of about 3 ergs/cm^2 sec. This is an enormous amount of energy flux and is about ten orders of magnitude larger than the observed energy flux in electromagnetic waves! The basic difference is the duration of the two signals; gravitational wave signal will last a few milliseconds, whereas an electromagnetic signal lasts many days. This example provides us with a useful numerical formula for the energy flux:

$$F = 3 \left(\frac{f}{1\text{kHz}} \right)^2 \left(\frac{h}{10^{-22}} \right)^2 \frac{\text{ergs}}{\text{cm}^2\text{sec}}, \tag{2.26}$$

from which one can easily estimate the flux on earth, given the amplitude (on earth) and the frequency of the waves.

2.4 Generation of gravitational waves

As we have mentioned earlier, when the gravitational field is strong there are a number on nonlinear effects that influence the generation and propagation of gravitational waves. For example, nonlinear effects are significant during the last phases of black hole formation. The analytic description of such a dynamically changing spacetime is impossible, and until numerical relativity provides us with accurate estimates of the dynamics of gravitational fields under such extreme conditions we have to be content with order of magnitude estimates. Furthermore, there are differences in the predictions of various relativistic theories of gravity in the case of high concentrations of rapidly varying energy distributions. However, all metric theories of gravity, as long as they admit the correct Newtonian limit, make similar predictions for the total amount of gravitational radiation emitted by "weak" gravitational wave sources, that is, sources where the energy content is small enough to produce only small deformations of the flat spacetime and where all motions are slow compared to the velocity of light.

Let us now try to understand the nature of gravitational radiation, by starting from the production of electromagnetic radiation. Electromagnetic radiation emitted by slowly varying charge distributions can be decomposed into a series of multipoles, where the amplitude of the 2^ℓ-pole ($\ell = 0, 1, 2, ...$) contains a small factor a^ℓ, with a equal to the ratio of the diameter of the source to the typical wavelength, namely, a number typically much smaller than 1. From this point of view the strongest electromagnetic radiation would be expected for monopolar radiation ($\ell = 0$), but this is completely absent, because the electromagnetic monopole moment is proportional to the total charge, which does not change with time (it is a conserved quantity). Therefore, electromagnetic radiation consists only of $\ell \geq 1$ multipoles, the strongest being the electric dipole radiation ($\ell = 1$), followed by the weaker magnetic dipole and electric quadrupole radiation ($\ell = 2$). One could proceed with a similar analysis for gravitational waves and by following the same arguments show that mass conservation (which is equivalent to charge conservation in electromagnetic theory) will exclude monopole radiation. Also, the rate of change of the mass dipole moment is proportional to the linear momentum of the system, which is a conserved quantity, and therefore there cannot be any mass dipole radiation in Einsteins relativity theory. The next strongest form of electromagnetic radiation is the magnetic dipole. For the case of gravity, the change of the magnetic dipole is proportional to the angular momentum of the system, which is also a conserved quantity and thus there is no dipolar gravitational radiation of *any* sort. It follows that gravitational radiation is of quadrupolar or higher nature and is directly linked to the quadrupole moment of the mass distribution.

As early as 1918, Einstein derived the quadrupole formula for gravitational radiation. This formula states that the wave amplitude h_{ij} is proportional to the second time derivative of the quadrupole moment of the source:

$$h_{ij} = \frac{2}{r}\frac{G}{c^4}\ddot{Q}_{ij}^{TT}\left(t - \frac{r}{c}\right) \tag{2.27}$$

where

$$Q_{ij}^{TT}(x) = \int \rho \left(x^i x^j - \frac{1}{3}\delta^{ij} r^2 \right) d^3x \tag{2.28}$$

is the quadrupole moment in the TT gauge, evaluated at the retarded time $t - r/c$ and ρ is the matter density in a volume element d^3x at the position x^i. This result is quite accurate for all sources, as long as the reduced wavelength $\tilde{\lambda} = \lambda/2\pi$ is much longer than the source size R. It should be pointed out that the above result can be derived via a quite cumbersome calculation in which we solve the wave equation (2.6) with a source term $T_{\mu\nu}$ on the right-hand side. In the course of such a derivation, a number of assumptions must be used. In particular, the observer must be located at a distance $r \gg \tilde{\lambda}$, far greater than the reduced wavelength (in what is called the radiation zone) and $T_{\mu\nu}$ must not change very quickly.

Using the formulae (2.24) and (2.25) for the energy carried by gravitational waves, one can derive the luminosity in gravitational waves as a function of the third-order time derivative of the quadrupole moment tensor. This is the quadrupole formula

$$L_{GW} = -\frac{dE}{dt} = \frac{1}{5}\frac{G}{c^5}\langle \frac{\partial^3 Q_{ij}}{\partial t^3} \, \frac{\partial^3 Q_{ij}}{\partial t^3} \rangle. \tag{2.29}$$

Based on this formula, we derive some additional formulas, which provide order of magnitude estimates for the amplitude of the gravitational waves and the corresponding power output of a source. First, the quadrupole moment of a system is approximately equal to the mass M of the part of the system that moves, times the square of the size R of the system. This means that the third-order time derivative of the quadrupole moment is

$$\frac{\partial^3 Q_{ij}}{\partial t^3} \sim \frac{MR^2}{T^3} \sim \frac{Mv^2}{T} \sim \frac{E_{\mathrm{ns}}}{T}, \tag{2.30}$$

where v is the mean velocity of the moving parts, E_{ns} is the kinetic energy of the component of the source's internal motion which is non spherical, and T is the time scale for a mass to move from one side of the system to the other. The time scale (or period) is actually proportional to the inverse of the square root of the mean density of the system

$$T \sim \sqrt{R^3/GM}. \tag{2.31}$$

This relation provides a rough estimate of the characteristic frequency of the system $f = 2\pi/T$. Then, the luminosity of gravitational waves of a given source is approximately

$$L_{GW} \sim \frac{G}{c^5}\left(\frac{M}{R}\right)^5 \sim \frac{G}{c^5}\left(\frac{M}{R}\right)^2 v^6 \sim \frac{c^5}{G}\left(\frac{R_{\rm Sch}}{R}\right)^2 \left(\frac{v}{c}\right)^6 \tag{2.32}$$

where $R_{\rm Sch} = 2GM/c^2$ is the Schwarzschild radius of the source. It is obvious that the maximum value of the luminosity in gravitational waves can be achieved if the source's dimensions are of the order of its Schwarzschild radius and the typical velocities of the components of the system are of the order of the speed of light. This explains why we expect the best gravitational wave sources to be highly relativistic compact objects. The above formula sets also an upper limit on the power emitted by a source, which for $R \sim R_{\rm Sch}$ and $v \sim c$ is

$$L_{GW} \sim c^5/G = 3.6 \times 10^{59} \text{ergs/sec}. \tag{2.33}$$

This is an immense power, often called the *luminosity of the universe*.

Using the above order-of-magnitude estimates, we can get a rough estimate of the amplitude of gravitational waves at a distance r from the source:

$$h \sim \frac{G}{c^4}\frac{E_{\rm ns}}{r} \sim \frac{G}{c^4}\frac{\varepsilon E_{\rm kin}}{r} \tag{2.34}$$

where $\varepsilon E_{\rm kin}$ (with $0 \leq \varepsilon \leq 1$), is the fraction of kinetic energy of the source that is able to produce gravitational waves. The factor ε is a measure of the asymmetry of the source and implies that only a time varying quadrupole moment will emit gravitational waves. For example, even if a huge amount of kinetic energy is involved in a given explosion and/or implosion, if the event takes place in a spherically symmetric manner, there will be no gravitational radiation.

Another formula for the amplitude of gravitational waves relation can be derived from the flux formula (2.26). If, for example, we consider an event (perhaps a supernovae explosion) at the Virgo cluster during which the energy equivalent of $10^{-4}M_\odot$ is released in gravitational waves at a frequency of 1 kHz, and with signal duration of the order of 1 msec, the amplitude of the gravitational waves on Earth will be

$$h \approx 10^{-22}\left(\frac{E_{GW}}{10^{-4}M_\odot}\right)^{1/2}\left(\frac{f}{1\text{kHz}}\right)^{-1}\left(\frac{\tau}{1\text{msec}}\right)^{-1/2}\left(\frac{r}{15\text{Mpc}}\right)^{-1}. \tag{2.35}$$

For a detector with arm length of 4 km we are looking for changes in the arm length of the order of

$$\Delta\ell = h \cdot \ell = 10^{-22} \cdot 4\text{km} = 4 \times 10^{-17}!!!$$

These numbers shows why experimenters are trying so hard to build ultra-sensitive detectors and explains why all detection efforts till today were not successful.

Finally, it is useful to know the damping time, that is, the time it takes for a source to transform a fraction $1/e$ of its energy into gravitational radiation. One can obtain a rough estimate from the following formula

$$\tau = \frac{E_{\text{kin}}}{L_{\text{GW}}} \sim \frac{1}{c} R \left(\frac{R}{R_{\text{Sch}}}\right)^3. \tag{2.36}$$

For example, for a non-radially oscillating neutron star with a mass of roughly $1.4\text{M}_\odot$ and a radius of 12Km, the damping time will be of the order of ~50msec. Also, by using formula (2.31), we get an estimate for the frequency of oscillation which is directly related to the frequency of the emitted gravitational waves, roughly 2kHz for the above case.

2.5 Rotating binary system

Among the most interesting sources of gravitational waves are binaries. The inspiralling of such systems, consisting of black holes or neutron stars, is, as we will discuss later, the most promising source for the gravitational wave detectors. Binary systems are also the sources of gravitational waves whose dynamics we understand the best. If we assume that the two bodies making up the binary lie in the $x-y$ plane and their orbits are circular (see Figure 3), then the only non-vanishing components of the quadrupole tensor are

$$Q_{xx} = -Q_{yy} = \frac{1}{2}\mu a^2 \cos 2\Omega t, \quad \text{and} \quad Q_{xy} = Q_{yx} = \frac{1}{2}\mu a^2 \sin 2\Omega t, \tag{2.37}$$

where Ω is the orbital angular velocity, $\mu = M_1 M_2 / M$ is the reduced mass of the system and $M = M_1 + M_2$ its total mass.

According to equation (2.29) the gravitational radiation luminosity of the system is

$$L^{\text{GW}} = \frac{32}{5}\frac{G}{c^5}\mu^2 a^4 \Omega^6 = \frac{32}{5}\frac{G^4}{c^5}\frac{M^3\mu^2}{a^5}, \tag{2.38}$$

where, in order to obtain the last part of the relation, we have used Kepler's third law, $\Omega^2 = GM/a^3$. As the gravitating system loses energy by emitting radiation, the distance between the two bodies shrinks at a rate

$$\frac{da}{dt} = -\frac{64}{5}\frac{G^3}{c^5}\frac{\mu M^2}{a^3}, \tag{2.39}$$

Figure 3: A system of two bodies orbiting around their common center of gravity. Binary systems are the best sources of gravitational waves.

and the orbital frequency increases accordingly ($\dot{T}/T = 1.5\dot{a}/a$). If, for example, the present separation of the two stars is a_0, then the binary system will coalesce after a time

$$\tau = \frac{5}{256} \frac{c^5}{G^3} \frac{a_0^4}{\mu M^4} \tag{2.40}$$

Finally, the amplitude of the gravitational waves is

$$h = 5 \times 10^{-22} \left(\frac{M}{2.8M_\odot}\right)^{2/3} \left(\frac{\mu}{0.7M_\odot}\right) \left(\frac{f}{100\text{Hz}}\right)^{2/3} \left(\frac{15\text{Mpc}}{r}\right). \tag{2.41}$$

In all these formulae we have assumed that the orbits are circular. In general, the orbits of the two bodies are approximately ellipses, but it has been shown that long before the coalescence of the two bodies, the orbits become circular, at least for long-lived binaries, due to gravitational radiation. Also, the amplitude of the emitted gravitational waves depends on the angle between the line of sight and the axis of angular momentum; formula (2.41) refers to an observer along the axis of the orbital angular momentum. The complete formula for the amplitude contains angular factors of order 1. The relative strength of the two polarizations depends on that angle as well.

If three or more detectors observe the same signal it is possible to reconstruct the full waveform and deduce many details of the orbit of the binary system.

As an example, we will provide some details of the well-studied pulsar PSR 1913+16 (the Hulse-Taylor pulsar), which is expected to coalesce after $\sim 3.5 \times 10^8$ years. The binary system is roughly 5kpc away from Earth, the masses of the two neutron stars are estimated to be $\sim$1.4M$_\odot$ each, and the present period of the system is $\sim$7h and 45min. The predicted rate of period change is $\dot{T} = -2.4 \times 10^{-12}$sec/sec, while the corresponding observed value is in excellent agreement with the predictions, i.e., $\dot{T} = (-2.30 \pm 0.22) \times 10^{-12}$sec/sec; finally the present amplitude of gravitational waves is of the order of $h \sim 10^{-23}$ at a frequency of $\sim 7 \times 10^{-5}$Hz.

Figure 4: A pair of masses joined by a spring can be viewed as the simplest conceivable detector.

3. DETECTION OF GRAVITATIONAL WAVES

The first attempt to detect gravitational waves was undertaken by the pioneer Joseph Weber during the early 1960s. He developed the first resonant mass detector and inspired many other physicists to build new detectors and to explore from a theoretical viewpoint possible cosmic sources of gravitational radiation.

A pair of masses joined by a spring can be viewed as the simplest conceivable detector; see Figure 4. In practice, a cylindrical massive metal bar or even a massive sphere is used instead of this simple system. When a gravitational wave hits such a device, it causes the bar to vibrate. By monitoring this vibration, we can reconstruct the true waveform. The next step, following the idea of resonant mass detectors, was the replacement of the spring by pendulums. In this new detector the motions induced by a passing-by gravitational wave would be detected by monitoring, via laser interferometry, the relative change in the distance of two freely suspended bodies. The use of interferometry is probably the most decisive step in our attempt to detect gravitational wave signals. In what follows, we will discuss both resonant bars and laser interferometric detectors.

Although the basic principle of such detectors is very simple, the sensitivity of detectors is limited by various sources of noise. The internal noise of the detectors can be Gaussian or non-Gaussian. The non-Gaussian noise may occur several times per day such as strain releases in the suspension systems which isolate the detector from any environmental mechanical source of noise, and the only way to remove this type of noise is via comparisons of the data streams from various detectors. The so-called Gaussian noise obeys the probability distribution of Gaussian statistics and can be characterized by a *spectral density* $S_n(f)$. The observed signal at the output of a detector consists of the true gravitational wave strain h and Gaussian noise. The optimal method to detect a gravitational wave signal leads to the following signal-to-noise ratio:

$$\left(\frac{S}{N}\right)^2_{\mathrm{opt}} = 2\int_0^\infty \frac{|\tilde{h}(f)|^2}{S_n(f)} df, \tag{3.1}$$

where $\tilde{h}(f)$ is the Fourier transform of the signal waveform. It is clear from this expression that the sensitivity of gravitational wave detectors is limited by noise.

3.1 Resonant detectors

Suppose that a gravitational wave propagating along the z axis with pure (+) polarization impinges on an idealized detector, two masses joined by a spring along the x axis as in Figure 4. We will assume that $h_{\mu\nu}$ describes the strain produced in the spacetime by the passing wave. We will try to calculate the amplitude of the oscillations induced on the spring detector by the wave and the amount of energy absorbed by this detector. The tidal force induced on the detector is given by equation (2.23), and the masses will move according to the following equation of motion:

$$\ddot{\xi} + \dot{\xi}/\tau + \omega_0^2 \xi = -\frac{1}{2}\omega^2 L h_+ e^{i\omega t}, \tag{3.2}$$

where ω_0 is the natural vibration frequency of our detector, τ is the damping time of the oscillator due to frictional forces, L is the separation between the two masses and ξ is the relative change in the distance of the two masses. The gravitational wave plays the role of the driving force for our ideal oscillator, and the solution to the above equation is

$$\xi = \frac{\frac{1}{2}\omega^2 L h_+ e^{i\omega t}}{\omega_0^2 - \omega^2 + i\omega/\tau} \tag{3.3}$$

If the frequency ω of the impinging wave is near the natural frequency ω_0 of the oscillator (near resonance) the detector is excited into large-amplitude motions and it rings like a bell. Actually, in the case of $\omega = \omega_0$, we get the maximum amplitude $\xi_{\max} = \omega_0 \tau L H_+/2$. Since the size of our detector L and the amplitude of the gravitational waves h_+ are fixed, large-amplitude motions can be achieved only by increasing the quality factor $Q = \omega_0 \tau$ of the detector. In practice, the frequency of the detector is fixed by its size and the only improvement we can get is by choosing the type of material so that long relaxation times are achieved.

The *cross section* is a measure of the interception ability of a detector. For the special case of resonance, the average cross section of our test detector, assuming any possible direction of the wave, is

$$\sigma = \frac{32\pi}{15}\frac{G}{c^3}\omega_0 Q M L^2. \tag{3.4}$$

This formula is general; it applies even if we replace our toy detector with a massive metal cylinder, for example Weber's first detector. That detector had the following characteristics: Mass M=1410 kg, length L=1.5 m, diameter 66 cm, resonant frequency ω_0=1660Hz, and quality factor $Q = \omega_0\tau = 2 \times 10^5$. For these values the calculated cross section is roughly $3 \times 10^{-19}\text{cm}^2$, which is quite small, and even worse, it can be reached only when the frequency of the impinging wave is very close to resonance frequency (the typical resonance width is usually of the order of 0.1-1 Hz).

In reality, the efficiency of a resonant bar detector depends on several other parameters. Here, we will discuss only the more fundamental ones. Assuming perfect

Figure 5: A graph of NAUTILUS in Frascati near Rome. Nautilus is probably the most sensitive resonant detector available.

isolation of the resonant bar detector from any external source of noise (acoustical, seismic, electromagnetic), the thermal noise is the only factor limiting our ability to detect gravitational waves. Thus, in order to detect a signal, the energy deposited by the gravitational wave every τ seconds should be larger than the energy kT due to thermal fluctuations. This leads to a formula for the minimum detectable energy flux of gravitational waves, which, following equation (2.25), leads into a minimum detectable strain amplitude

$$h_{\min} \leq \frac{1}{\omega_0 L Q}\sqrt{\frac{15kT}{M}} \tag{3.5}$$

For Weber's detector, at room temperature this yields a minimum detectable strain of the order of 10^{-20}. However, this estimate of the minimum sensitivity applies only to gravitational waves whose duration is at least as long as the damping time of the bar's vibrations and whose frequency perfectly matches the resonant frequency of the detector. For bursts or for periodic signals which are off-resonance (with regard to the detector) the sensitivity of a resonant bar detector decreases further by several orders of magnitude.

In reality, modern resonant bar detectors are quite complicated devices, consisting of a solid metallic cylinder weighing a few tons and suspended *in vacuo* by a cable that is wrapped under its center of gravity (Figure 5). This suspension system protects the antenna from external mechanical shocks. The whole system is cooled down to temperatures of a few kelvins or even millikelvins. To monitor the vibrations of the bar, piezoelectric transducers (or the more modern capacitive ones) are attached to the bar. The transducers convert the bar's mechanical energy into electrical energy. The signal is amplified by an ultra-low-frequency amplifier, by using a device called a SQUID (Super-conducting QUantum Interference Device) before it becomes available for data analysis. Transducers and amplifiers of electronic signals require careful design to achieve low noise combined with adequate signal transfer.

The above description of the resonant bar detectors shows that, in order to achieve high sensitivity, one has to:

1. *Create more massive antennas.* Today, most of the antennas are about 50% more massive than the early Webers antenna. There are studies and research plans for future construction of spherical antennae weighing up to tenths of tons.

2. *Obtain higher quality factor Q.* Modern antennas generally use aluminum alloy 5056 ($Q \sim 4 \times 10^7$); although niobium (which is used in the NIOBE detector) is even better ($Q \sim 10^8$) but much more expensive. Silicon or sapphire bars would enhance the quality factor even more, but experimenters must first find a way to produce large single pieces of these crystals.

3. *Lower the temperature of the antenna as much as possible.* Advanced cryogenic techniques have been used and the resonant bar detectors are probably the coolest places in the Universe. Typical cooling temperatures for the most advanced antennae are below the temperature of liquid helium.

4. *Achieve strong coupling between the antenna and the electronics and low electrical noise.* The bar detectors include the best available technology related to transducers and integrate the most recent advances in SQUID technology.

Since the early 1990s, a number of resonant bar detectors have been in nearly continuous operation in several places around the world. They have achieved sensitivities of a few times 10^{-21}, but still there has been no clear evidence of gravitational wave detection. As we will discuss later, they will have a good chance of detecting a gravitational wave signal from a supernova explosion in our galaxy, although, this is a rather rare event (2-5 per century).

The technical specifications of the most sensitive cryogenic bar detectors in operation are as follows:

- ALLEGRO (Baton Rouge, USA) Mass 2296 Kg (Aluminium 5056), length 3 m, bar temperature 4.2 K, mode frequency 896 Hz.

- AURIGA (Legrano, Italy) Mass 2230 Kg (Aluminium 5056), length 2.9 m, bar temperature 0.2 K, mode frequency 913 Hz.

- EXPLORER (CERN, Switzerland) Mass 2270 Kg (Aluminium 5056), length 3 m, bar temperature 2.6K, mode frequency 906Hz.

- NAUTILUS (Frascati, Italy) Mass 2260 Kg (Aluminium 5056), length 3 m, bar temperature 0.1 K, mode frequency 908 Hz.
- NIOBE (Perth, Australia) Mass 1500 Kg (Niobium), length 1.5 m, bar temperature 5K, mode frequency 695Hz.

Also, there are plans for construction of *massive spherical resonant detectors*, the advantages of which will be their high mass, their broader sensitivity (up to 100-200 Hz) and their omnidirectional sensitivity. In a spherical detector, five modes at a time will be excited, which is equivalent to five independent detectors oriented in different ways. This offers the opportunity, in the case of detection, to obtain direct information about the polarization of the wave and the direction to the source. A prototype spherical detector is already in operation in Leiden, Netherlands (mini-GRAIL)[7] with 1 m diameter and mode frequency $\sim$3.2 kHz. Two other detectors of the same size are under construction in Italy (Sfera) and Brazil (Shenberg).

3.2 Beam detectors

3.2.1 Laser Interferometers

A laser interferometer is an alternative gravitational wave detector that offers the possibility of very high sensitivities over a broad frequency band. Originally, the idea was to construct a new type of resonant detector with much larger dimensions. As one can realize from the relations (3.4) and (3.5), the longer the resonant detector is the more sensitive it becomes. One could then try to measure the relative change in the distance of two well-separated masses by monitoring their separation via a laser beam that continuously bounces back and forth between them. (This technique is actually used in searching for gravitational waves by using the so-called Doppler tracking technique, where a distant interplanetary spacecraft is monitored from earth through a microwave tracking link; the earth and spacecraft act as free particles.) Soon, it was realized that it is much easier to use laser light to measure relative changes in the lengths of two perpendicular arms; see Figure 6. Gravitational waves that are propagating perpendicular to the plane of the interferometer will increase the length of one arm of the interferometer, and at the same time will shorten the other arm, and vice versa. This technique of monitoring the waves is based on Michelson interferometry. L-shaped interferometers are particularly suited to the detection of gravitational waves due to their quadrupolar nature.

Figure 6 shows a schematic design of a Michelson interferometer; the three masses M_0, M_1 and M_2 are freely suspended. Note that the resonant frequencies of these pendulums should be much smaller than the frequencies of the waves that we are supposed to detect since the pendulums are supposed to behave like free masses. Mirrors are attached to M_1 and M_2 and the mirror attached on mass M_0 splits the light (beam splitter) into two perpendicular directions. The light is reflected on the

Figure 6: A schematic design of a Michelson interferometer.

two corner mirrors and returns back to the beam splitter. The splitter now half-transmits and half-reflects each one of the beams. One part of each beam goes back to the laser, while the other parts are combined to reach the photodetector where the fringe pattern is monitored. If a gravitational wave slightly changes the lengths of the two arms, the fringe pattern will change, and so by monitoring the changes of the fringe pattern one can measure the changes in the arm lengths and consequently monitor the incoming gravitational radiation [1].

Let us consider an impinging gravitational wave with amplitude h and $(+)$ polarization, propagating perpendicular to the plane of the detector. We will further assume that the frequency is much higher than the resonant frequency of the pendulums and the wavelength is much longer than the arm length of our detector. Such a wave will generate a change of $\Delta L \sim hL/2$ in the arm length along the x-direction and an opposite change in the arm length along the y-direction, according to equations (2.19) and (2.20). The total difference in length between the two arms will be

$$\frac{\Delta L}{L} \sim h. \tag{3.6}$$

For a gravitational wave with amplitude $h \sim 10^{-21}$ and detector arm- length 4 km (such as LIGO), this will induce a change in the arm-length of about $\Delta L \sim 10^{-16}$. In the general case, when a gravitational wave with arbitrary polarization impinges on the detector from a random direction, the above formula will be modified by some angular coefficients of order 1.

[1] In practice, things are arranged so that when there is no actual change in the arms, all light that returns on the beam splitter from the corner mirrors is sent back into the laser, and only if there is some motion of the masses there is an output at the photodetector

If the light bounces a few times between the mirrors before it is collected in the photodiode, the effective arm length of the detector is increased considerably, and the measured variations of the arm lengths will be increased accordingly. This is a quite efficient procedure for making the arm length longer. For example, a gravitational wave at a frequency of 100 Hz has a wavelength of 3000 Km, and if we assume 100 bounces of the laser beam in the arms of the detector, the effective arm-length of the detector is 100 times larger than the actual arm-length, but still this is 10 times smaller than the wavelength of the incoming wave. The optical cavity that is created between the mirrors of the detector is known as a Fabry-Perot cavity and is used in modern interferometers.

In the remainder of this paragraph we will focus on the Gaussian sources of noise and their expected influence on the sensitivity of laser intereferometers.

a. Photon shot noise. When a gravitational wave produces a change ΔL in the arm-length, the phase difference between the two light beams changes by an amount $\Delta\phi = 2b\Delta L/\tilde{\lambda}$, where $\tilde{\lambda}$ is the reduced wavelength of the laser light ($\sim 10^{-8}$cm) and b is the number of bounces of the light in each arm. It is expected that a detectable gravitational wave will produce a phase shift of the order of 10^{-9}rad. The precision of the measurements, though, is ultimately restricted by fluctuations in the fringe pattern due to fluctuations in the number of detected photons. The number of photons that reach the detector is proportional to the intensity of the laser beam and can be estimated via the relation $N = N_0 \sin^2(\Delta\phi/2)$, where N_0 is the number of photons that the laser supplies and N is the number of detected photons. Inversion of this equation leads to an estimation of the relative change of the arm lengths ΔL by measuring the number of the emerging photons N. However, there are statistical fluctuations in the population of photons, which are proportional to the square root of the number of photons. This implies an uncertainty in the measurement of the arm length

$$\delta(\Delta L) \sim \frac{\tilde{\lambda}}{2b\sqrt{N_0}} \tag{3.7}$$

Thus, the minimum gravitational wave amplitude that we can measure is

$$h_{\min} = \frac{\delta(\Delta L)}{L} = \frac{\Delta L}{L} \sim \frac{\tilde{\lambda}}{bLN_0^{1/2}} \sim \frac{1}{bL}\left(\frac{\hbar c\tilde{\lambda}}{\tau I_0}\right)^{1/2}, \tag{3.8}$$

where I_0 is the intensity of the laser light ($\sim$5-10 W) and τ is the duration of the measurement. This limitation in the detector's sensitivity due to the photon counting uncertainty is known as *photon shot noise*. For a typical laser interferometer the photon shot noise is the dominant source of noise for frequencies above 200 Hz, while its power spectral density $S_n(f)$ for frequencies 100-200Hz is of the order of $\sim 3\times 10^{-23}\sqrt{\mathrm{Hz}}$.

b. Radiation pressure noise: According to formula (3.8), the sensitivity of a detector can be increased by increasing the intensity of the laser. However, a very powerful laser produces a large radiation pressure on the mirrors. Then an uncertainty in the measurement of the momentum deposited on the mirrors leads to a proportional uncertainty in the position of the mirrors or, equivalently, in the measured change in the arm-lengths. Then, the minimum detectable strain is limited by

$$h_{\rm min} \sim \frac{\tau}{m}\frac{b}{L}\left(\frac{\tau\hbar I_0}{c\tilde{\lambda}}\right)^{1/2} , \tag{3.9}$$

where m is the mass of the mirrors.

As we have seen, the photon shot noise decreases as the laser power increases, while the inverse is true for the noise due to radiation pressure fluctuations. If we try to minimize these two types of noise with respect to the laser power, we get a minimum detectable strain for the optimal power via the very simple relation

$$h_{\rm min} \sim \frac{1}{L}\left(\frac{\tau\hbar}{m}\right)^{1/2} \tag{3.10}$$

which for the LIGO detector (where the mass of the mirrors is $\sim$100 kg and the arm length is 4 km), for observation time of 1 ms, gives $h_{\rm min} \approx 10^{-23}$.

c. Quantum limit. An additional source of uncertainty in the measurements is set by Heisenberg's principle, which says that the knowledge of the position and the momentum of a body is restricted from the relation $\Delta x \cdot \Delta p \geq \hbar$. For an observation that lasts some time τ, the smallest measurable displacement of a mirror of mass m is ΔL; assuming that the momentum uncertainty is $\Delta p \approx m \cdot \Delta L/\tau$, we get a minimum detectable strain due to quantum uncertainties

$$h_{\rm min} = \frac{\Delta L}{L} \sim \frac{1}{L}\left(\frac{\tau\hbar}{m}\right)^{1/2} . \tag{3.11}$$

Surprisingly, this is identical to the optimal limit that we calculated earlier for the other two types of noise. The standard quantum limit does set a fundamental limit on the sensitivity of beam detectors. An interesting feature of the quantum limit is that it depends only on a single parameter, the mass of the mirrors.

d. Seismic noise. At frequencies below 60 Hz, the noise in the interferometers is dominated by seismic noise. This noise is due to geological activity of the earth, and human sources, e.g., traffic and explosions. The vibrations of the ground couple to the mirrors via the wire suspensions which support them. This effect is strongly suppressed by properly designed suspension systems. Still, seismic noise is very difficult to eliminate at frequencies below 5-10 Hz.

e. Residual gas-phase noise. The statistical fluctuations of the residual gas density induce a fluctuation of the refraction index and consequently of the monitored phase shift. Hence, the residual gas pressure through which the laser beams travel should be extremely low. For this reason the laser beams are enclosed in pipes over their entire length. Inside the pipes a high vacuum of the order of 10^{-9} Torr guarantees elimination of this type of noise.

Prototype laser interferometric detectors have been already constructed in the USA, Germany and UK more than 15 years ago. These detectors have an arm length of a few tens of meters and they have achieved sensitivities of the order of $h \sim 10^{-19}$. The construction of the new generation of laser interferometric detectors is near completion and some of them have already collected data. It is expected that in a year or so they will be in full operation. The US project named LIGO [8](Laser Interferometer Gravitational Observatory) consists of two detectors with arm length of 4 Km, one in Hanford, Washington, and one in Livingston, Louisiana. The detector in Hanford includes, in the same vacuum system, a second detector with arm length of 2 km. The detectors are already in operation and they have recently achieved the designed sensitivity.

The Italian/French EGO (VIRGO) detector [9] of arm-length 3 km at Cascina near Pisa is designed to have better sensitivity at lower frequencies is about to commence operation next year. GEO600 is the German/British detector build in Hannover [10]. It has 600 m arm length and is collecting data in collaboration with LIGO [11]. The TAMA300[12] detector in Tokyo has arm length of 300 m and it was the first major interferometric detector in operation. There are already plans for improving the sensitivities of all the above detectors and the construction of new interferometers in the near future.

3.2.2 Space Detectors

Both bars and laser interferometers are high-frequency detectors, but there are a number of interesting gravitational waves sources which emit signals at lower frequencies. The seismic noise provides an insurmountable obstacle in any earth-based experiment and the only way to surpass this barrier is to fly a laser interferometer in space. LISA [13] (Laser Interferometer Space Antenna) is such a system. It has been proposed by European and American scientists and has been adopted by ESA (European Space Agency) as a cornerstone mission, while recently NASA joined the effort and the launch date is expected to be around 2011.

LISA will consist of three identical drag-free spacecrafts forming an equilateral triangle with one spacecrafts at each vertex (Figure 7). The distance between the two vertices (the arm length) is 5×10^6 km. The spacecrafts will be placed into the same heliocentric orbit as earth, but about 20^o behind earth. The equilateral triangle will be inclined at an angle of 60^o with respect to Earth's orbital plane. The three spacecraft would track each other optically by using laser beams. Because of the diffraction losses it is not feasible to reflect the beams back and forth as is done

Figure 7: A schematic design of the space interferometer LISA.

with LIGO. Instead, each spacecraft will have its own laser. The lasers will be phase locked to each other, achieving the same kind of phase coherence as LIGO does with mirrors. The configuration will function as three, partially independent and partially redundant, gravitational wave interferometers.

At frequencies $f \geq 10^{-3}$, LISA's noise is mainly due to photon shot noise. The sensitivity curve steepens at $f \sim 3 \times 10^{-2}$ Hz because at larger frequencies the gravitational waves period is shorter than the round-trip light travel time in each arm. For $f \leq \times 10^{-2}$ Hz, the noise is due to buffeting-induced random motions of the spacecraft, and cannot be removed by the drag- compensation system. LISA's sensitivity is roughly the same as that of LIGO, but at 10^5 times lower frequency. Since the gravitational waves energy flux scales as $F \sim f^2 h^2$, this corresponds to 10^{10} times better energy sensitivity than LIGO.

3.2.3 Satellite Tracking

The Doppler delay of communication signals between the earth-based stations and spacecraft underlies another type of gravitational wave detector. A radio signal of frequency f_0 is transmitted to a spacecraft and is coherently transported back to earth, where it is received and its frequency is measured with a highly stable clock (typically a hydrogen maser). The relative change $\Delta f / f_0$ as function of time is monitored. A gravitational wave propagating through the solar system causes small perturbations in $\Delta f / f_0$. The relative shift in the frequency of the signals is proportional to the amplitude of gravitational waves. With this technique, broad-band searches are possible in the millihertz frequency band, and thanks to the very stable atomic clocks, it is possible to achieve sensitivities of order $h_{\min} \sim 10^{-13} - 10^{-15}$. Noise sources that affect the sensitivity of Doppler tracking experiments can be divided into two broad classes a) instrumental and b) related to propagation. At the high-frequency end of the band accessible to Doppler tracking, thermal noise dominates over all other noise sources, typically at about 0.1 Hz. Among all other sources of instrumental noise (transmitter and receiver, mechanical stability of the antenna, stability of the spacecraft etc), clock noise has been shown to be the most important instrumental source of frequency fluctuations. The propagation noise is due to fluctuations in the index of refraction of the troposphere, ionosphere, and the interplanetary solar plasma. Both NASA and ESA have performed such measurements and there is a continued effort in this direction.

3.2.4 Pulsar Timing

Pulsars are extremely stable clocks and by measuring irregularities in their pulses we expect to set upper limits on the background gravitational waves (see next section). If an observer monitors simultaneously two or more pulsars, the correlation of their signals could be used to detect gravitational waves. Since such observation requires timescales of the order of 1 year, this means that the waves have to be of extremely low frequencies.

4. ASTRONOMICAL SOURCES OF GRAV. WAVES

The new generation of gravitational wave detectors (LIGO, VIRGO) have very good chances of detecting gravitational waves, but until these expectations are fulfilled, we can only make educated guesses as to the possible astronomical sources of gravitational waves. The detectability of these sources depends on three parameters: their intrinsic gravitational wave luminosity, their event rate, and their distance from the Earth. The luminosity can be approximately estimated via the quadrupole formula that we discussed earlier. Even though there are certain restrictions in its applicability (weak field, slow motion), it provides a very good order-of-magnitude estimate for the expected gravitational wave flux on Earth. The rate, at which various events with high luminosity in gravitational waves take place is extrapolated from astronomical observations in the electromagnetic spectrum. Still, there might be a number of *gravitationally luminous sources*, for example binary black holes, for which we have no direct observations in the electromagnetic spectrum. Finally, the amplitude of gravitational wave signals decreases as one over the distance to the source. Thus, a signal from a supernova explosion might be clearly detectable if the event takes place in our galaxy (2-3 events per century), but it is highly unlikely to be detected if the supernova explosion occurs at far greater distances, of order 100 Mpc, where the event rate is high and at least a few events per day take place. All three factors have to be taken into account when discussing sources of gravitational waves.

It was mentioned earlier that the frequency of gravitational waves is proportional to the square root of the mean density of the emitting system; this is approximately true for any gravitating system. For example, neutron stars usually have masses around 1.4$M_\odot$ and radii in the order of 10 km; thus if we use these numbers in the relation $f \sim \sqrt{GM/R^3}$ we find that an oscillating neutron star will emit gravitational waves primarily at frequencies of 2-3 kHz. By analogy, a black-hole with a mass $\sim 100 M_\odot$, will have a radius of ∼300 km and the natural oscillation frequency will be $\sim$ 100 Hz. Finally, for a binary system Kepler's law (see section 2.5) provides a direct and accurate estimation of the frequency of the emitted gravitational waves. For two 1.4$M_\odot$ neutron stars orbiting around each other at a distance of 160 Km, Keplers law predicts an orbital frequency of 50 Hz, which leads to an observed gravitational wave frequency of 100 Hz.

4.1 Radiation from gravitational collapse

Type II supernovae are associated with the core collapse of a massive star together with a shock-driven expansion of a luminous shell which leaves behind a rapidly rotating neutron star or a black hole, if the core has mass of $> 2-3M_\odot$. The typical signal from such an explosion is broadband and peaked at around 1 kHz. Detection of such a signal was the goal of detector development over the last three decades. However, we still know little about the efficiency with which this process produces gravitational waves. For example, an exactly spherical collapse will not produce any gravitational radiation at all. The key question is what is the kinetic energy of the nonspherical motions, since the gravitational wave amplitude is proportional to this [equation (2.30). After 30 years of theoretical and numerical attempts to simulate gravitational collapse, there is still no great progress in understanding the efficiency of this process in producing gravitational waves. For a conservative estimate of the energy in non-spherical motions during the collapse, relation (2.32) leads to events of an amplitude detectable in our galaxy, even by bar detectors. The next generation of laser interferometers would be able to detect such signals from Virgo cluster at a rate of a few events per month.

The main source for deviations from spherical or axial symmetry during the collapse is the angular momentum. During the contraction phase, the angular momentum is conserved, and the star spins up to rotational periods of the order of 1 msec. In this case, consequent processes with large luminosity might take place in this newly born neutron star. A number of instabilities, such as the so-called *bar mode instability* and the *r-mode instability*, may occur which radiate copious amounts of gravitational radiation immediately after the initial burst. Gravitational wave signals from these rotationally induced stellar instabilities are detectable from sources in our galaxy and are marginally detectable if the event takes place in the nearby cluster of about 2500 galaxies, the Virgo cluster, 15 Mpc away from the Earth. Additionally, there will be weaker but extremely useful signals due to subsequent oscillations of the neutron star; *f*, *p* and *w* modes are some of the main patterns of oscillations (normal modes) of the neutron star that observers might search for.

These modes have been studied in detail and once detected in the signal, they would provide a sensitive probe of the neutron star structure and its supranuclear equation of state. Detectors with high sensitivity in the kHz band will be needed in order to fully develop this so-called gravitational wave asteroseismology.

If the collapsing central core is unable to drive off its surrounding envelope, then the collapse continues and finally a black hole forms. In this case the instabilities and oscillations that we discussed above are absent and the newly formed black hole radiates away, within a few milliseconds, any deviations from axisymmetry and ends up as a rotating or *Kerr* black hole. The characteristic oscillations of black holes (normal modes) are well studied, and this unique ringing down of a black hole could be used as a direct probe of their existence. The frequency of the signal is inversely proportional to the black hole mass. For example, it has been stated earlier that a $100\mathrm{M}_\odot$ black hole will oscillate at a frequency of $\sim$100 Hz (an ideal source for LIGO), while a supermassive one with mass $10^7 M_\odot$, which might be excited by an infalling star, will ring down at a frequency of 10^{-3} Hz (an ideal source for LISA). The analysis of such a signal should reveal directly the two parameters that characterize any (uncharged) black hole; namely its mass and angular momentum.

4.2 Radiation from binary systems

Binary systems are the best sources of gravitational waves because they emit copious amounts or gravitational radiation, and for a given system we know exactly what is the amplitude and frequency of the gravitational waves in terms of the masses of the two bodies and their separation (see section 2.5). If a binary system emits detectable gravitational radiation in the bandwidth of our detectors, we can easily identify the parameters of the system. According to the formulas of section 2.5, the observed frequency change will be

$$\dot{f} \sim f^{11/3} \mathcal{M}^{5/3}, \tag{4.1}$$

and the corresponding amplitude will be

$$h \sim \frac{\mathcal{M}^{5/3} f^{2/3}}{r} = \frac{\dot{f}}{r f^3}, \tag{4.2}$$

where $\mathcal{M}^{5/3} = \mu M^{2/3}$ is a combination of the total and reduced mass of the system, called *chirp mass.* Since both frequency f and its rate of change $\dot{f}$ are measurable quantities, we can immediately compute the chirp mass (from the first relation), thus obtaining a measure of the masses involved. The second relation provides a direct estimate of the distance of the source. These relations have been derived using the Newtonian theory to describe the orbit of the system and the quadrupole formula for the emission of gravitational waves. Post-Newtonian theory inclusion of the most important relativistic corrections in the description of the orbit can provide more accurate estimates of the individual masses of the components of the binary system.

When analyzing the data of periodic signals the *effective amplitude* is not the amplitude of the signal alone but $h_c = \sqrt{n} \cdot h$, where n is the number of cycles of the signal within the frequency range where the detector is sensitive. A system consisting of two typical neutron stars will be detectable by LIGO when the frequency of the gravitational waves is ~10Hz until the final coalescence around 1000Hz. This process will last for about 15 min and the total number of observed cycles will be of the order of 10^4, which leads to an enhancement of the detectability by a factor 100. Binary neutron star systems and binary black hole systems with masses of the order of $50 M_\odot$ are the primary sources for LIGO. Given the anticipated sensitivity of LIGO, binary black hole systems are the most promising sources and could be detected as far as 200 Mpc away. The event rate with the present estimated sensitivity of LIGO is probably a few events per year, but future improvement of detector sensitivity (the LIGO II phase) could lead to the detection of at least one event per month. Supermassive black hole systems of a few million solar masses are the primary source for LISA. These binary systems are rare, but due to the huge amount of energy released, they should be detectable from as far as the boundaries of the observable universe.

4.3 Radiation from spinning neutron stars

A perfectly axisymmetric rotating body does not emit any gravitational radiation. Neutron stars are axisymmetric configurations, but small deviations cannot be ruled out. Irregularities in the crust (perhaps imprinted at the time of crust formation), strains that have built up as the stars have spun down, off-axis magnetic fields, and/or accretion could distort the axisymmetry. A bump that might be created at the surface of a neutron star spinning with frequency f will produce gravitational waves at a frequency of $2f$ and such a neutron star will be a weak but continuous and almost monochromatic source of gravitational waves. The radiated energy comes at the expense of the rotational energy of the star, which leads to a spin down of the star. If gravitational wave emission contributes considerably to the observed spin down of pulsars, then we can estimate the amount of the emitted energy. The corresponding amplitude of gravitational waves from nearby pulsars (a few kpc away) is of the order of $h \sim 10^{-25} - 10^{-26}$, which is extremely small. If we accumulate data for sufficiently long time, e.g., 1 month, then the effective amplitude, which increases as the square root of the number of cycles, could easily go up to the order of $h_c \sim 10^{-22}$. We must admit that at present we are extremely ignorant of the degree of asymmetry in rotating neutron stars, and these estimates are probably very optimistic. On the other hand, if we do not observe gravitational radiation from a given pulsar we can place a constraint on the amount of non axisymmetry of the star.

4.4 Cosmological gravitational waves

One of the strongest pieces of evidence in favor of the Big Bang scenario is the 2.7 K cosmic microwave background radiation. This thermal radiation first bathed the universe around 380,000 years after the Big Bang. By contrast, the gravitational radiation background anticipated by theorists was produced at Planck times, i.e., at 10^{-32} sec or earlier after the Big Bang. Such gravitational waves have travelled almost unimpeded through the universe since they were generated. The observation of cosmological gravitational waves will be one of the most important contributions of gravitational wave astronomy. These primordial gravitational waves will be, in a sense, another source of noise for our detectors and so they will have to be much stronger than any other internal detector noise in order to be detected. Otherwise, confidence in detecting such primordial gravitational waves could be gained by using a system of two detectors and cross-correlating their outputs. The two LIGO detectors are well placed for such a correlation.

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