

NEUTRON STARS: ROTATIONAL & MAGNETIC FIELD INSTABILITIES

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Abstract. *In this short review we present recent results on the dynamics of neutron stars and their magnetic fields. We discuss the progress that has been done in understanding the rotational instabilities with emphasis to the one due to the f -mode, the possibility of using gravitational wave detection in constraining the parameters of neutron stars and revealing the equation of state as well as the detectability of gravitational waves produced during the unstable phase of the life of a neutron star. In addition we study the dynamics of extremely strong magnetic fields that have been observed in a class of neutron stars (magnetars). Magnetic fields of that strength are responsible for highly energetic phenomena (giant flares) and we demonstrate that the analysis of the emitted electromagnetic radiation can lead in constraining the parameters of neutron stars. Furthermore, we present our results from the study of such violent phenomena in association with the emission of gravitational radiation*

1 INTRODUCTION

Neutron stars are among the most interesting astronomical objects since their understanding involves all branches of physics such as general relativity, quantum mechanics, quantum chromodynamics, nuclear physics at supra nuclear densities, elasticity theory and magnetohydrodynamics (MHD). The observations over the last three decades in x-rays, gamma rays and radio have provided enormous sets of data that astrophysicists and nuclear physicists are analyzing and modeling. The detection of gravitational waves in the next 3-5 years will provide an alternative way in studying the universe and obviously these highly relativistic objects. In a series of articles over the last five years we made an effort in modeling the observations in the electromagnetic spectrum in an attempt to understand the dynamics of the phenomena (giant flares) and the magnetic field instabilities involved [59, 98, 100, 58]. Our aim was also to use the data from existing observations in the electromagnetic spectrum in order to constraint the parameters of the neutron stars [86, 94, 87, 80, 85, 27, 46, 28, 29] our ultimate plan is to provide waveforms and empirical formulae in order to help in the detection of the events and the analysis of the forthcoming data by the gravitational wave detectors via our studies of the instabilities induced by rotation, the expected gravitational wave spectra and the amplitudes and forms of their signal [72, 42, 51, 43, 55, 99, 52, 44, 41, 70].

2 Oscillations and Instabilities of Neutron Stars

During the birth of a proto-neutron star or the merging of two older compact stars, violent non-radial oscillations may be excited, resulting in the emission of significant amounts of gravitational radiation [8]. The detection of gravitational waves from oscillating neutron stars will allow the study of their interior, in the same way as helioseismology provides information about the interior of the Sun. It is expected that the identification of specific pulsation frequencies in the observational data will reveal the true properties of matter at densities that cannot be probed today by any other experiment. Thus, there is significant interest in this problem also from the nuclear-physics community since the discovery of the true properties of matter at extremely high energies will allow significant advances in theoretical physics and possibly new discoveries and applications will follow. The recent works on gravitational wave asteroseismology [42, 43, 55, 44], provided new empirical relationships for mode-frequencies and damping times of the quadrupolar f -mode for rapidly rotating neutron stars, extending earlier studies for the non-rotating case [10, 11, 53].

As it has been originally suggested by Chandrasekhar [23] and verified by Friedman & Schutz [38] certain non-axisymmetric pulsation modes may grow exponentially in rotating stars; this is due to the emission of gravitational waves and is called CFS instability. Exploring this type of instability in rapidly rotating stars turned out to be very difficult. In linear perturbation theory for example, rapid rotation was never treated properly until recently; almost all formulations of the relevant perturbation equations were prone to numerical instabilities either at the surface or along the rotation axis of the

neutron star. Thus, it was not surprising that the first results for the oscillations of rapidly rotating stars were derived using evolutions of the non-linear equations [89, 34, 50]. Still all these studies were purely axisymmetric and thus the effects of rotation on the spectra was present only for very high rotation rates. Rotational instabilities are driven by non-axisymmetric modes and thus these first 2D calculations were not of much use for their study. The only non-linear 3D study of the non-axisymmetric oscillations is by Zink et. al. [99].

2.1 Gravitational Wave Asteroseismology

The original suggestions about gravitational wave asteroseismology has been supported by many complementary works which studied specific features of oscillation spectra for various compact objects, such as typical neutron stars [15, 88, 16, 90, 91, 26, 96, 60], but also for strange [81, 82, 17] or superfluid stars [6, 7]. More recently, it has also been suggested that one may use asteroseismology to find the imprints of scalar or even vector components of gravity [83, 84, 79, 78] as well as the possibility for pressure anisotropies [35]. It should be noted here that all previous studies have been performed for non-rotating relativistic stars. The treatment of rotation was always a problem in general relativity and thus the majority of the studies for the oscillation spectra of fast-rotating compact stars was done mainly in Newtonian theory which gives only qualitative answers.

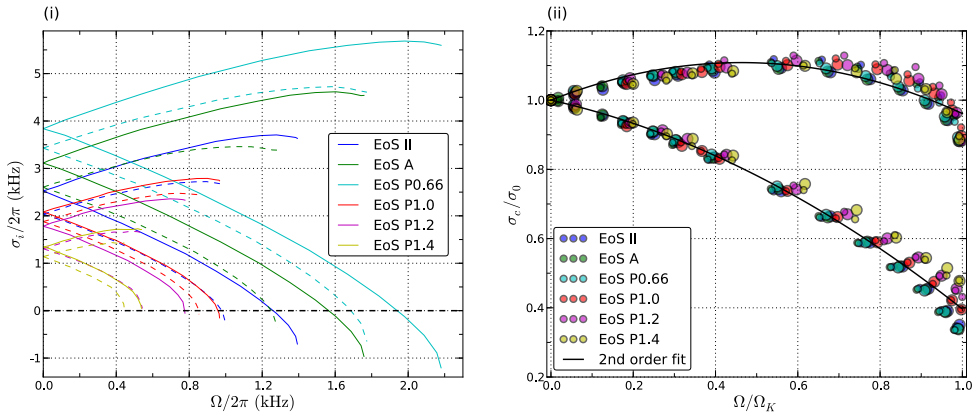


Figure 1: Co- and counterrotating branches of the different polytropic EoS. *Panel (i)*: Mode frequencies in the inertial frame; the solid line of each EoS depicts the more compact sequence while the dashed line traces the less compact configurations. *Panel (ii)*: Normalized mode frequencies and fitting curve in the comoving frame; the larger circles represent again the more compact models while the small circles stand for the less compact ones.

In the last four years there was significant progress in the study of non-axisymmetric perturbations of rapidly rotating neutron stars. For the first time it was possible to calculate in GR the oscillation spectra of fast rotating relativistic stars by using the linearized form of the fluid equations in the so-called Cowling approximation. Thus the effect of fast rotation on f - and r -modes has been demonstrated while the critical points for the onset of the f -mode (CFS) instability have been derived [42]. In addition it has been demonstrated that there is a way to derive empirical relations connecting the oscillation frequencies with the rotation of the stars [44]. These studies has been recently extended to g -modes [43] and to differentially rotating neutron stars [55]. For non-axisymmetric oscillations, the frequencies of co- and counter-rotating modes split in the presence of rotation and there is a critical rotation rate for which these frequencies vanish for an inertial observer, see Figure 1.

Based on the data of our simulations, we proposed the following relationships [44]

$$\frac{\sigma_c^s}{\sigma_0} = 1.0 - 0.27 \left(\frac{\Omega}{\Omega_K} \right) - 0.34 \left(\frac{\Omega}{\Omega_K} \right)^2 \quad \text{for the always stable } m = -2 \quad (1)$$

$$\frac{\sigma_c^u}{\sigma_0} = 1.0 + 0.47 \left(\frac{\Omega}{\Omega_K} \right) - 0.51 \left(\frac{\Omega}{\Omega_K} \right)^2 \quad \text{for the potentially unstable } m = 2 \quad (2)$$

where σ_0 is the frequency in the nonrotating limit and Ω_K represents the Kepler-limit.

Similar relations can be found for the damping/growth times of the two f -mode branches. What is needed for the potentially unstable branch is a quantity that also changes its sign when a mode becomes prone to the CFS-instability and which is a monotonic function of the rotation rate. The f -mode frequency in the inertial frame σ_i exactly conforms to these requirements. Since $dE/dt \sim \sigma^6$ and for any oscillation $E \sim \sigma^2$, we can also make an estimation on how the damping times depend on the mode frequency thus we expect $\frac{1}{\tau} \sim \frac{dE/dt}{E} \sim \sigma^4$.

Using least-square methods fit we finally yield [44], see also Figure

$$\frac{\tau_0}{\tau} = \text{sgn}(\sigma_i^u) 0.256 \left(\frac{\sigma_i^u}{\sigma_0} \right)^4 \times \left[1 + 0.048 \left(\frac{\sigma_i^u}{\sigma_0} \right) + 0.359 \left(\frac{\sigma_i^u}{\sigma_0} \right)^2 \right]^4, \quad (3)$$

where $\text{sgn}(x)$ is the sign function. For the corotating branch, using a somehow different fitting we get

$$\frac{\tau}{\tau_0} = -0.656 \times \left[1 - 7.33 \left(\frac{\sigma_c^s}{\sigma_0} \right) + 14.07 \left(\frac{\sigma_c^s}{\sigma_0} \right)^2 - 9.26 \left(\frac{\sigma_c^s}{\sigma_0} \right)^3 \right]. \quad (4)$$

Apart for the above relations for the frequency and the damping additional, model-independent relation for the frequency σ_0 and the damping time τ_0 of nonrotating configurations is needed, this information can be found in [11, 53].

The above relations demonstrate how one can do gravitational wave asteroseismology by using the frequencies and possibly the damping/growth times of the emitted

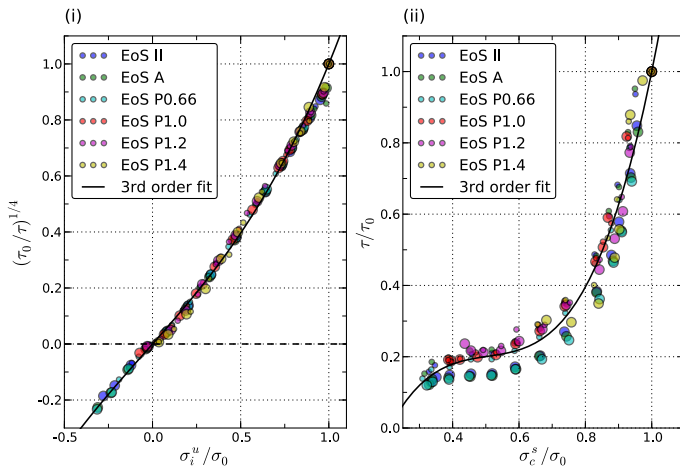


Figure 2: Model-independent relations for the damping time. Larger circles represent the more compact models while small circles stand for the less compact configurations of each EoS. *Panel (i)*: Damping times for the counterrotating branch, σ_i^u is the mode frequency in the inertial frame. *Panel (ii)*: Damping times for the corotating branch, σ_c^s is the mode frequency in the comoving frame.

waves from oscillating and rapidly rotating relativistic stars. In a realistic situation when an f -mode will be excited, it will be possible to detect the signal at least from galactic sources if the mode is CFS-stable and at least from sources in the Virgo cluster if it is unstable [56, 68, 77, 52]. This will be possible with the sensitivity of the advanced Virgo and LIGO detectors [4] and probably even more feasible with the next generation gravitational wave telescopes such as ET (Einstein Telescope) [73, 8].

2.2 Instabilities & Gravitational Waves

Among the several modes which can be driven unstable, the f - and r -modes are the most important due to their relatively short growth timescale. The r -mode instability has attracted more attention so far as it is CFS unstable at any rotation rate and may grow in about few tens of seconds in rapidly rotating stars. There is in fact an extensive literature which studied the r -mode instability for various stellar models and considered it as a possible candidate for limiting the star's rotation below the Kepler velocity (see for instance [12, 41] and references therein).

To date the evolution of the f -mode instability has been studied only for the $l = m = 2$ case with non-linear dynamical simulations in Newtonian gravity. These works used both ellipsoidal configurations [33, 56] and compressible stellar models with uniform [68] and differential rotation [77]. In these non-linear simulations, the effects of viscosity has been neglected and the bar mode instability is driven by a gravitational radia-

tion reaction term which has been incorporated in the Newtonian dynamical equations. In particular, the strength of this Post-Newtonian term has been artificially increased to shorten the instability evolution and make feasible its analysis within the simulation time. For typical neutron stars parameter these non-linear studies find that the gravitational-wave signal emitted during the bar mode instability may be detected by Advanced LIGO from a source located in the Virgo cluster. More importantly, while the star spins down the mode's frequency decreases toward the more sensitive frequency band (100 Hz) of the gravitational-wave detectors.

Here we present some recent results on the f -mode instability window and the evolution of this mode from its growth phase to saturation and its final damping, for more details [41, 70]. Our approach is based on perturbative calculations of the growth time and the frequency of the unstable mode as well as the damping due to bulk and shear viscosity. For the actual evolution of the mode we follow the procedure developed for the r -mode instability in [69, 9]. From the evolution equations for the mode energy, total angular momentum and temperature one can derive a system of ordinary differential equations to describe the mode's amplitude, the star's rotation and the thermal evolution. In our studies we also included the effects of viscosity, magnetic field, and finally consider the impact of an unstable r -mode on the f -mode instability. The coefficients of these evolution equations depend on the stellar model and mode properties.

We considered rapidly rotating and relativistic models with uniform rotation and extract the f -mode properties (frequency and eigenfunctions) from the time evolutions of the linearised dynamical equations. In particular, we simplify the problem by using the Cowling approximation, i.e. we neglect the space-time perturbations in our linearised problem. The accuracy of this approximation is to better than 20% for the quadrupole f -mode, but increases considerably for higher multipoles. In a second step, the mode's frequency and eigenfunctions are inserted in appropriate volume integrals which determine the viscous damping times and the gravitational radiation growth time [48].

The results that we show here are for two polytropic neutron star models with different compactness and maximum rotation limit. The first is a standard star with polytropic index $N = 1$ and baryonic mass $M_b = 1.4M_\odot$. The second is an $N = 2/3$ polytrope with $M_b = 1.6M_\odot$, which represents a supramassive star, i.e. a configuration that is stabilised by rotation and that in the non-rotating limit does not have a static stable configuration [30, 31, 76].

Considering several configurations we find that the gravitational-wave signal generated by an unstable f -mode may be potentially detected with the Einstein Telescope (ET) from a source located in the Virgo cluster. For instance, the gravitational characteristic strain generated by the $l = m = 4$ f -mode is shown in Figure 3 (left panel) for the $N = 1$ and $N = 2/3$ polytropic models with a relatively weak magnetic field, $B_p = 10^{11}$ G. This characteristic strain is determined by integrating in time the signal generated by an f -mode during the instability and setting a maximum saturation energy of $E \sim 10^{-6}M_\odot c^2$.

A general expectation was that superfluidity should further restrict the parameter

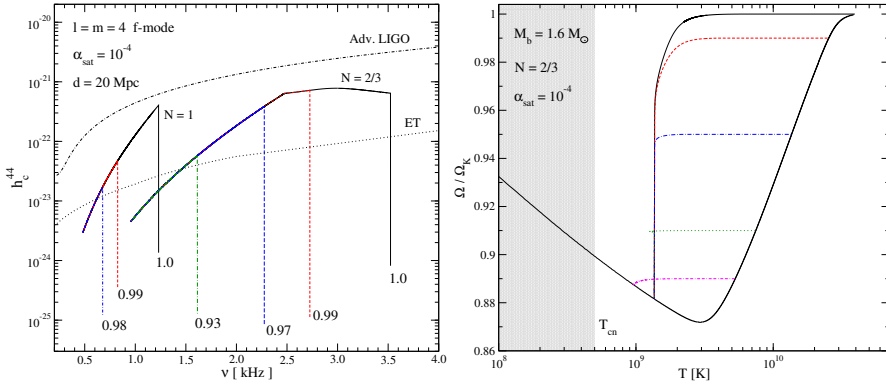


Figure 3: *Left panel:* Characteristic strain generated by the f -mode ($l = m = 4$) instability for a polytrope with $N = 1$ and $N = 2/3$. The source is located at 20 Mpc and the saturation amplitude of the f -mode is set to $\alpha_{sat} = 10^{-4}$. The number near the vertical lines denotes the initial rotation rate Ω/Ω_K . *Right panel:* Evolution of the $l = m = 4$ f -mode instability for a relativistic polytrope with $N = 1$, a baryonic mass of $M_b = 1.4 M_\odot$ and $B_p = 10^{11}$ G. The star enters the instability window at different spins. The shaded region in the right panel represents the temperature range where the neutrons and protons of the core are expected to be in a superfluid/superconducting state.

space of the instability. In fact, if a star cools down below the transition temperature where the neutrons of the core become superfluid, the f -mode instability should be efficiently suppressed by the mutual friction force [65]. However, for typical stellar parameters we find that the heat generated by shear viscosity during the saturation phase of an unstable f -mode counterbalance the neutrino cooling. In fact, our results show that an unstable star may follow a quasi isothermal trajectory within the instability window without cooling below the critical temperature of the core's neutrons. As a result the evolution time is longer and the star loses significantly more angular momentum via the emission of gravitational waves.

A rough estimation suggests that from the Virgo cluster we should expect about 30-60 supernova explosions per year, by assuming a rate of 2-3 events per century in our Galaxy. This means that if few of them leave behind a very rapidly spinning proto-neutron star, we might be able to detect these events from the gravitational radiation emitted during the f -mode instability. The number of these potential sources may be even more promising if we note that in more massive stars the gravitational-wave signal remains detectable for a period up to ten years.

3 Magnetars

Soft-gamma repeaters (SGRs) and anomalous X-ray pulsars (AXPs) are classes of neutron stars, collectively known as *magnetars*, that contain the most extreme magnetic fields in the Universe [37]. These exotic objects exhibit sporadic bursting and flaring events, which are commonly associated to dynamics of the magnetic field [36].

3.1 Magnetar QPOs

In the last few years, a great attention was devoted to the interpretation of the quasi-periodic oscillations (QPOs) discovered in the tail of giant flares in Soft Gamma Repeaters (SGRs). Those oscillations cover a wide range of frequencies going from some Hz up to a few kHz. Until now only in two SGRs it was possible to observe clearly QPOs: in the SGR 1806-20 and in the SGR 1900+14 (see [49], [95] for a review). In the SGR 1900+14 four frequencies have been detected: 28, 53, 84, and 155 Hz while in the SGR 1806-20 several QPOs have been discovered: 18, 26, 30, 92, 150, 625 and 1840 Hz. A recent reanalysis of the data for the SGR 1806-20 carried on by [46] with a different method, shows the presence of additional QPOs at 16.9, 21.4, 36.4, 59.0 and 116.3 Hz.

Understanding the nature of the QPOs will offer an unique opportunity to investigate the proprieties of neutron stars and to constrain the nuclear matter in neutron stars as well as the topology and strength of their magnetic field. For those reasons, since their discovery, QPOs have been the subject of an intensive study. The first hypothesis, in order to explain QPOs, was to consider pure shear modes of the crust, excited after the burst generated by giant flares. This idea proposed by [36] was investigated in several analytical and numerical works (see for example [86, 74]). Those works suggested that not all the lower QPOs could be explained as pure crustal frequencies. An alternative scenario has been then proposed by [62] and [45], involving global crust-core oscillations of the star. Levin showed with a toy model [63] that the QPOs spectrum should be continuum, with turning points that were called edges' of the continuum. Following this idea, a significant amount of work has been devoted in understanding global Alfvén oscillations in neutron star, using general relativistic models ([87], [27], [22]) or Newtonian model ([61]). All those works made clear that global Alfvén modes in a pure fluid neutron star could not to explain the small gap between the lower frequencies observed in QPOs.

A further step was then needed: to add a solid crust and to study the coupling of this latter with the fluid core, [54, 92], using a non-relativistic model, showed that the presence of the crust strongly influences the continuum spectrum. In particular, they found that near the edges of the continua other discrete Alfvén modes appear. This results was partially confirmed by [39] and [40], using a non-linear relativistic method: the authors could find some frequencies in the gaps between two near continua, however they stated that those oscillations were quickly damped. In a linear relativistic simulation, [28] it was confirmed the presence of those discrete Alfvén modes in the gap between two continua and showed that the crustal modes could live long if they are located in

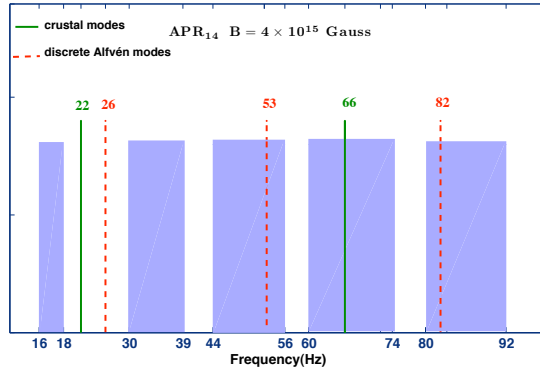


Figure 4: Identification of the frequencies of SGR 1806-20. We show that a stellar model with EoS APR with mass $M = 1.4M_{\odot}$ and radius $R = 11.57\text{km}$ can explain all the frequencies observed. The magnetic field strength is $4 \times 10^{15}\text{G}$ (from [28]).

those gaps, see Figure 4. The discrete Alfvén modes as well as the crustal modes in the gap between the first two continua could explain the small gap between the observed lower frequencies QPOs. However as pointed out in [93], it was still difficult to explain the higher frequencies as the 625Hz because at higher frequencies the continuum dominates, absorbing all the discrete frequencies, and in addition the continua overlap to each other: in this way it is not possible to isolate the edges of the continuum and to have a unique interpretation for the 625Hz QPO.

A possibility to explain the high frequency QPOs came from the study of polar oscillations in magnetars [85]. In this work it was shown that the polar Alfvén spectrum has a discrete nature and, in addition, the lower polar Alfvén modes have frequencies around few hundred Hz. However, it was not clear how this result could affect the axial spectrum. An alternative suggestion is that the few hundred Hz frequencies might be explained by introducing the effects of superfluidity [71].

In [29] we investigated the effect of the coupling between axial (torsional) oscillations and polar oscillations on the frequency spectrum. The neutron star model was composed of a fluid core and a solid crust and is permeated by a magnetic field with strength $B = 10^{16}\text{G}$ which acquires both poloidal and toroidal components. The toroidal component is the coupling pipeline between the axial and polar part of the spectrum. By using a two dimensional relativistic, linear code, we found that the axial spectrum is strongly modified by the presence of the coupling: its continuum feature is destroyed, leaving behind only the edges of the continuum. In this new picture, both Alfvén modes and crustal modes have a discrete nature and those latter, if out of resonance with the edges of the continuum, could live long enough to be seen as high frequency modes. *Due to the presence of the crust in our model, also crustal modes appear in the spectrum.* In the axial case, they are not anymore absorbed by the continuum so they are long-living modes.

The only case in which a damping of the crustal modes can be observed is if they are in resonance with the edges of the continuum. In this case the energy leaks from the crust to the core with a velocity that depends on the strength of the magnetic field. The efficiency of the process depends on the magnetic field strength. Finally, in [29], an attempt was made to resolve the puzzle of the 625 Hz QPO: we found that in our model, although the spectrum is discrete, at higher frequencies it becomes too dense to allow an unambiguous interpretation of this high frequency mode.

3.2 Magnetars & Gravitational Waves

The strongest of the magnetar flares has been considered in the literature as potential multi-messenger gravitational wave sources, which has prompted targeted gravitational wave searches [14, 2, 3, 67, 1]. Presently, these searches have placed upper limits on gravitational wave energies of 1.4×10^{49} erg for the f -mode (i.e. in the k Hz frequency range), and 3.5×10^{44} erg for white noise around 100 – 200 Hz.

Comparatively little theoretical work has been done on the topic of magnetar flares as a viable source of gravitational radiation. In part this is due to a reasonably poor understanding of the internal dynamics of the objects during and immediately following a flare. Ioka [47] first investigated this question by looking at the maximum gravitational wave energy released by a change in moment of inertia induced by a dynamical rearrangement of the core magnetic field inside the star. This calculation placed an upper limit of about 10^{49} erg under ideal conditions, including optimistic values of the internal magnetic field. More recently, Corsi & Owen [32] found similar values to be possible under more generic conditions, still tapping into the full energy reservoir associated with an instantaneous change in the magnetic potential energy of the star. In contrast, Levin & van Hoven [64] did not find f -mode detection to be very likely in the near future. Their model was based on some mechanism external to the star triggering the internal f -modes.

Here we present a short review on the recent numerical work aimed at understanding the magnetic field instabilities and the related emission of gravitational waves caused by a large-scale rearrangement of the internal magnetic field [100]. In particular, we utilise magnetic field instabilities to trigger global reconfigurations of the magnetic field, and determine the gravitational wave output for such models. Recently, Ciolfi *et al.* [25] performed numerical simulations as our earlier paper [59], concluding that giant flares *could* give rise to observable gravitational radiation. However, they utilised a stellar model with a surface magnetic field strength some sixty times the strongest observed to reach these conclusions. We have shown [98] that the gravitational wave energy emitted in these events is a highly nonlinear function of the magnetic field strength, implying f -mode detection is unlikely. In this review, a conglomeration of our three recent papers [59, 100, 58], is provided.

We study the time evolution of the ideal MHD equations in general relativity under the Cowling approximation utilising the three-dimensional GRMHD code THOR [101] and the GPU code HORIZON [98]. The MHD portion of the code utilises the conservative

formalism outlined in [13], with hyperbolic divergence cleaning employed following the prescription in [5]. In our simulations we used a Cartesian grid with 120^3 grid points. Resolution studies have been completed using 90^3 , 150^3 and 200^3 grid points, finding consistent phenomenology. The outer boundary of the star is located approximately 1.4 times the stellar radius (at the closest point), and we adopt Dirichlet boundary conditions for the evolution of the magnetic field at this outer boundary. We have performed various tests with different boundary conditions and again found consistent phenomenology. The spectral code LORENE is utilised to create initial conditions, which are self-consistent solutions of the Einstein-Maxwell field equations in ideal MHD with purely poloidal magnetic fields [18]. Our fiducial model is a non-rotating star with polytropic equation of state with $N = 1$ and $K = 100$. This gives a stellar model with gravitational mass $M = 1.3 M_\odot$ and equatorial radius $R = 12.6$ km.

3.2.1 Hydromagnetic Instabilities

Purely poloidal fields are known to be dynamically unstable to the kink instability from local, linear studies in Newtonian physics [66]. Such studies have been confirmed with global, linearised numerical evolutions [57] and in full non-linear studies [19]. Such configurations were also shown to be unstable in fully nonlinear relativistic calculations in [59] – some details of which we repeat here.

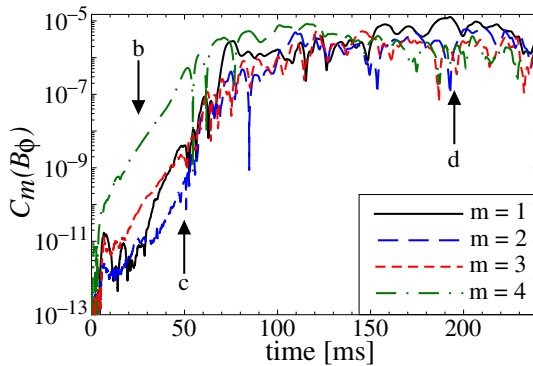


Figure 5: Evolution of $C_m(B_\phi) = \frac{1}{2\pi} \int_0^{2\pi} B_\phi(\omega, \phi, z=0) e^{im\phi} d\phi$, as a function of time for a model with initial surface magnetic field of $B_{\text{surf}} = 8.8 \times 10^{15}$ G, yielding an Alfvén crossing time of 5.0 ms. The arrows represent the times of the three-dimensional snapshots plotted in figure 6. This figure was originally published in [59]

An instability can be seen in all modes after the first couple of Alfvén crossing times. Each mode grows exponentially by approximately six orders of magnitude over many subsequent Alfvén crossing times. Saturation of the modes is seen to occur after about 75 ms, at which point the simulation evolves to a pseudo-equilibrium state. We have

evolved such simulations to $400 \text{ ms} \equiv 80 t_A$, where t_A is the Alfvén crossing time, with little variation in the equilibrium configuration following the first hundred or so milliseconds. The equilibrium configurations are discussed in more detail in refs. [59, 100, 58].

In Figures 6 we present three-dimensional plots of the magnetic field at various instances throughout the evolution. The instability acts near the neutral line, which is the line where $B = 0$ around the equatorial plane. In our simulations this is located at approximately two-thirds of the stellar radius. For clarity we have plotted red magnetic field lines seeded near the neutral line in the equatorial plane. Also plotted are black field lines seeded in the equatorial plane interior to the neutral line. The blue volume rendering is an isopycnic surface of $\rho = 0.37\rho_c$, where ρ_c is the central rest-mass density. This surface lies at a radius of approximately 50% of the stellar radius, which is well inside the neutral line.

Figure 6a shows the initial data imported from the LORENE spectral code. The domain of our grid is larger than that plotted here, and field lines are truncated at the surface of the star for clarity in the figure.

Figure 6b shows the evolution after $25 \text{ ms} \equiv 5 t_A$. The onset of the “sausage” or “varicose” mode [66], involving a change in the cross-sectional area of a flux tube around the neutral line, is clearly visible. This is strongest in the $m = 4$ mode, which is a result of the transient excitation at the beginning of our simulation (for more details we refer the reader to ref. [59]). It is worth noting that, while this transient reduces with increasing grid resolution, the presence of the varicose mode is an inherent characteristic of the system.

Figure 6c shows the point at which the “kink” instability begins to visually dominate the system, which is after $50 \text{ ms} = 10 t_A$. The kink mode acts perpendicularly to the gravitational field, in accordance with the prediction of ref. [66]. While one can still see the presence of the varicose mode in this snapshot, this point represents the non-linear development of the instability where the change in field structure is of similar order to the background field.

Figure 6d shows the simulation after $195 \text{ ms} = 39 t_A$. This is a typical snapshot many Alfvén timescales after the non-linear saturation of the unstable modes. In some ways, this figure is consistent with the “twisted-torus” configurations seen in the non-linear evolutions of refs. [21, 20], and in the semi-analytic equilibrium calculations of refs. [97, 24]. Figure 6d shows about half of the star is well approximated by a twisted-torus, however the remainder of the star also exhibits large non-axisymmetric structures.

3.2.2 Gravitational Wave Emissions

The preceding simulations provide an invaluable tool for predicting gravitational wave emissions from magnetised neutron stars. In particular, we utilise the hydromagnetic instabilities to mimic a global magnetic field reconfiguring that may occur during, or immediately following, a magnetar flare. While the entire magnetic field may not

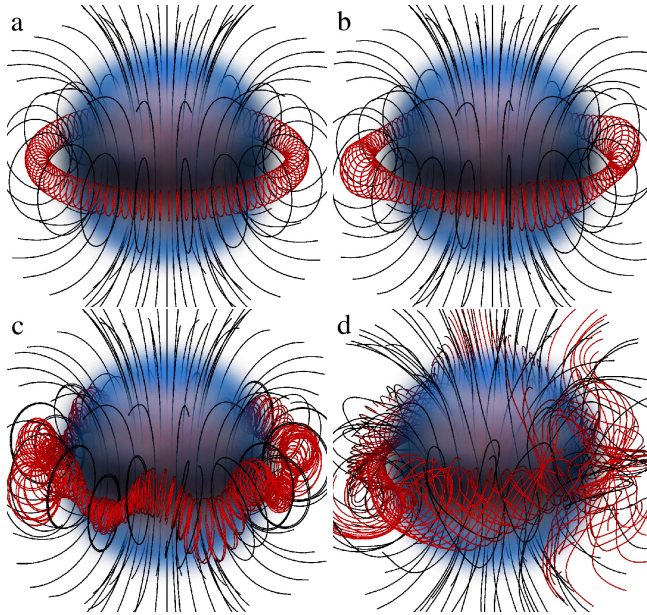


Figure 6: Time evolution of model in figure 5 with an Alfvén crossing time of 5.0 ms. The figures are (a) $t = 0$ ms, (b) $t = 25$ ms, (c) $t = 50$ ms and (d) $t = 195$ ms. The red magnetic field lines are seeded on the equatorial plane close to the neutral line to more clearly visualise the instabilities, while the black magnetic field lines are seeded on the equatorial plane interior to the neutral line. The volume rendering is an isopycnic surface at 37% of the central rest-mass density which is shown only to provide contrast with the field lines. A full movie of this simulation lasting 400 ms can be seen at www.tat.physik.uni-tuebingen.de/~tat/grmhd. These figures were originally published in [59]

rearrange in a realistic magnetar, rearranging the entire field gives us an *upper limit* of gravitational wave excitation in one of these events.

Our simulations exhibit negligible gravitational wave strain (i.e. $h_{\times} \lesssim 10^{-27}$ measured at 10 kpc) for the initial period of the simulation. During the non-linear phase of the instability, typically around $20 t_A$ into the evolution, the gravitational wave strain grows until the instability saturates, at which point the strain signal remains roughly constant. It is worth noting that the dominant mechanism for damping the f -mode is through gravitational wave emissions, which occurs on a timescale of order 100 – 200 ms [11]. As we are working in the Cowling approximation, we do not see such damping, and hence the gravitational wave amplitude remains nearly constant throughout the evolution of the system.

For the gravitational wave amplitude as a function of surface magnetic field strength

we find an approximate power-law relation between h and B_{surf} given by

$$h_{\times}^{\text{max}} = 8.5 \times 10^{-28} \left(\frac{10 \text{ kpc}}{d} \right) \left(\frac{R}{10 \text{ km}} \right)^{4.8} \left(\frac{M}{M_{\odot}} \right)^{1.8} \left(\frac{B_{\text{pole}}}{10^{15} \text{ G}} \right)^{2.9} \quad (5)$$

It is worth noting that the 2.9 exponent is an approximate value we find from this particular set of simulations. In general, this number will vary with the equation of state and magnetic field configuration, however an exponent of approximately 3 does agree with theoretical predictions. More details can be found in detail in our latest article [58].

Most of the energy in the aforementioned signal is in the f -mode. Assuming a gravitational wave damping time of approximately 100 ms [11], we find a corresponding power-law relation for the energy emitted in gravitational wave radiation via the f -mode to be

$$E_{\text{GW}} = 1.7 \times 10^{36} \left(\frac{R}{10 \text{ km}} \right)^{9.6} \left(\frac{M}{M_{\odot}} \right)^{3.6} \left(\frac{B_{\text{pole}}}{10^{15} \text{ G}} \right)^{5.8} \text{ erg}. \quad (6)$$

We can see from the above two relations that the gravitational wave amplitude and energies are highly non-linear functions of the surface magnetic field strength. Moreover, for typical magnetar field strengths of order 10^{15} G, one finds strains *below* 10^{-25} and energies lower than 10^{40} erg for a source at 10 kpc, even if we assume a catastrophic global restructuring of the field to be associated with a giant flare.

We provide the signal-to-noise ratios for different detector sensitivity curves in Figure 7. Here we have plotted the signal amplitude, $\sqrt{T}|\tilde{h}(f)|$, where T is the damping time of the oscillation and $\tilde{h}(f)$ is the Fourier transform of h_{\times} , as a function of the frequency for various magnetic field strength models. At approximately 1.8 kHz we show the signal amplitude for the f -mode as excited by the hydromagnetic instability assuming a damping time of $50 \text{ ms} \leq T \leq 200 \text{ ms}$. In the lower part of the spectrum we plot the other maximal mode seen in the Fourier transform, assuming a damping time between $10 \text{ ms} \leq T \leq 1 \text{ s}$. Over this we plot the entire spectrum (assuming $T = 100 \text{ ms}$) for two models with $B_{\text{surf}} = 8.8 \times 10^{15} \text{ G}$ and $B_{\text{surf}} = 1.8 \times 10^{16} \text{ G}$. Finally, we also plot the root of the noise power spectral density, $\sqrt{|S_h(f)|}$, as a function of frequency for the LIGO, AdvLIGO and ET detectors [75].

The amplitude signal-to-noise ratio, defined by $\sqrt{T}|\tilde{h}(f)|/\sqrt{|S_h(f)|}$, can be read off the graph as the ratio between the signal and the noise curve for the respective detector. The main conclusion from this figure is this: Assuming a giant flare is associated with a catastrophic large-scale rearrangement of the core magnetic field, *the gravitational wave signal associated with f -modes are not observable with present or near-future gravitational wave observatories* [100].

We do note the presence of lower frequency modes in the 10–100 Hz region of the spectrum. These are Alfvén modes, whereby the restoring force is provided by the magnetic field itself. The dominant damping mechanism for these modes is largely unknown, although it is generally expected that they will last significantly longer than

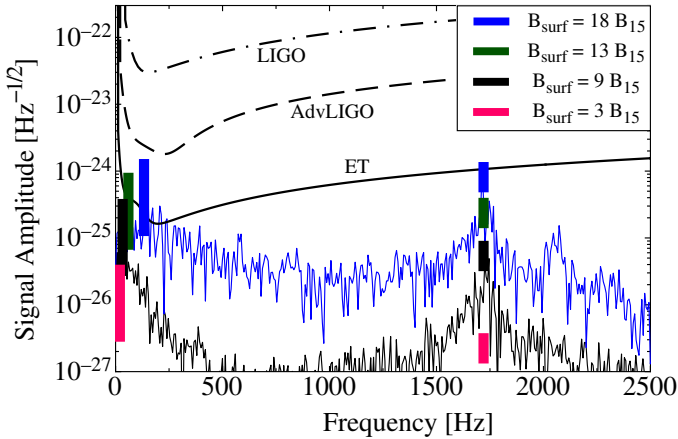


Figure 7: Signal amplitude, $\sqrt{T}|\tilde{h}(f)|$ against oscillation frequency. The colored boxes on the right represent the maximum for the f -mode assuming a constant periodic source lasting between 50 and 200 ms. The colored boxes on the left represent the maximum mode seen in the Fourier transform for any given frequency below the f -mode frequency assuming a constant periodic source lasting between 10 ms and 1 s. These scale with \sqrt{T} , implying one can easily extrapolate to alternative values of the damping time. We have further plotted the entire spectrum (assuming a damping time of 100 ms) for both the $B_{\text{surf}} = 1.8 \times 10^{16}$ G model (blue line) and $B_{\text{surf}} = 8.8 \times 10^{15}$ G model (black line). This figure was originally published in [100].

the f -mode signal. For this reason we have plotted them in figure 7 with damping times between 10 ms and 1 s, although we note they may be excited for significantly longer than this. These modes are an exciting new prospect for gravitational wave detection [100], which we discuss in significantly more detail in a forthcoming article.

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