Relativistic Equations of State for hot matter and neutron star dynamics

A. Chorozidou, Th. Gaitanos

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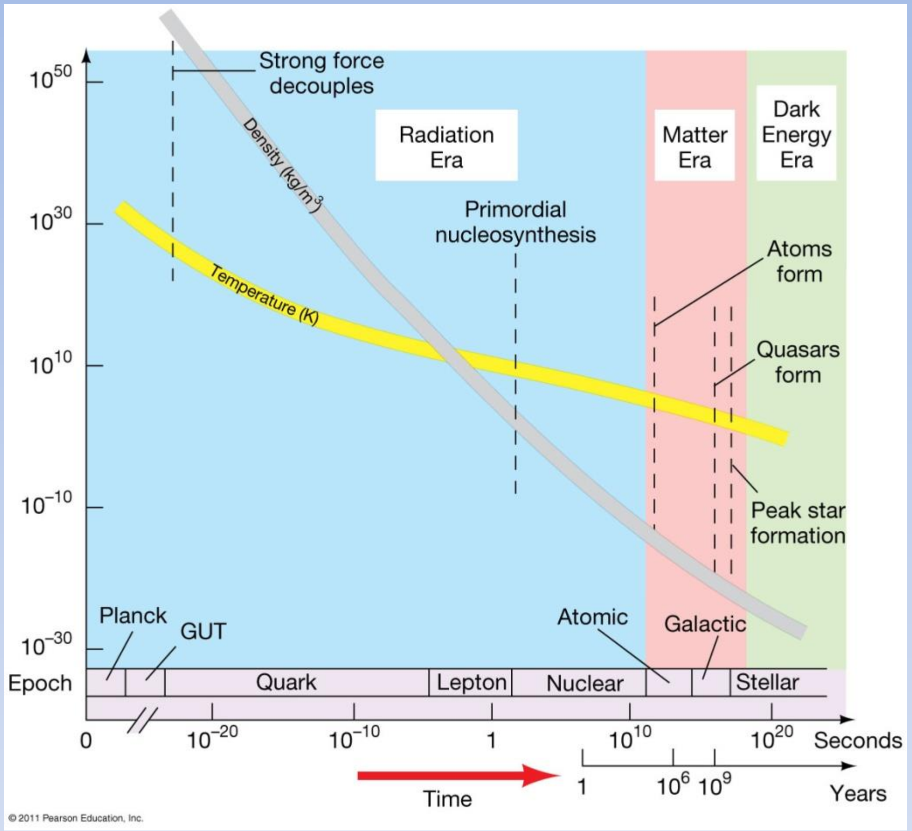
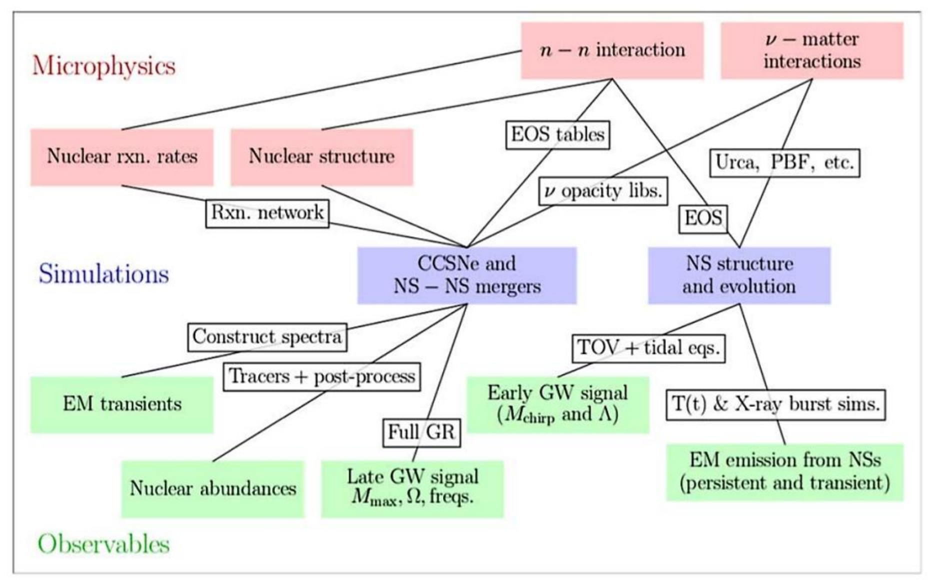
Introduction…

Importance of studying Neutron Stars (NS) for fundamental questions of physics

- How does GR work for such systems?
- Is the GW speed equal to c? If yes, why?
- What was the state of matter when universe was very hot and dense?
- Is our current understanding of strong interaction really correct?



The richness of the study of NS

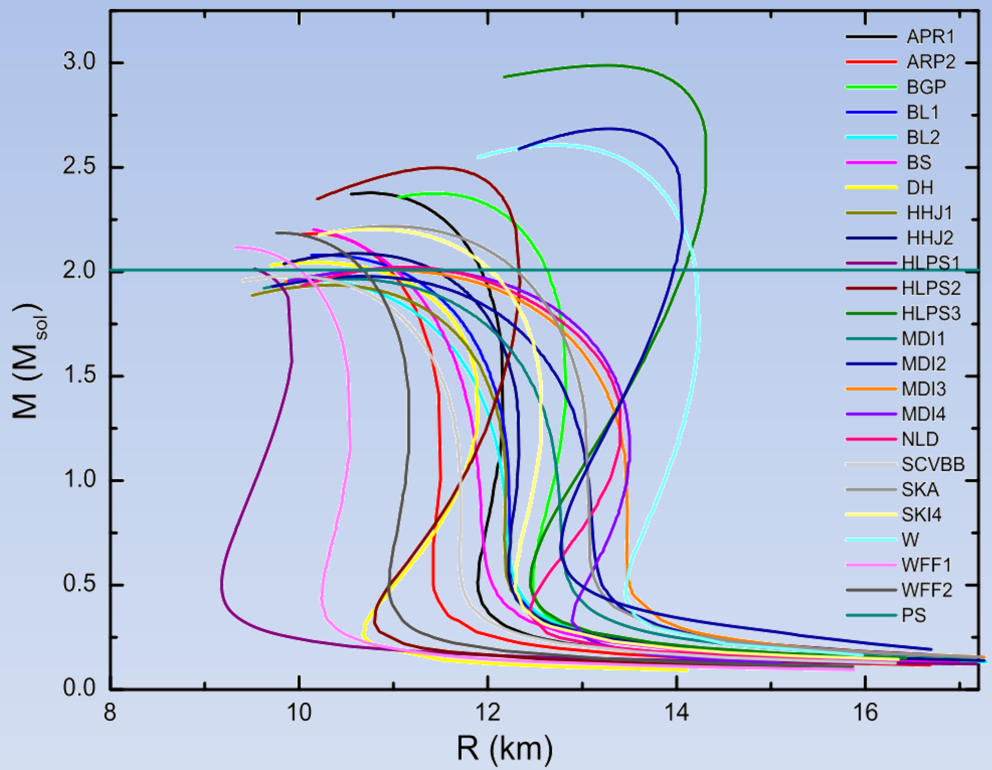


A diagram representing the connections between microphysical models and astrophysical observations

From macro-features (M_{max} , surface T, pulsar glitches) —> info about the internal (super dense, isospin-asymmetric, superfluid, superconductive, bulk hadronic matter)

Description of NS

Mass, Radius and Equation of State (EoS)



model of nuclear matter -> EoS ->TOV equation -> M(R)

Modelling dense nuclear matter - EoS

•challenge: model that describes both dense & normal matter
•in dense systems particles have high energy=>relativistic effects

The description must incorporate:
1. The general properties of QM
2. Lorentz covariance
3. EM gauge invariance
4. Microscopic causality within a many-body system

The framework: R-QFT based on a local, Lorentz-invariance Lagrangian density.

This is **QHD (Walecka model)**. It is based on meson exchange formalism.

Unknown parameters: coupling constants between the different meson & nucleon fields. They are determined by fitting the calculated properties of nuclei and nuclear matter (ρ_{sat},E_b,a_{sym},K) to experimental values

T=0 approximation

$$\rho = 2\rho_0$$

$$n = \frac{\rho}{m_N} = 2n_0 \approx 0.34\, fm^{-3} \rightarrow \bar{l} \approx n^{-1/3} \approx 1.4\, fm$$

$$\lambda_T \approx \frac{h}{\sqrt{m_N k_B T}} \approx 4 \cdot 10^6 T^{-1/2}\, fm$$

$$*\ for T \approx 10^9 K, \lambda_T \succ 10^2\, fm$$

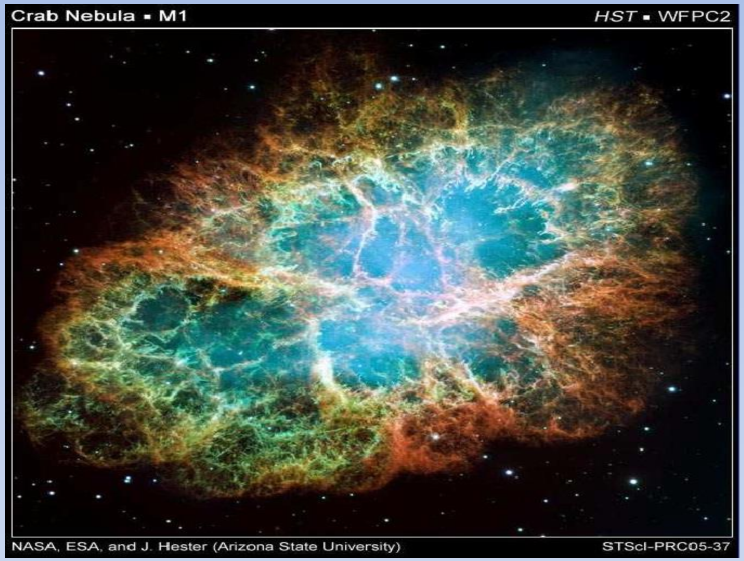
$$\Rightarrow \lambda_T >> \bar{l}$$

- QM needed
- The energy of the particles is mainly due to the degeneracy and not thermal anymore!

Important for astrophysics

explore EoS far beyond saturation (high ρ, high τ-asymm, high T)

Neutron stars



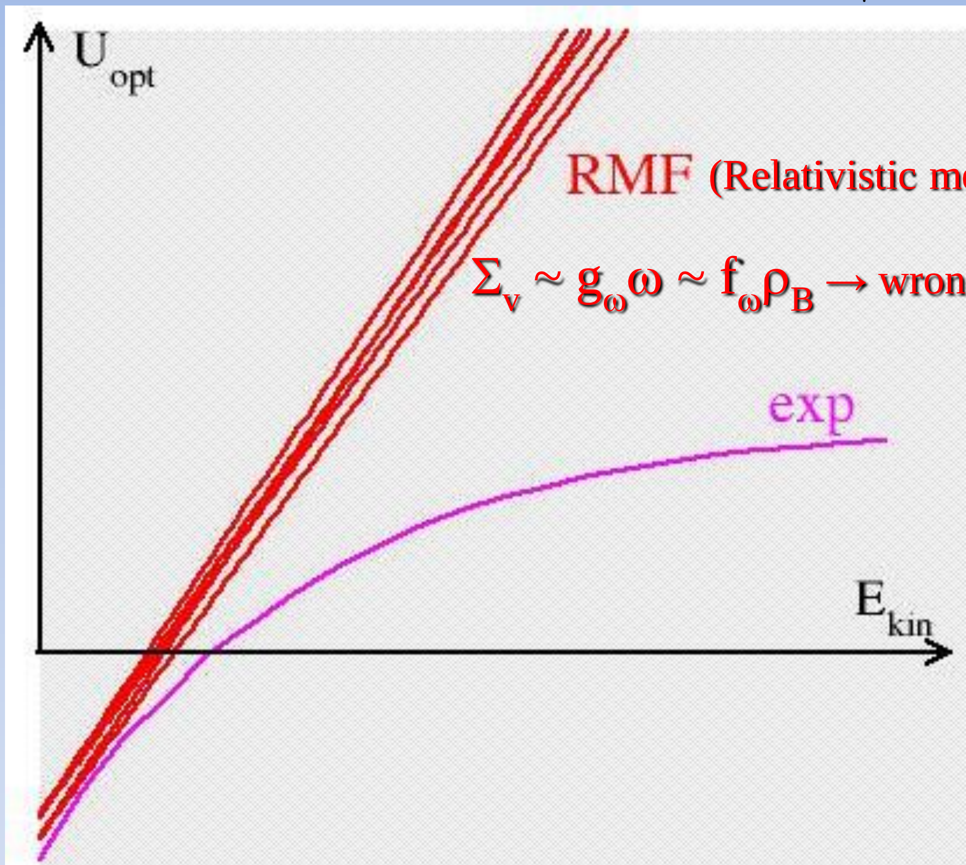
Densities in static NS: $\rho \sim (8\text{-}10)\rho_0$

- In high-density matter (+kinematics) → particles with high-momenta p
- Not only density dependence, but also momentum dependence (MD) essential

The optical potential

In-medium proton Schrödinger-equivalent Re(U_{opt})

$$U_{\text{opt}} = \frac{E}{m}\Sigma_v - \Sigma_s + \frac{1}{2m}\left(\Sigma_s^2 - \Sigma_v^2\right)$$



RMF (Relativistic mean-field)

$\Sigma_v \sim g_\omega \omega \sim f_\omega \rho_B \rightarrow$ wrong divergent MD at high ρ_B rsp. high momenta

DBHF & Dirac-phenomenology for nucleons:
Proton-opt. pot. well known
saturating fields (particular vector) with rising p

Solutions so far:

→ non-local (Hartree-Fock) contributions to RMF (Hartree) mean-field
Weber, Blättel, Cassing et al., Nucl. Phys. A539 (1992) 713

→ first-order derivative coupling terms into the interaction Lagrangian
S. Typel, Phys. Rev. C71, 064301 (2005)

The Non-Linear Derivative (NLD) model...

NLD Lagrangian : as in conventional QHD

$$\mathcal{L} = \frac{1}{2}\left[\overline{\Psi}\gamma_\mu i\overrightarrow{\partial}^\mu\Psi - \overline{\Psi}i\overleftarrow{\partial}^\mu\gamma_\mu\Psi\right] - m\overline{\Psi}\Psi - \frac{1}{2}m_\sigma^2\sigma^2 + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - U(\sigma)$$
$$+\frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m_\rho^2\vec{\rho}_\mu\vec{\rho}^\mu - \frac{1}{4}\vec{G}_{\mu\nu}\vec{G}^{\mu\nu}$$
$$+\mathcal{L}_{int}$$

Interaction Lagrangian : as in conventional QHD + non-linear derivative operators

$$\mathcal{L}_{int} = \frac{g_\sigma}{2}\left[\overline{\Psi}\overleftarrow{\mathcal{D}}\Psi\sigma + \sigma\overline{\Psi}\overrightarrow{\mathcal{D}}\Psi\right] - \frac{g_\omega}{2}\left[\overline{\Psi}\overleftarrow{\mathcal{D}}\gamma^\mu\Psi\omega_\mu + \omega_\mu\overline{\Psi}\gamma^\mu\overrightarrow{\mathcal{D}}\Psi\right]$$
$$-\frac{g_\rho}{2}\left[\overline{\Psi}\overleftarrow{\mathcal{D}}\gamma^\mu\vec{\tau}\Psi\vec{\rho}_\mu + \vec{\rho}_\mu\overline{\Psi}\vec{\tau}\gamma^\mu\overrightarrow{\mathcal{D}}\Psi\right]$$

Non-linear derivative operators : Taylor expansion of partial derivatives ξ

$$\overrightarrow{\mathcal{D}} := \mathcal{D}\left(\overrightarrow{\xi}\right) = \sum_{j=0}^{n\to\infty}\frac{\partial^j}{\partial\overrightarrow{\xi}^j}\mathcal{D}|_{\overrightarrow{\xi}\to 0}\frac{\overrightarrow{\xi}^j}{j!}$$

$$\overrightarrow{\xi} = -\frac{v^\alpha i\overrightarrow{\partial}_\alpha}{\Lambda}$$

cut-off, will regulate the high-momentum tail of RMF fields

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Features of NLD model…

$$\Sigma^{\mu}_{vi} = g_{\omega}\omega^{\mu}\mathcal{D} + g_{\rho}\tau_i\rho^{\mu}\mathcal{D} \ , \ \Sigma_{si} = g_{\sigma}\sigma\mathcal{D}$$

meson-field equations

$$m_{\sigma}^2\sigma + \frac{\partial U}{\partial\sigma} = g_{\sigma}\sum_{i=p,n}\left\langle\overline{\Psi}_i\mathcal{D}\Psi_i\right\rangle = g_{\sigma}\rho_s$$

$$m_{\omega}^2\omega = g_{\omega}\sum_{i=p,n}\left\langle\overline{\Psi}_i\gamma^0\mathcal{D}\Psi_i\right\rangle = g_{\omega}\rho_0$$

$$m_{\rho}^2\rho = g_{\rho}\sum_{i=p,n}\tau_i\left\langle\overline{\Psi}_i\gamma^0\mathcal{D}\Psi_i\right\rangle = g_{\rho}\rho_I$$

Equation of State (EoS)

$$\varepsilon = \sum_{i=p,n}\frac{\kappa}{(2\pi)^3}\int_{|\vec{p}|\le p_{F_i}} d^3p\, E(\vec{p}) - \langle\mathcal{L}\rangle$$

$$P = \frac{1}{3}\sum_{i=p,n}\frac{\kappa}{(2\pi)^3}\int_{|\vec{p}|\le p_{F_i}} d^3p\,\frac{\vec{\Pi}_i\cdot\vec{p}}{\Pi_i^0} + \langle\mathcal{L}\rangle$$

cut-off Λ regulates

1) DD & MD of selfenergies

2) DD of meson-field sources

(particularly for ω-field)

3) fully thermodynamic consistent

(important for neutron stars)

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NLD results: saturation…

Parameters

	$\vec{\mathcal{D}}$	cut-off	Λ_s [GeV]	Λ_v [GeV]	g_σ	g_ω	g_ρ	b [fm^{-1}]	c	m_σ [GeV]	m_ω [GeV]	m_ρ [GeV]
NLD	$\frac{1}{1+\sum_{j=1}^{4}\left(\zeta_j^\alpha i \vec{\partial}_\alpha\right)^2}$	$\frac{\Lambda^2}{\Lambda^2+\vec{p}^2}$	0.95	1.125	10.08	10.13	3.50	15.341	−14.735	0.592	0.782	0.763

Comparison with other models

Model	ρ_{sat} [fm^{-3}]	E_b [MeV/A]	K [MeV]	a_{sym} [MeV]	L [MeV]	K_{sym} [MeV]	K_{asy} [MeV]	
NLD	0.156	−15.30	251	30	81	−28	−514	
NL3*	0.150	−16.31	258	38.68	125.7	104.08	−650.12	→ Lalazissis
DD	0.149	−16.02	240	31.60	56	−95.30	−431.30	→ Typel
D^3C	0.151	−15.98	232.5	31.90	59.30	−74.7	−430.50	
DBHF	0.185	−15.60	290	33.35	71.10	−27.1	−453.70	→ Li, Machleidt, Brockmann
	0.181	−16.15	230	34.20	71	87.36	−340	→ Fuchs
empirical	0.167 ± 0.019	-16 ± 1	230 ± 10	31.1 ± 1.9	88 ± 25	–	-550 ± 100	

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monopole form

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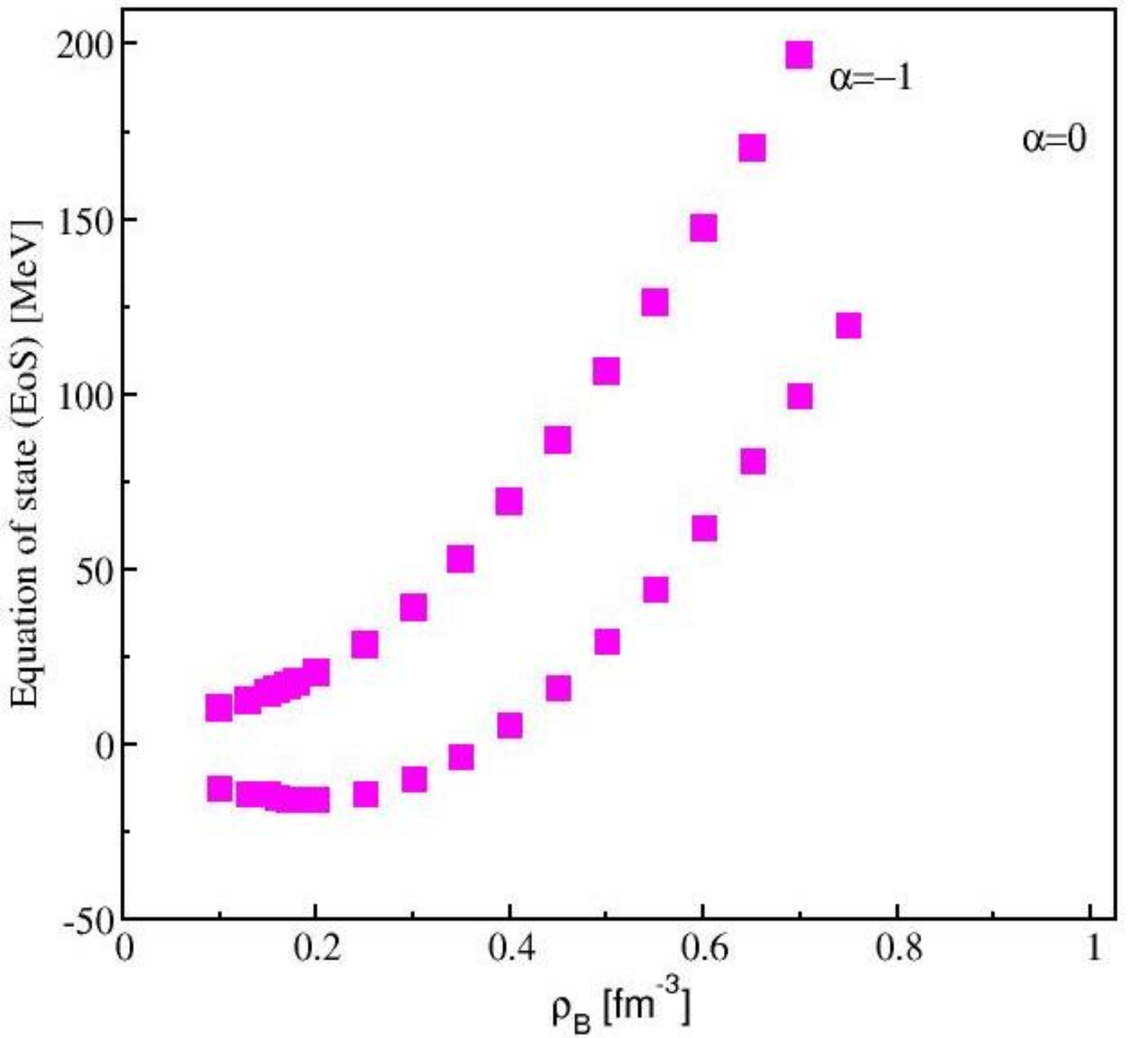
soft EoS at ρ_{sat},
but stiff at high ρ relevant for NS!

NLD results: EoS...



$$\alpha = \frac{\rho_p - \rho_n}{\rho}$$

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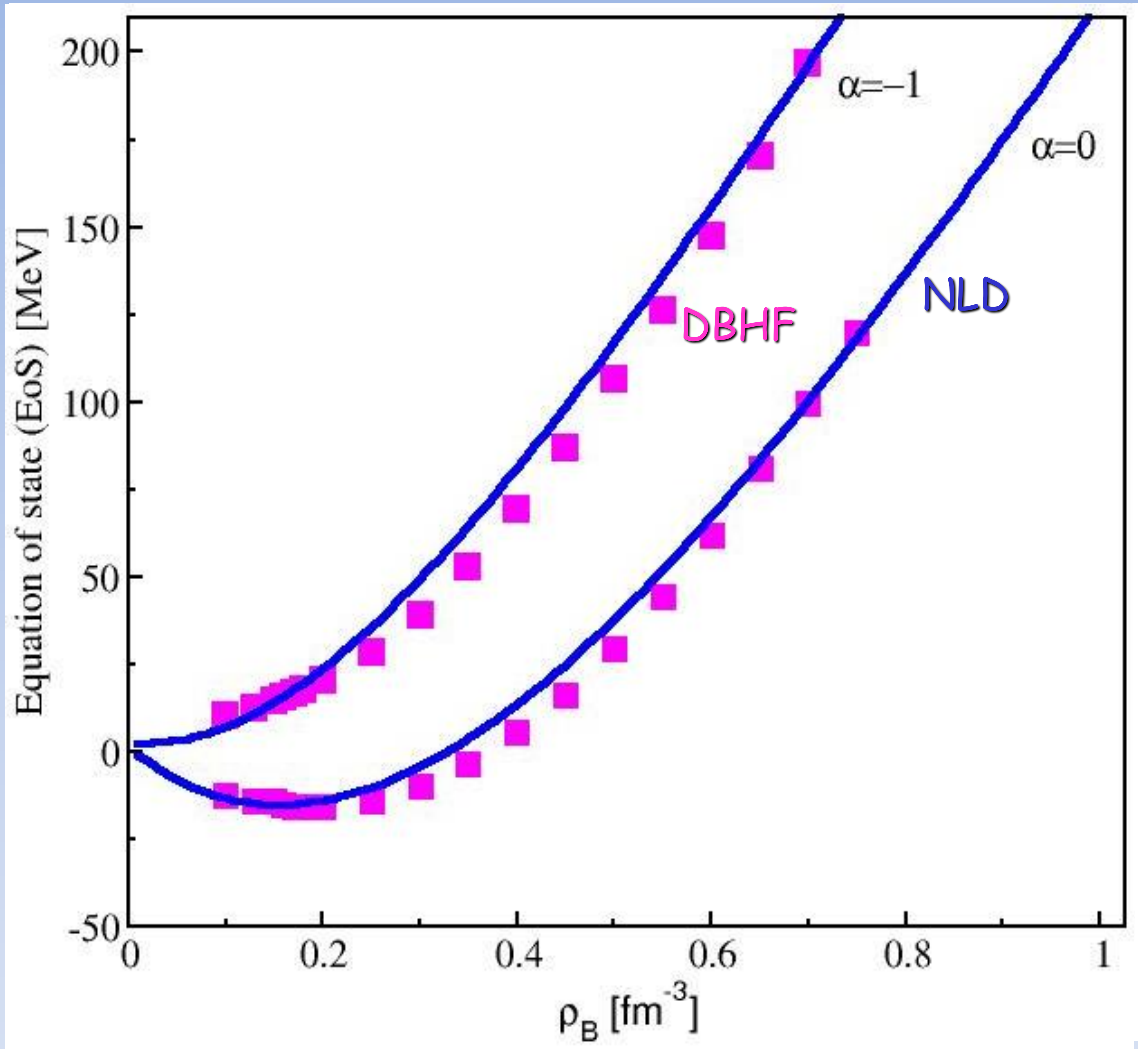
NLD results: EoS...



Remarkable comparison with microscopic DBHF!

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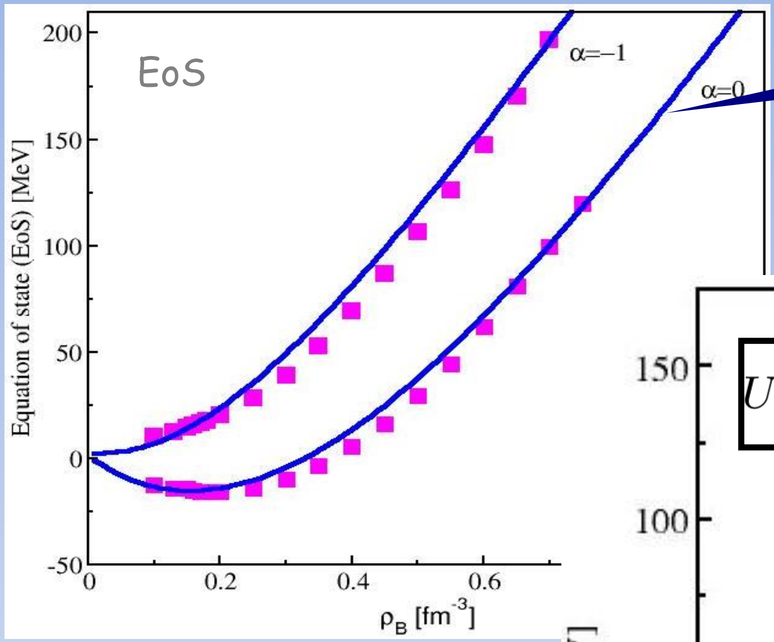
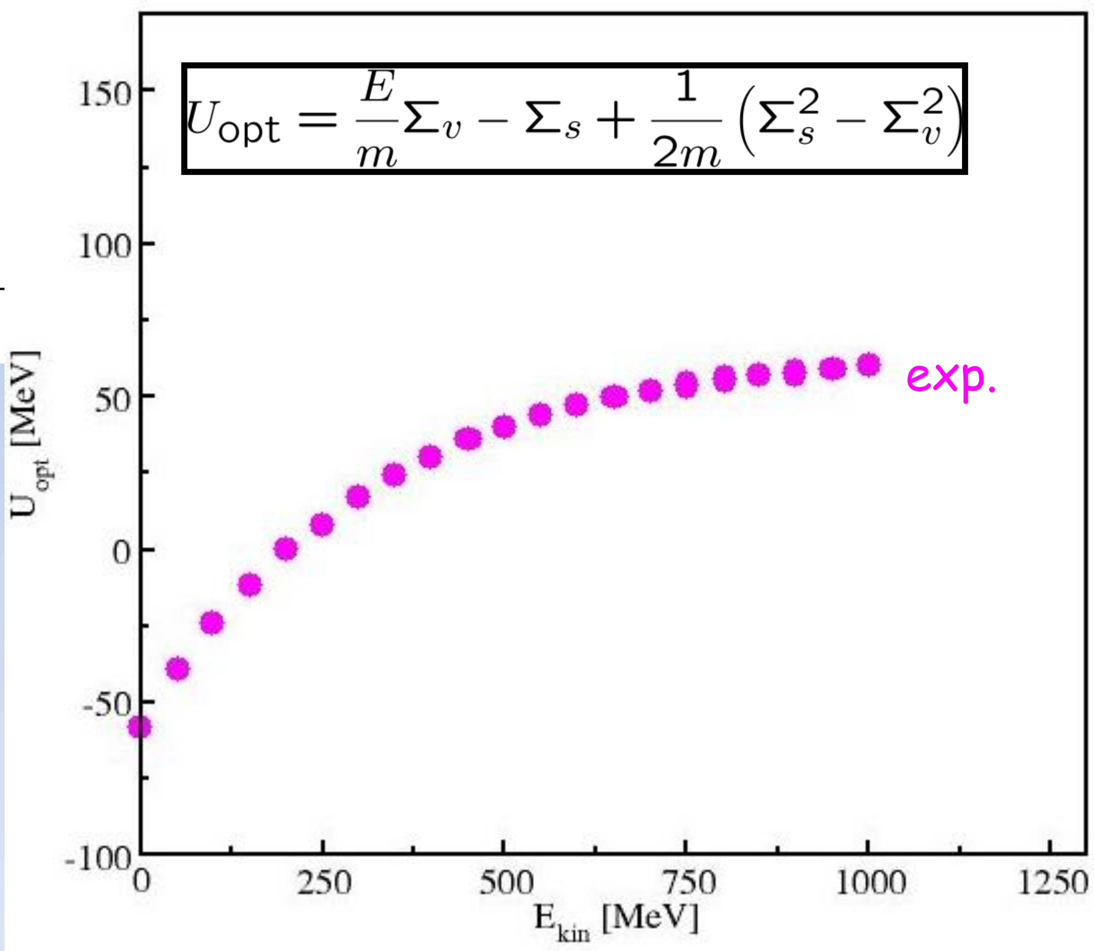
NLD results: MD & optical potentials...



high ρ → high momenta

In-medium proton SEP (real part)



$$U_{\mathrm{opt}} = \frac{E}{m}\Sigma_v - \Sigma_s + \frac{1}{2m}\left(\Sigma_s^2 - \Sigma_v^2\right)$$

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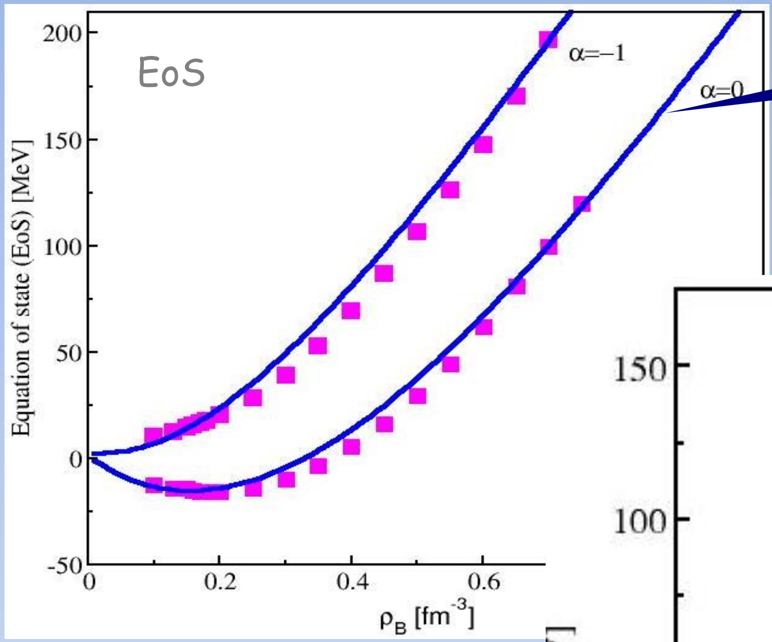

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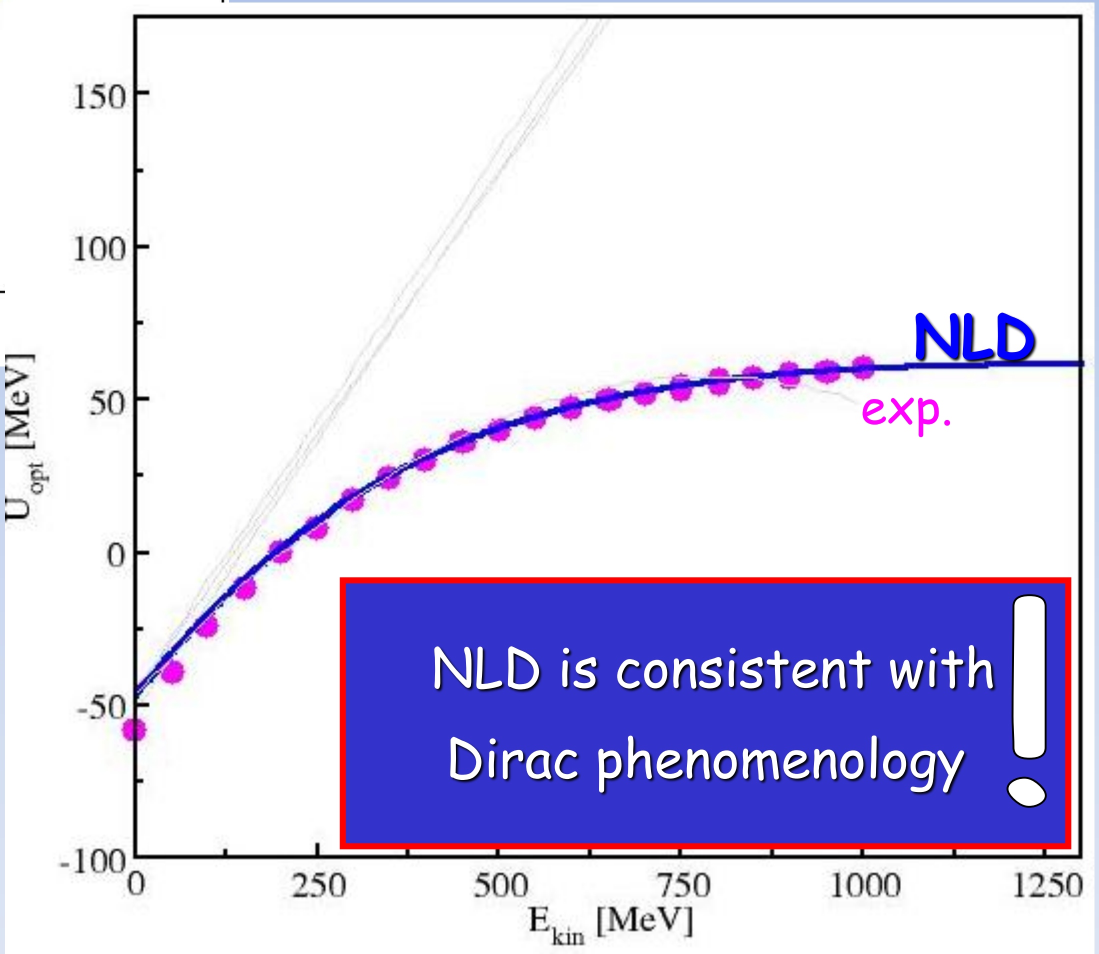

NLD is consistent with Dirac phenomenology !

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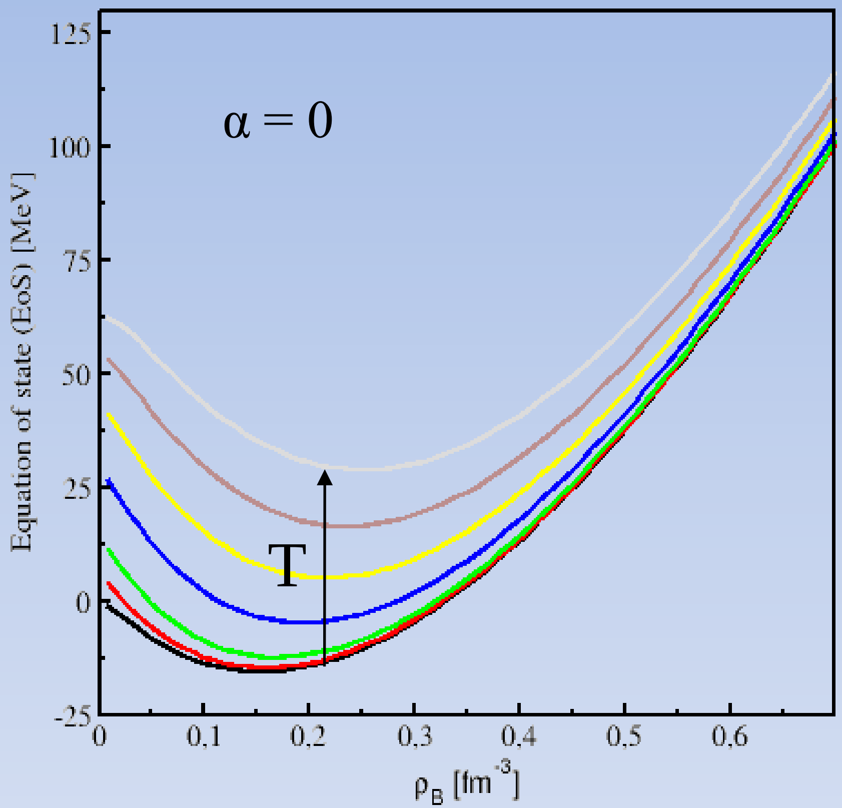
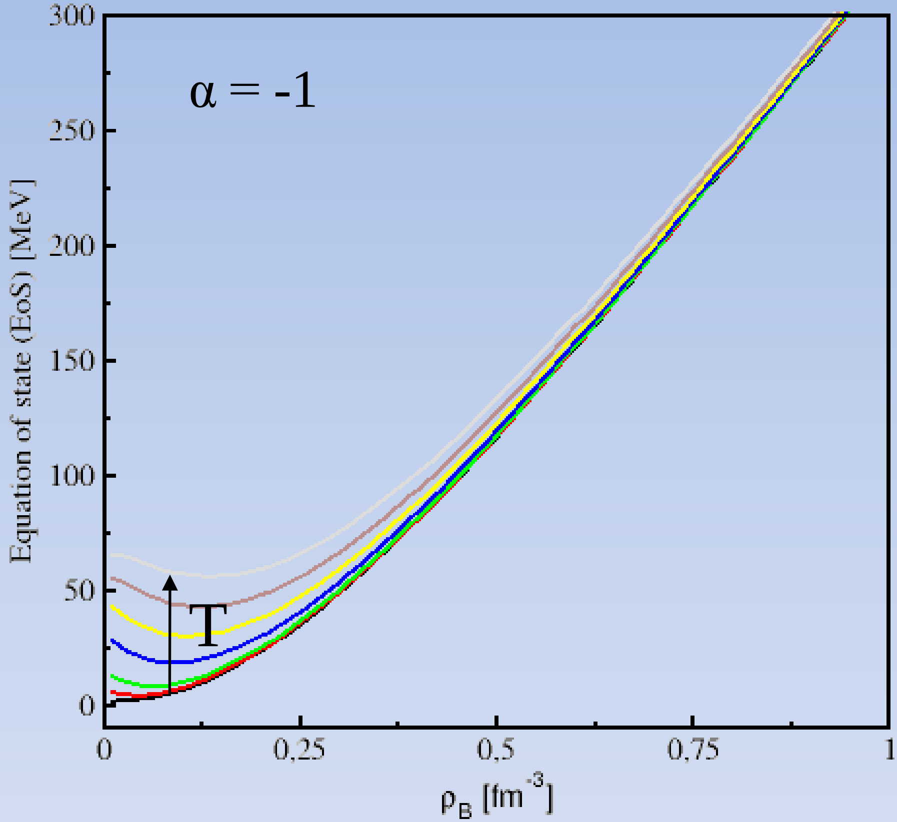
NLD predictions: hot matter…





Final remarks & outlook…

NLD model

→ keeping simplicity (RMF) to describe complexity (**non-linear ρ & p dependences)**

→ realized by covariant introduction of regulators on a Lagrangian level

→ in NLD: cut-off Λ regulates high ρ- & p-components of mean-fields

NLD Results

→ EoS soft at low ρ (K~250 MeV), but stiff at high ρ

remarkable agreement with microscopic DBHF

→ Correct MD for in-medium proton (!) and (!) antiproton interactions

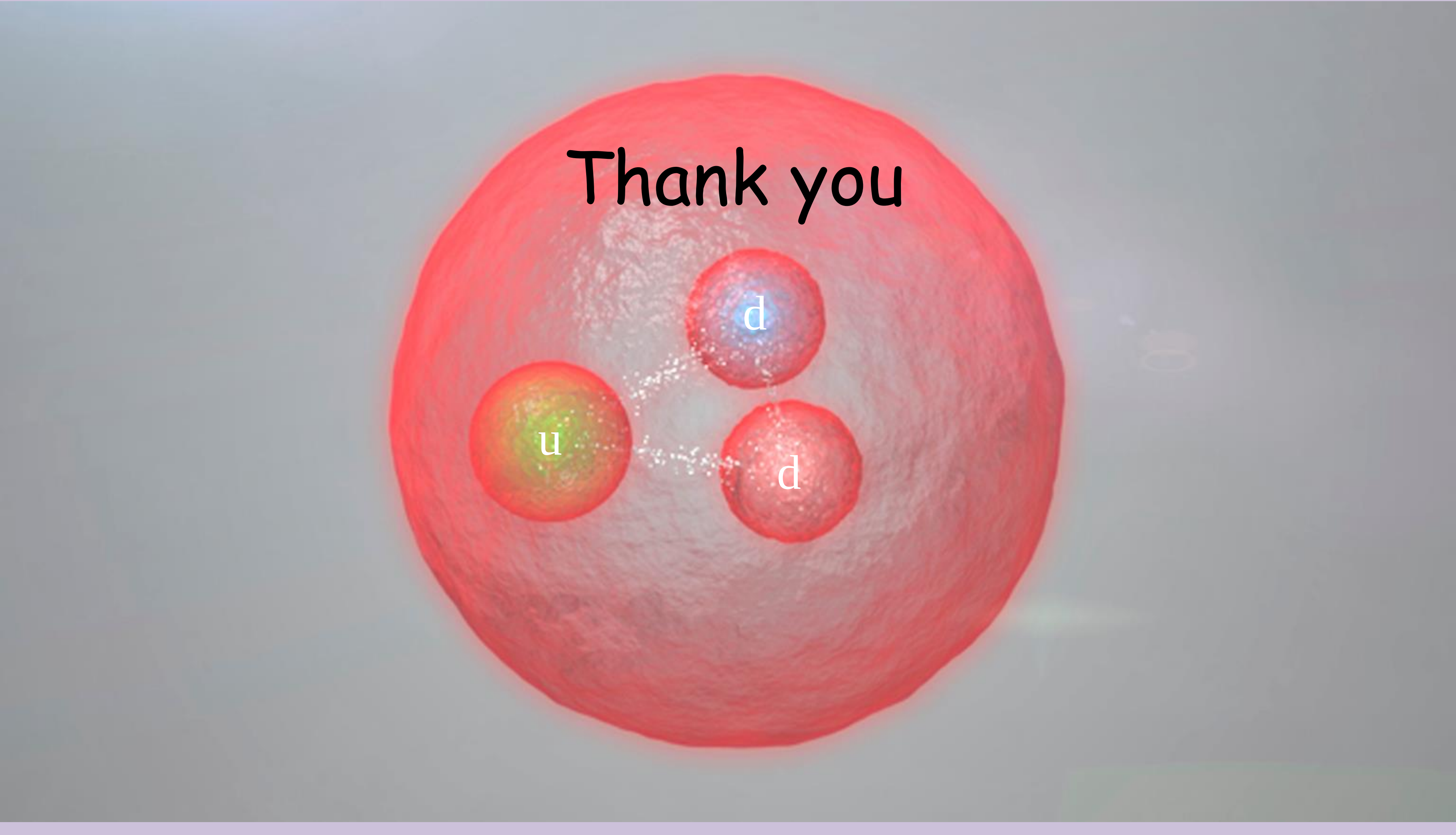
→ compatible with all recent observations of high-ρ EoS & NS

Under progress developments

→ HADES experiment for π+A induced reactions

→ many exp. new data on Λ-production

→ comparison with theory through transport simulations in progress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