Strong Cosmic Censorship with a positive cosmological constant

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Neutron Stars in the GW era

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centra

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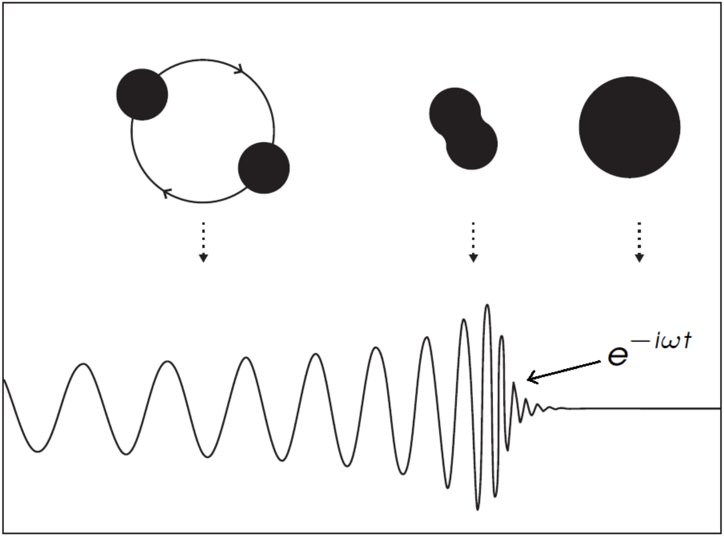
based on: **Phys. Rev. Lett. 120 (2018), 031103**
JHEP 1910 (2019) 280

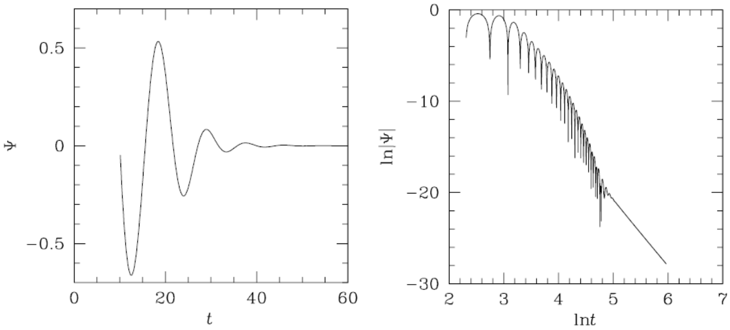
The waveform of a binary coalescence



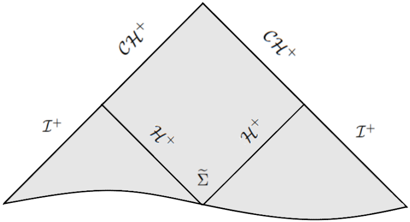
The ringdown waveform of a black hole

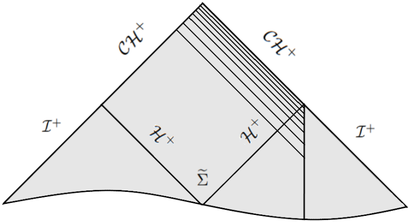
The **characteristic damped oscillations** that a perturbed black hole undergoes are called **quasinormal modes (QNMs)** $\rightarrow \omega = \omega_R + i\omega_I$

- **time dependence of perturbation** $\rightarrow e^{-i\omega t} \sim e^{-i\omega_R t} e^{\omega_I t}$
- **characteristic frequency** of the ringdown $\rightarrow \omega_R$
- **decay rate** of the ringdown $\rightarrow \omega_I < 0$



The Reissner-Nordström solution





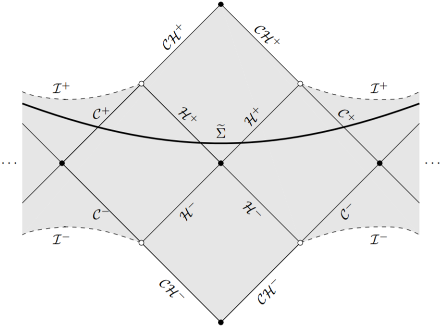
Exterior

$\phi \sim v^{-p}$, at late times

Interior

$\rho_{CH^+} \sim v^{-2(p-1)} e^{2\kappa_- v}$

The Reissner-Nordström-de Sitter solution



$$\phi \sim e^{-\alpha t},\ t \to \infty^{1,2}, \alpha = \inf\{-\mathrm{Im}(\omega)\}$$

We consider a **RNdS BH**, where

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_2^2,$$

$$f(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{\Lambda r^2}{3},$$

with three horizons $r_- < r_+ < r_c$.

The **surface gravity** of each horizon is

$$\kappa_h = \frac{1}{2}|f'(r_h)|,\ h \in \{-,+,c\}.$$

[1] P. Hintz, A. Vasy, **1606.04014**
[2] P. Hintz, **1612.04489**

Quasinormal modes and Strong Cosmic Censorship

The **Klein-Gordon equation** for **charged scalar fields** can be recast to

$$\frac{d^2\psi}{dr_*^2}+\left[\left(\omega-\frac{qQ}{r}\right)^2-V(r)\right]\psi=0,$$

where $dr_* = dr/f(r)$ the tortoise coordinate and

$$V(r)=f(r)\left(\mu^2+\frac{\ell(\ell+1)}{r^2}+\frac{f'(r)}{r}\right),\qquad \ell=0,\,1,\,2\ldots$$

To obtain the **QNMs** we impose the **boundary conditions**

$$\psi\sim\begin{cases} e^{-i(\omega-qQ/r_+)r_*}, & r_*\to-\infty \quad (r\to r_+)\\ e^{i(\omega-qQ/r_c)r_*}, & r_*\to\infty \quad (r\to r_c)\end{cases}$$

Strong Cosmic Censorship $\leftrightarrow$ $\beta\equiv\frac{\alpha}{\kappa_-}<\frac{1}{2}$

QNMs of RNdS BHs

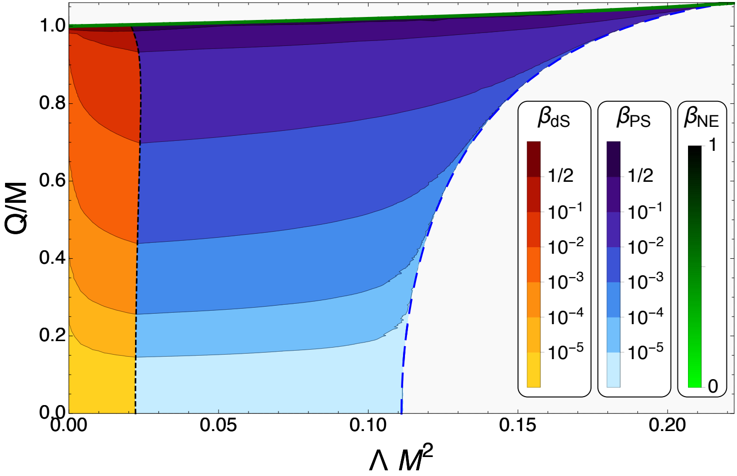
Numerical methods

- WKB method
- Matrix methods
- Pseudo-spectral methods
- Time domain integration

The 3 families

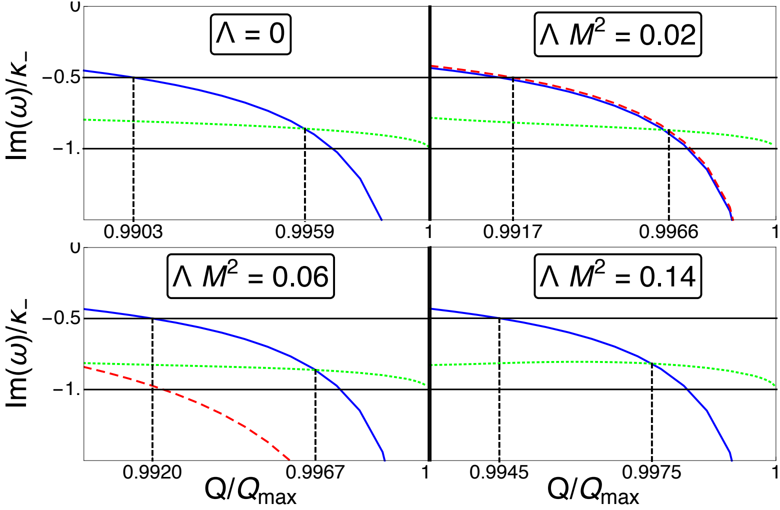
- Photon sphere modes
- de Sitter modes
- Near-extremal modes

Neutral scalar fields in RNdS



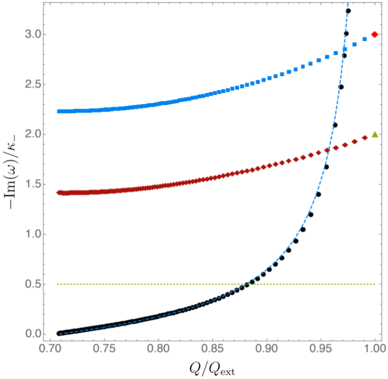
V. Cardoso, J. L. Costa, KD, P. Hintz, A. Jansen, **Phys. Rev. Lett. 120 (2018), 031103**

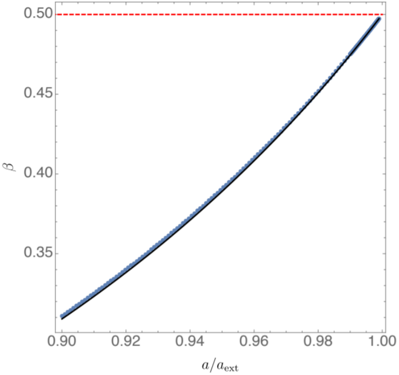
Neutral scalar fields in RNdS



V. Cardoso, J. L. Costa, KD, P. Hintz, A. Jansen, **Phys. Rev. Lett. 120 (2018), 031103**

Gravitational perturbations of RNdS[1] and Kerr-dS[2]





[1] O. J. C. Dias, H. S. Reall, J. E. Santos, **JHEP 1810 (2018) 001**

[2] O. J. C. Dias, F. C. Eperon, H. S. Reall, J. E. Santos, **Phys. Rev. D 97 (2018), 104060**

Neutral scalar fields in Horndeski theory

In the particular **Horndeski theory** the action reads

$$S = \int_{\mathscr{M}} d^4x\sqrt{-g}\left(\frac{\mathscr{R}-2\Lambda}{16\pi} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\left(g^{\mu\nu} + \eta G^{\mu\nu}\right)\partial_\mu\phi\partial_\nu\phi\right),$$

with $G_{\mu\nu} \equiv \mathscr{R}_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\mathscr{R}$ the **Einstein tensor**. The equations of motion are:

$$\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}\left(g^{\mu\nu} + \eta G^{\mu\nu}\right)\partial_\nu\phi\right) = 0,$$

$$dF = d\star F = 0,$$

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi T_{\mu\nu},$$

where

$$T_{\mu\nu} = T^{(s)}_{\mu\nu} + T^{(em)}_{\mu\nu} + \eta\Theta_{\mu\nu},$$

with

$$T^{(s)}_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - \frac{1}{2}g_{\mu\nu}\partial_\alpha\phi\partial^\alpha\phi,$$

$$T^{(em)}_{\mu\nu} = F^{\alpha}{}_{\mu}F_{\mu\alpha} - \frac{1}{4}g_{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}.$$

Neutral scalar fields on RNdS in Horndeski theory

The new component of the **modified energy-momentum tensor** is:

$$\Theta_{\mu\nu} = -\frac{1}{2}\partial_\mu\phi\partial_\nu\phi\mathscr{R} + 2\partial_\alpha\phi\partial_{(\mu}\phi\mathscr{R}^\alpha{}_{\nu)} - \frac{1}{2}G_{\mu\nu}\left(\partial\phi\right)^2 + \nabla^\alpha\phi\nabla^\beta\phi\mathscr{R}_{\mu\alpha\nu\beta}$$
$$+ \nabla_\mu\nabla^\alpha\phi\nabla_\nu\nabla_\alpha\phi - \nabla_\mu\nabla_\nu\phi\Box\phi$$
$$+ \frac{1}{2}g_{\mu\nu}\left[-\nabla^\alpha\nabla^\beta\phi\nabla_\alpha\nabla_\beta\phi + (\Box\phi)^2 - 2\partial_\alpha\phi\partial_\beta\phi\mathscr{R}^{\alpha\beta}\right].$$

The **QNM equation** is

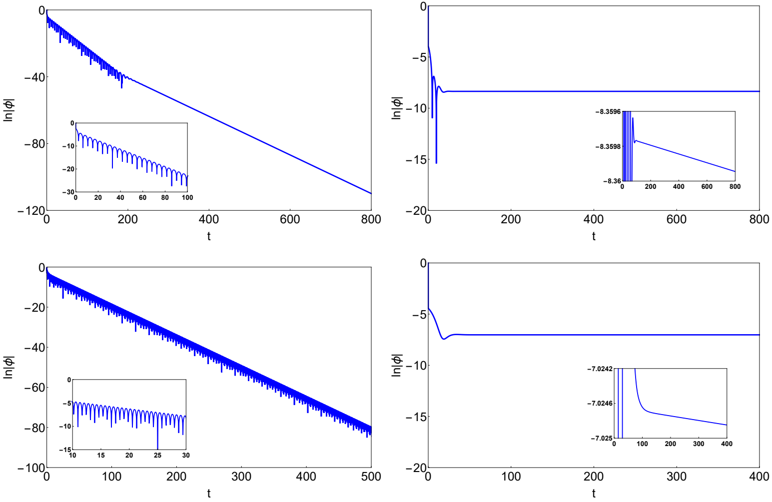
$$\frac{d^2\psi}{dr_*^2} + (\omega^2 - V(r))\psi = 0,$$

with $k = 1 - \eta\left(\Lambda + Q^2/r^4\right)$ and

$$V(r) = f(r)\left(f(r)\frac{2rkk'' + 4kk' - r(k')^2}{4rk^2} + f'(r)\frac{2k + rk'}{2rk} + \left(1 + \frac{2Q^2\eta}{r^4 k}\right)\frac{\ell(\ell+1)}{r^2}\right).$$

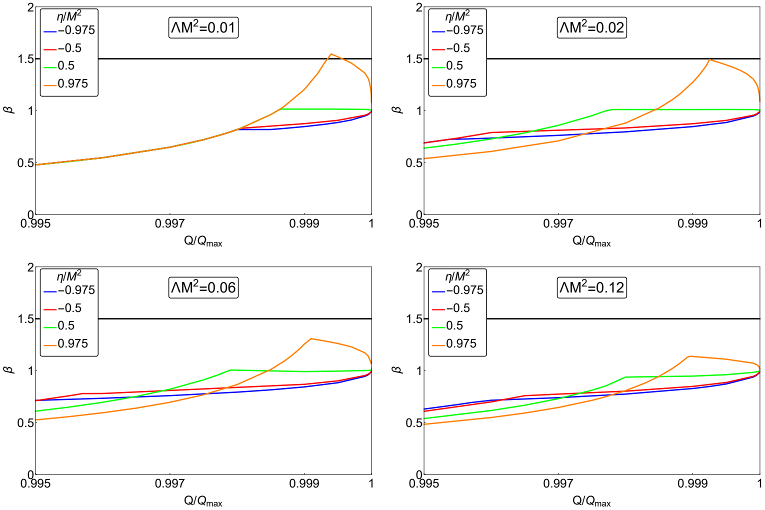
Strong Cosmic Censorship in the particular theory $\leftrightarrow$ $\beta \equiv \frac{\alpha}{\kappa_-} < \frac{3}{2}$

Late-time behavior of scalar fields on RNdS in Horndeski theory



KD, R. D. B. Fontana, F. Mena, E. Papantonopoulos, **JHEP 1910 (2019) 280**

Strong Cosmic Censorship in Horndeski theory



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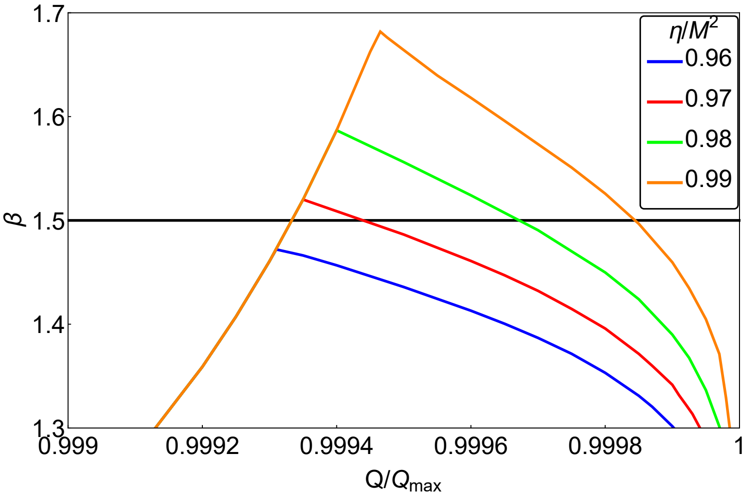
Strong Cosmic Censorship in Horndeski theory



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Summary & Conclusions

β **controls** the stability of Cauchy horizons and **the fate of Strong Cosmic Censorship**.

If β is larger than a specified value, the Cauchy horizon can be stabilized and **Strong Cosmic Censorship may be violated**.

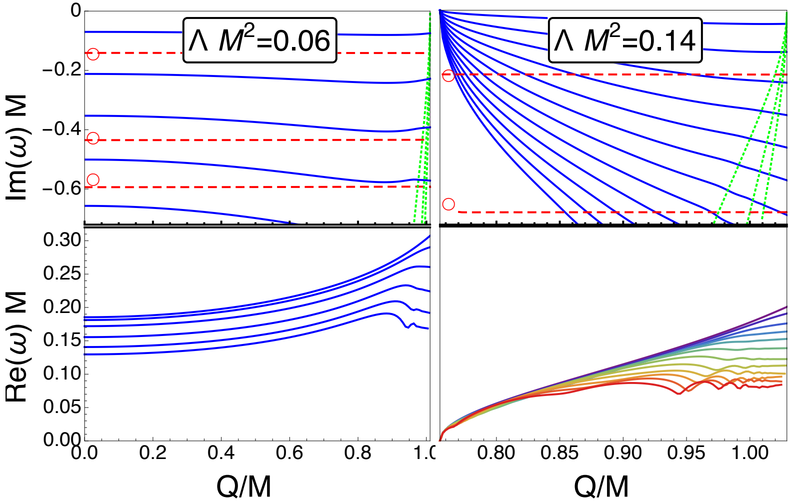
By studying **scalar perturbations** propagating on Reissner-Nordström-de Sitter in **General Relativity** we find regions of the parameter space for which $\beta > \mathbf{1/2}$.

By studying **scalar perturbations** propagating on Reissner-Nordström-de Sitter in **Horndeski theory** we find regions of the parameter space for which $\beta > \mathbf{3/2}$.

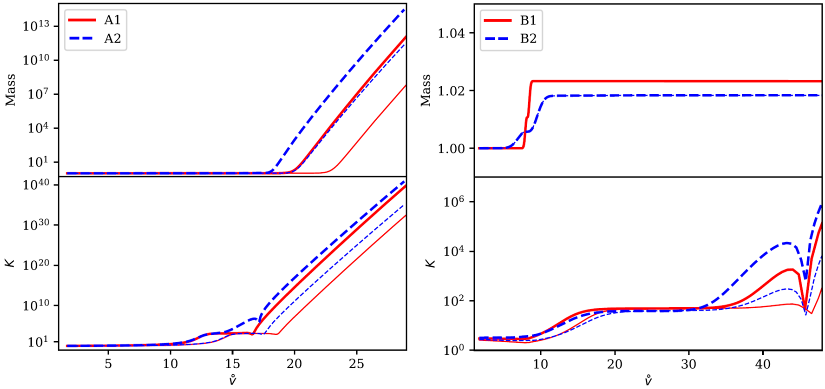
Our results indicate a potential **failure of determinism in General Relativity and beyond**.

Appendix

Neutral scalar QNMs in RNdS



V. Cardoso, J. L. Costa, KD, P. Hintz, A. Jansen, **Phys. Rev. Lett. 120 (2018), 031103**



R. Luna, M. Zilhão, V. Cardoso, J. L. Costa, J. Natario, **Phys. Rev. D 99 (2019), 064014**

A matrix method for quasinormal modes

The **master equation** can be recast to

$$\tau(x,\omega)\phi''(x)+\lambda(x,\omega)\phi'(x)+s(x,\omega)\phi(x)=0,$$

under suitable transformations. To **discretize the derivatives** of the equation we use the **Taylor expansion**

$$\phi(x)=\phi(x_0)+(x-x_0)\phi'(x_0)+\frac{1}{2}(x-x_0)^2\phi''(x_0)+\frac{1}{3!}(x-x_0)^3\phi'''(x_0)+\dots$$

Using **the data points** $(x_1,x_2,\dots,x_N)$ we get

$$\Phi=MD,$$

where

$$\Phi=\left(\phi(x_1),\phi(x_2),\phi(x_3),\phi(x_4),\dots\right)^T,$$

$$M=\begin{pmatrix} 1 & x_1-x_0 & \frac{(x_1-x_0)^2}{2} & \frac{(x_1-x_0)^3}{3!} & \frac{(x_1-x_0)^4}{4!} & \dots \\ 1 & x_2-x_0 & \frac{(x_2-x_0)^2}{2} & \frac{(x_2-x_0)^3}{3!} & \frac{(x_2-x_0)^4}{4!} & \dots \\ 1 & x_3-x_0 & \frac{(x_3-x_0)^2}{2} & \frac{(x_3-x_0)^3}{3!} & \frac{(x_3-x_0)^4}{4!} & \dots \\ 1 & x_4-x_0 & \frac{(x_4-x_0)^2}{2} & \frac{(x_4-x_0)^3}{3!} & \frac{(x_4-x_0)^4}{4!} & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \end{pmatrix},$$

and

$$D=\left(\phi(x_0),\phi'(x_0),\phi''(x_0),\phi'''(x_0),\dots\right)^T.$$

A matrix method for quasinormal modes

To calculate specific elements of D, we can use **Cramer's rule**

$$D_i = \frac{\det(M_i)}{\det(M)},$$

where M_i is the matrix formed by replacing the $i-$th column of M by the column vector Φ, e.g.

$$\phi''(x_0) = \frac{\det(M_3)}{\det(M)}$$

We end up with the **matrix equation**

$$\mathscr{M}\Phi = 0,$$

where $\mathscr{M}$ is a **square matrix that depends on** ω and $\Phi = (\phi(x_1), \phi(x_2), \ldots, \phi(x_N))^T$. To find the eigenvalues we require

$$\det(\mathscr{M}) = 0.$$

QNMSpectral package: A pseudo-spectral method

QNMSpectral is a *Mathematica* package which solves QNM equations of the form

$$\tau(x,\omega)\phi''(x)+\lambda(x,\omega)\phi'(x)+s(x,\omega)\phi(x)=0,$$

The **pseudo-spectral method** utilized is using a specific grid of points to approximate a function as

$$f(x)\sim\sum_{j=0}^{N}f(x_j)L_j(x),\qquad L_j(x)=\prod_{j=0,j\neq i}^{N}\frac{x-x_j}{x_i-x_j}.$$

To calculate **derivatives of the function**

$$D^{(1)}_{j\,i}=L_j^{'}(x_i).$$

The master equation is expressed as a **generalized eigenvalue problem**

$$(M_0+\omega M_1)\phi=0,$$

and solved numerically.

Weak solutions of the field equations

Consider a **scalar field** ϕ and the **RNdS metric** $g_{\mu\nu}$ satisfying

$$\mathscr{R}_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \mathscr{R} + g_{\mu\nu} \Lambda = 8\pi T_{\mu\nu},$$

where $T_{\mu\nu} \sim (\partial\phi)^2$ and $\mathscr{R}, \mathscr{R}_{\mu\nu} \sim \Gamma^2 + \partial\Gamma$. A **weak solution** must satisfy

$$\int_{\mathscr{V}} (\Gamma^2 + \partial\Gamma + g_{\mu\nu}\Lambda)\psi = 8\pi \int_{\mathscr{V}} (\partial\phi)^2 \psi$$

At the Cauchy horizon $f(r) \sim |r - r_-|$ and

$$r_* = \int \frac{dr}{f(r)} \sim -\frac{\ln|r - r_-|}{2\kappa_-},$$

therefore the Klein-Gordon equation admits **solutions of the asymptotic form**

$$\phi \sim e^{-i\omega(u+2r_*)} \sim e^{-i\omega u} |r - r_-|^{i\omega/\kappa_-} \sim e^{-i\omega u} |r - r_-|^{i\omega_R/\kappa_-} |r - r_-|^{\beta}$$

where $\beta = \alpha/\kappa_-$ and $\omega = \omega_R + i\omega_I$. **For weak solutions to exist** we require $\Gamma \in L^2_{\text{loc}}$ and $\phi \in H^1_{\text{loc}}$, that is

$$\int_{\mathscr{V}} (\partial_r \phi)^2 \sim \frac{|r - r_-|^{2\beta - 1}}{2\beta - 1} < \infty \qquad \Rightarrow \qquad \beta > \frac{1}{2}$$

Scalar QNMs of RNdS BHs: the 3 families

Photon Sphere Modes

- **Dominant mode for $n = 0$, $\ell \to \infty$**

de Sitter Modes

- Deformations of pure dS spacetime QNMs

$$\omega_{n=0,\mathrm{pure\,dS}} / \kappa_c^{\mathrm{dS}} = -i\ell \,,$$
$$\omega_{n\neq 0,\mathrm{pure\,dS}} / \kappa_c^{\mathrm{dS}} = -i(\ell + n + 1) \,.$$

- **Dominant mode for $n = 0$, $\ell = 1$**

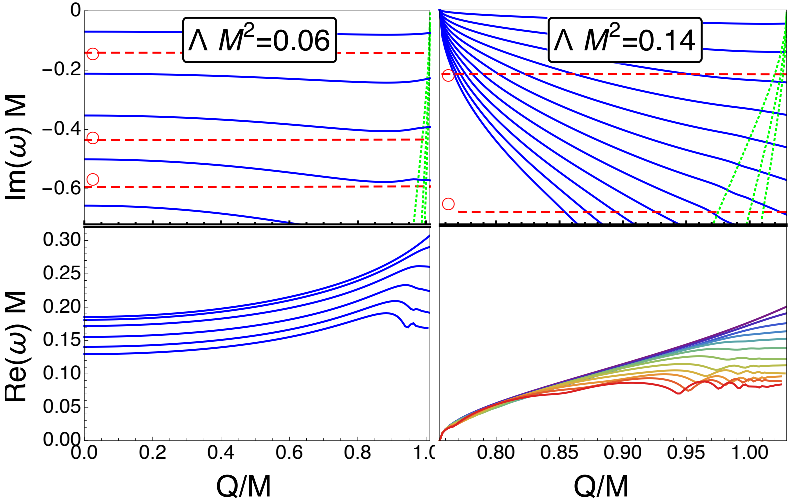
Near-Extremal Modes

- In the near-extremal limit $r_- \approx r_+$

$$\omega_{\mathrm{NE}} \approx \frac{qQ}{r_-} - i\left(n + \frac{1}{2} + \sqrt{\left(\ell + \frac{1}{2}\right)^2 + M^2\left(\mu^2 - q^2\right)}\right)\kappa_-$$

- **Dominant mode for $n = 0$, $\ell = 0$**

Scalar QNMs of RNdS



V. Cardoso, J. L. Costa, KD, P. Hintz, A. Jansen, **Phys. Rev. Lett. 120 (2018), 031103**

Charged fermionic perturbations of RNdS BHs

The **Dirac equation** for **charged fermions** can be recast to the coupled system

$$\frac{dF^+}{d\tilde{r}_*} - WF^+ + \omega F^- = 0,$$
$$\frac{dF^-}{d\tilde{r}_*} + WF^- - \omega F^+ = 0,$$

can be **decoupled** to

$$\frac{d^2F^+}{d\tilde{r}_*^2} + \left(\omega^2 - V_+\right) F^+ = 0,$$
$$\frac{d^2F^-}{d\tilde{r}_*^2} + \left(\omega^2 - V_-\right) F^- = 0,$$

with

$$V_\pm = W^2 \pm \frac{dW}{d\tilde{r}_*}, \qquad W = \frac{\xi\sqrt{f(r)}}{r\left(1 - \frac{qQ}{r\omega}\right)}, \qquad d\tilde{r}_* = \frac{\left(1 - \frac{qQ}{r\omega}\right)}{f(r)} dr,$$

and $\xi = \pm 1, \pm 2, \dots$.

To obtain the **QNMs** we impose the **boundary conditions**

$$F^\pm \sim \begin{cases} e^{-i\omega\tilde{r}_*}, & \tilde{r}_* \to -\infty \quad (r \to r_+) \\ e^{i\omega\tilde{r}_*}, & \tilde{r}_* \to \infty \quad (r \to r_c) \end{cases}$$

Charged fermionic QNMs of RNdS BHs: the 3 families

Photon Sphere Modes

- Complex modes that can be traced back to the photon sphere
- **Dominant mode in the region of interest for $n = 0$, $\xi = 1$**

de Sitter Modes

- Purely imaginary modes for $q = 0$, independent of Q
- Deformations of pure dS spacetime Dirac QNMs

$$\omega_{pure\,\mathrm{dS}}/\kappa_c^{\mathrm{dS}} = -i(\xi + n + \frac{1}{2})\,.$$

- **Dominant mode for $n = 0$, $\xi = 1$**

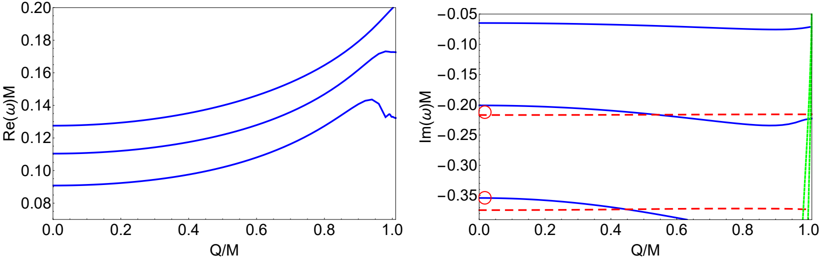
Near-Extremal Modes

- Purely imaginary modes for $q = 0$
- In the extremal limit $r_- = r_+$

$$\omega_{\mathrm{NE}} = \frac{qQ}{r_-} - i(\xi + n + \frac{1}{2})\kappa_- = \frac{qQ}{r_+} - i(\xi + n + \frac{1}{2})\kappa_+$$

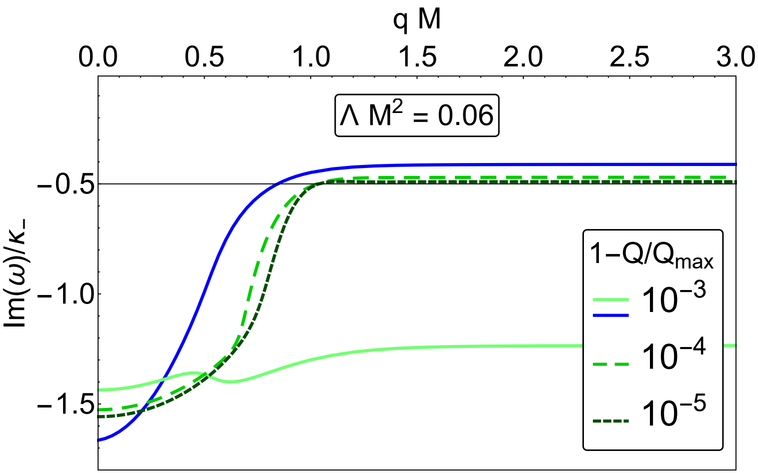
- **Dominant mode for $n = 0$, $\xi = 1$**

Fermionic QNMs of RNdS



KD, **Phys.Lett. B 795 (2019), 211-219**

Dominant charged fermionic modes and SCC



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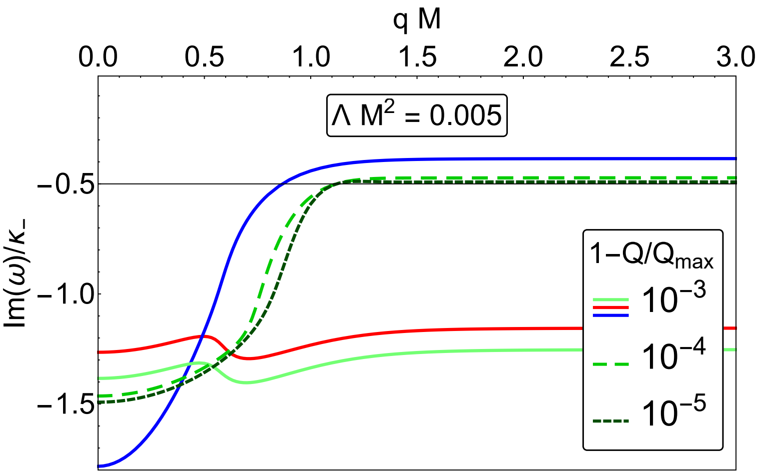
Dominant charged fermionic modes and SCC



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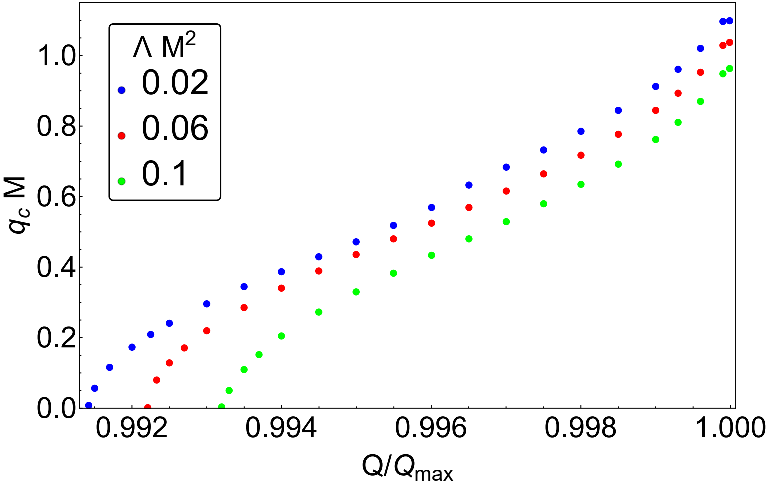
Behavior of the critical fermionic charge



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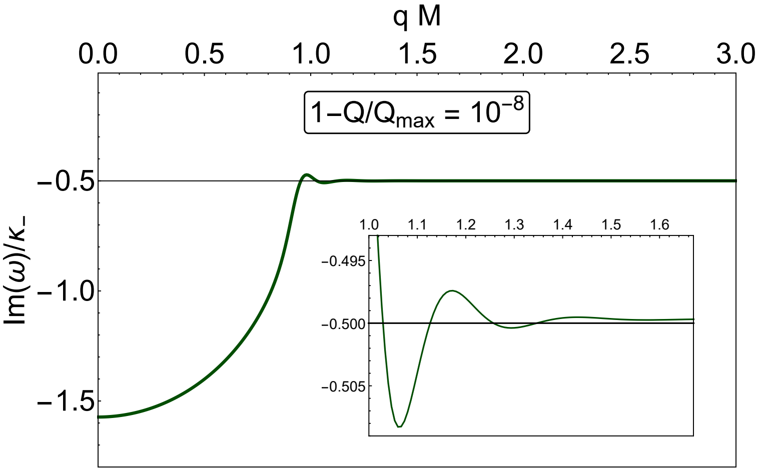
QNM "oscillations" and SCC



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