Speed of sound constraints in dense nuclear matter: applications on the bulk properties of rapidly-rotating neutron stars[1]

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Thursday 14th of November 2019

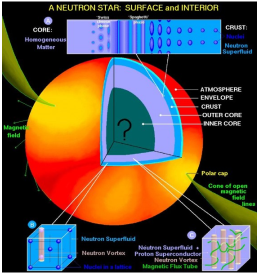
Neutron Stars in the GW Era
13-15 November 2019 Theoretical Astrophysics (TAT) Eberhard Karls University of Tuebingen, Germany

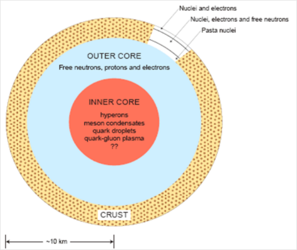
[1] This work was supported by the the DAAD Germany-Greece grant ID 57340132.

Outline of the talk

- Short introduction for neutron stars (NSs)
- Why are so interesting the rapidly (maximally) rotating stars?
- Discussion about the speed of sound bounds
- Maximum mass (and frequency) configurations for rotating NSs
- The stability condition for rotating NS
- Presentation of the effects on the bulk properties (mass, radius, period (frequency), Kerr parameter, moment of inertia)
- Conclusions and perspectives

Bulk neutron star properties





- Mean density: $\sim 5 \times 10^{14} \mathrm{g/cm^3}$
- Radius: $10 - 15$ km
- Mass: $1 - 2\ M_\odot$, very recently: $M_{\max} = 2.14^{+0.10}_{-0.09}\ M_\odot$
- Frequency: theoretically ≥ 2000 Hz, the maximum measured 716 Hz
- Magnetic field: $10^{12} - 10^{18}\mathrm{Gauss}$ (magnetars)
- Temperature: For cold NS $\leq MeV$ $(\leq 10^{10}\ K)$

Why we study rapidly rotating neutron stars?

- Rotating neutron stars offers much more information compared to static ones
- Basic bulk properties including: Kepler frequency (upper allowed limit of rotation), moment of inertia, Kerr parameter, deformation, braking index
- All of them depend sensitively on employed equation of state (and consequently to its possible bounds)
- The remnant of the GW170817 merger leads to differentially rotating neutron star close to mass-sending limit (Kepler frequency)
- Moreover, the case of a millisecond pulsar (in a binary system) which acquired angular momentum by accretion and becoming a maximally rotating pulsar with measured mass

Speed of sound bounds

- Weinberg showed that the speed of sound is much less than the speed of light for a cold nonrelativistic fluid. It increases with temperature, but it does not exceed the value $c/\sqrt{3}$ at the limit of very high temperatures
- Lattimer provided theoretical arguments that the causal limit is too extreme because the highly compressed hadronic matter may convert asymptotically to free quark matter where the speed of sound is $c/\sqrt{3}$
- Recently, Bedaque and Steiner have provided simple arguments that support the limit $c/\sqrt{3}$ in nonrelativistic and/or weakly coupled theories. This was demonstrated in several classes of strongly coupled theories with gravity duals
- Finally, it is worthy to mention that in a previous analysis Olson employed a phenomenological approach in the context of the kinetic theory, to determine an upper limit for the maximum mass of NS. In this approach the upper bound of the speed of sound appears to be less than the speed of light

The suggested speed of sound bounds

The pressure P, the energy density $\mathcal{E}$ the density ρ and the baryon density n in nuclear mater are given as follow:

$$\mathcal{E} = n\left(E + mc^2\right) = \rho c^2 \qquad \text{and} \quad P = n\frac{\mathrm{d}\mathcal{E}}{\mathrm{d}n} - \mathcal{E}$$

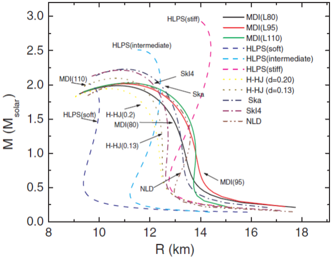
The adiabatic speed of sound is defined as

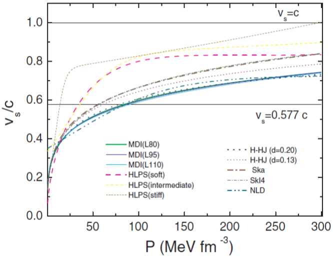
$$\frac{v_s}{c} = \sqrt{\left(\frac{\partial P}{\partial \mathcal{E}}\right)_S}$$

where S is the entropy per baryon. In the present work we consider the following three upper bounds for the speed of sound

1. $\frac{v_s}{c} \leq 1$: causality limit from special relativity
2. $\frac{v_s}{c} \leq \frac{1}{\sqrt{3}}$: from QCD and other theories
3. $\frac{v_s}{c} \leq \left(\frac{\mathcal{E} - P/3}{P + \mathcal{E}}\right)^{1/2}$: from relativistic kinetic theory

Speed of sound for various equations of state



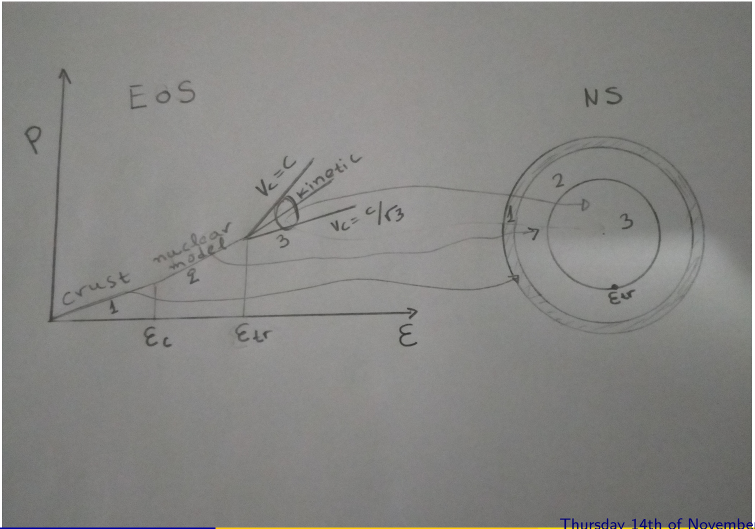


Basic assumptions for the maximum mass configuration

Hartle pointed out that causality arguments are not enough to allow a determination of the upper bound of the speed of sound. We consider the following four assumptions:

1. The matter of the neutron star is a perfect fluid described by a one-parameter equation of state between the pressure P and the energy density $\mathcal{E}$
2. The energy density $\mathcal{E}$ is non-negative (because of the attractive character of gravitational forces)
3. The matter is microscopically stable, which is ensured by the conditions $P \geq 0$ and $dP/d\mathcal{E} \geq 0$
4. Below a critical baryon density $n_{\rm tr}$ the equation of state is well known.

Schematic representation of the employed equation of state



Speed of sound bounds-Causality limits

We have constructed the maximum mass configuration (we consider that to a very good accuracy this configuration is identified with the maximum permitted rotational frequency (Keplerian velocity) by considering the following two structures for the neutron star EoS:

a) *Maximum angular velocity for known low-density EoS*

The EoS is given through the ansatz

$$P(\mathcal{E}) = \begin{cases} P_{crust}(\mathcal{E}), & \mathcal{E} \leq \mathcal{E}_{\mathrm{c-edge}} \\ P_{NM}(\mathcal{E}), & \mathcal{E}_{\mathrm{c-edge}} \leq \mathcal{E} \leq \mathcal{E}_{\mathrm{tr}} \\ \left(\frac{v_S}{c}\right)^2 (\mathcal{E} - \mathcal{E}_{\mathrm{tr}}) + P_{NM}(\mathcal{E}_{\mathrm{tr}}), & \mathcal{E}_{\mathrm{tr}} \leq \mathcal{E}. \end{cases}$$

The EoS that we used are based on the EoS produced by the data from Akmal, Pandharipande and Ravenhall (APR) and the MDI (momentum dependent interacting) model

Speed of sound bounds-Relativistic kinetic theory

b) *Maximum angular velocity from the relativistic kinetic theory*

$$P(n) = \begin{cases} P_{crust}(n), & n \le n_{\mathrm{c-edge}} \\ P_{NM}(n), & n_{\mathrm{c-edge}} \le n \le n_{\mathrm{tr}} \\ \mathcal{C}_1 n^{a_1}(a_1 - 1) + \mathcal{C}_2 n^{a_2}(a_2 - 1), & n_{\mathrm{tr}} \le n. \end{cases}$$

In addition, in this case the maximally stiff EoS fulfills the following expression

$$\left(\frac{v_s}{c}\right)^2 = \frac{\mathcal{E} - P/3}{P + \mathcal{E}}$$

Continuity approach

We proceeded with the matching of the EoSs on the transition density considering that above $n_{\rm tr}$ the speed of sound is parameterized as follows

$$\frac{v_s}{c} = \left(a - c_1 \exp\left[-\frac{(n - c_2)}{w^2} \right] \right)^{1/2}, \quad a = 1, 1/3$$

where the parameters c_1, c_2 and w are fit to the speed of sound and its derivative at $n_{\rm tr}$, and also to the demands $v_{\rm s}(n_{\rm tr}) = [c, c/\sqrt{3}]$. The EoS for $n \geq n_{\rm tr}$ can be constructed with the help of the following recipe

$$\mathcal{E}_{i+1} = \mathcal{E}_i + \Delta\mathcal{E}, \quad P_{i+1} = P_i + \left(\frac{v_s}{c}(n_i) \right)^2 \Delta\mathcal{E},$$

$$\Delta\mathcal{E} = \Delta n \left(\frac{\mathcal{E}_i + P_i}{n_i} \right)$$

$$\Delta n = n_{i+1} - n_i$$

Rapidly-rotating neutron stars

In the framework of General Relativity, rapidly-rotating neutron stars can be described a) by the stationary axisymmetric spacetime metric

$$ds^2 = -e^{2\nu}dt^2 + e^{2\psi}\left(d\phi - \omega dt\right)^2 + e^{2\mu}\left(dr^2 + r^2 d\theta^2\right)$$

where the metric functions ν, ψ, ω and μ depend only on the coordinates r and θ, and b) the matter inside the neutron star. If we neglect sources of non-isotropic stresses, viscous stresses and heat transport, then the matter inside the neutron star can be fully described by the stress-energy tensor and modeled as a perfect fluid,

$$T^{\alpha\beta} = (\varepsilon + P)\, u^{\alpha} u^{\beta} + P g^{\alpha\beta}$$

where u^{α} is the fluid's 4-velocity. The energy density and pressure is denoted as ε and P.

The stability condition

It has been shown by Friedman and coworkers that in a constant angular momentum sequence, the turning-point of a sequence of configurations with increasing central density, separates the secular stable from unstable configuration and consequently, the condition

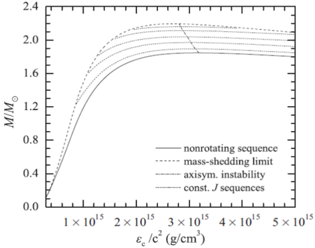
$$\frac{\partial M(\mathcal{E}_c, J)}{\partial \mathcal{E}_c} \,|_{J=\mathrm{constant}} = 0$$

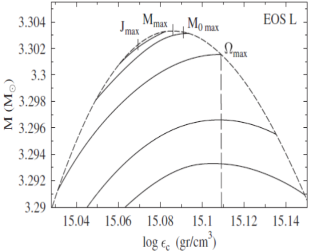
where $\mathcal{E}_c$ is the energy density in the center of the neutron star and J is the angular momentum, defines the possible maximum mass.
The Keplerian angular momentum is obtained as a self-consistency condition in the solution of Einstein's equations for a rotating neutron star. In this case an approximate expression has been obtained according to

$$\Omega_{\mathrm{max}} = \mathcal{F}_{\mathrm{max}} \left(\frac{G M_{\mathrm{max}}}{R^3_{\mathrm{max}}} \right)^{1/2}$$

The factor $\mathcal{F}_{\mathrm{max}}$ depends on the various approximations.

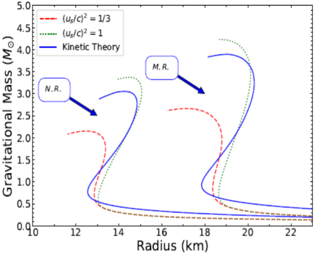
Mass-central density diagram for maximally rotating neutron stars[2]





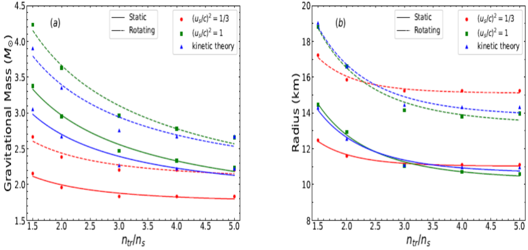
[2] Them pictures are taken from: J. Friedman and N. Stergioulas, Rotating Relativistic Stars (Cambridge University Press, Cambridge, 2013).

Mass-radius diagram for maximally rotating neutron stars



Mass - Radius diagram for the various speed of sound bounds and for transition density equals to $1.5n_{\mathrm{s}}$. Both non-rotating (N.R.) and maximally-rotating (M.R.) configurations are presented and noted on the figure. The $c/\sqrt{3}$ bound is presented with the dashed lines, the c bound with the dotted lines and the one from the relativistic kinetic theory with the solid lines.

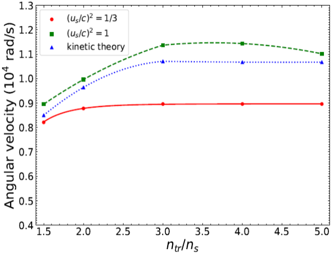
Constraints on mass and radius



$$\frac{M_{\max}}{M_\odot} = \alpha_1 \coth\left[\alpha_2\left(\frac{n_{\rm tr}}{n_{\rm s}}\right)^{1/2}\right] \qquad \frac{M_{\max}}{M_\odot} = k\sqrt{\frac{1}{n_{\rm tr}/n_{\rm s}}} \approx \frac{\alpha_1}{\alpha_2}\sqrt{\frac{1}{n_{\rm tr}/n_{\rm s}}}$$

$$\frac{R}{km} = \alpha_3 \coth\left[\alpha_4\left(\frac{n_{\rm tr}}{n_{\rm s}}\right)^{0.8}\right]$$

Kepler frequency (mass-sending limit)

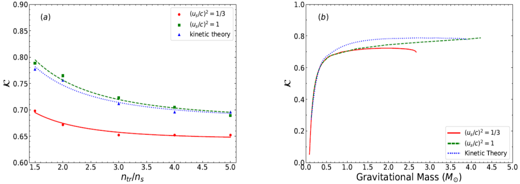


Dependence of the angular velocity on the transition density at the maximum mass configuration for the various speed of sound bounds. The data at the maximum mass configuration are presented with circles for the $v_s/c = 1/\sqrt{3}$ bound, squares for the $v_s/c = 1$ bound and triangles for the one from the relativistic kinetic theory.

Kerr parameter

$$\mathcal{K} = \frac{cJ}{GM^2}$$

It can lead to possible limits on the spin frequency and can be a criteria for determining the final fate of the collapse of a rotating compact star



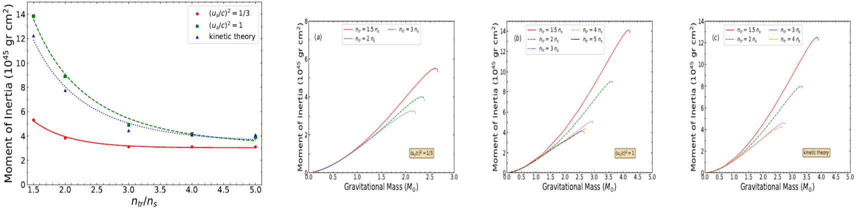
The dependence of the Kerr parameter on the transition density can be described accurately by the formula

$$\mathcal{K} = \alpha_7 \coth\left[\alpha_8\left(\frac{n_{\mathrm{tr}}}{n_{\mathrm{s}}}\right)^{1/2}\right]$$

Moment of Inertia

It is given by the form

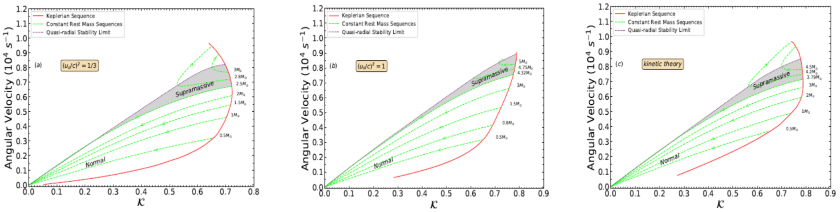
$$I = \frac{J}{\Omega}$$



$$I_{\rm max} = \alpha_5 \coth\left[\alpha_6 \left(\frac{n_{\rm tr}}{n_{\rm s}}\right)^{3/2}\right] \quad (10^{45}\ {\rm gr\ cm}^2)$$

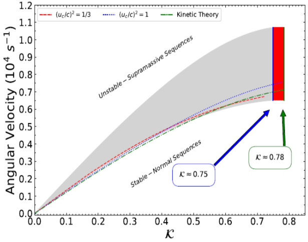
$$I_{\rm max} = k_2 \left(\frac{1}{n_{\rm tr}/n_{\rm s}}\right)^{k_3} \quad (10^{45}\ {\rm gr\ cm}^2)$$

Constant rest mass sequences



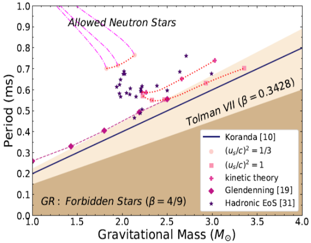
Normal and supramassive rest mass sequences as the dependence of the angular velocity on the Kerr parameter for the speed of sound bounds (a) $v_{\mathrm{s}}/c = 1/\sqrt{3}$, (b) $v_{\mathrm{s}}/c = 1$ and (c) the one from the relativistic kinetic theory at $n_{\mathrm{tr}} = 1.5 n_{\mathrm{s}}$. The red curves represent the Keplerian sequence. The constant rest mass sequences are presented with the green dashed lines while the quasi-radial stability limit with the purple dotted lines

Last stable rest mass sequences



Last stable rest mass sequence as the dependence of the angular velocity on the Kerr parameter for the various speed of sound bounds at $n_{\rm tr} = 1.5 n_{\rm s}$. Supramassive and normal areas are noted. The dashed line represent the $v_{\rm s}/c = 1/\sqrt{3}$ bound, the dotted line represent the $v_{\rm s}/c = 1$ bound and the dashed-dotted line represent the one from the relativistic kinetic theory. The gray area and Kerr limit 0.75 from our previous work and the red area from the present work are also presented. The maximum value at each case is also noted

Minimum Rational Period-Comparisons



$$P_{\min} = \mathcal{F}\left(\frac{M_\odot}{M_{\max}}\right)^{1/2}\left(\frac{R_{\max}}{10km}\right)^{3/2},\ P_{\min} = 0.79\left(\frac{0.1473}{\beta_{\max}}\right)^{3/2}\left(\frac{M_{\max}}{M_\odot}\right)$$

The data from Glendenning are also presented with diamonds. The blue curve is presented from Koranda *et al*. for comparison. The data for 23 hadronic realistic EoS are also presented with stars. Region forbidden from structure of General Relativity for $\beta = 4/9$ and the one from Tolman VII with $\beta = 0.3428$ are presented to guide the eye

Comparison from the different approaches

Table III. Comparison between the two methods presented in this study for the bulk properties of neutron stars at the maximum mass configuration for $n_{\mathrm{tr}} = 1.5 n_{\mathrm{s}}$. The symbol "c." correspond to the continuity method and the "dc." to the discontinuity one.

Speed of sound bounds	M ($M_{\odot}$)		R (km)		Ω ($10^4 s^{-1}$)		$\mathcal{K}$		I (10^{45} $gr\ cm^2$)	
	c.	dc.	c.	dc.	c.	dc.	c.	dc.	c.	dc.
c	4.231	4.229	18.812	18.800	0.895	0.896	0.788	0.786	13.850	13.799
$c/\sqrt{3}$	2.666	2.669	17.228	16.900	0.821	0.845	0.698	0.700	5.312	5.178

Conclusions and Perspectives

- Rotating neutron stars is an Astrophysical Laboratory to test dense nuclear matter properties (even richer than non-rotating neutron stars)
- Theoretical predictions of the bulk properties can be compared with observations and vice-versa. Observations may lead to confirm the bounds on the speed of sound
- The speed of sound bounds constrain in general the bulk neutron star properties
- In particular the bound $c/\sqrt{3}$ imposes strong constraints on mass, period (frequency), Kerr parameter and mainly in moment of inertia
- The bound $c/\sqrt{3}$ (compared the causality limit $v_s = c$)decreases $\sim 40\%$ the maximum mass, increases at least $\sim 15\%$ the period, decreases $\sim 15\%$ the Kerr parameter but mainly decreases 2.5 times the moment of inertia
- Further observations (a millisecond pulsar (in a binary system) acquired angular momentum by accretion and becoming a maximally rotating pulsar with measured mass) are necessary to confirm the predictions due to the speed of sound bounds.

THANK FOR YOUR ATTENTION