

IMAGE CREATION OF A THIN ACCRETION DISK AROUND JANIS-NEWMAN NAKED SINGULARITY

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Why the Image of compact objects is important?

- The specific features of the *images* can be used to extract information about the *physical properties* of the compact objects and to differentiate between gravitational theories.
- Using the deformation of the image shape of an isolated black hole we can evaluate its *spin* and *multipole moments*.
- We can detect the interaction of the black hole with an *external gravitation field*, since it leads to the formation of complicated lensing patterns with *multiple shadow images*.
- On the other side, because the shadow is not restricted to the presence of an *event horizon* we can investigate and shadows silhouette of *horizonless* objects by studying the light propagation in the vicinity of the photon region of *naked singularities* or *wormholes*.
- The experiments in the electromagnetic spectrum achieved a historical success when the Event Horizon Collaboration was able to provide the first image of the black hole shadow.

Static and spherically symmetric JNW Naked Singularity:

The static spherically symmetric Einstein massless scalar equations

$$R_{\mu\nu} = 8\pi\Phi_{,\mu}\Phi_{,\nu} \quad \Phi^{;\mu}_{;\mu} = 0,$$

the metric

$$ds^2 = - \left(1 - \frac{b}{r}\right)^\gamma dt^2 + \left(1 - \frac{b}{r}\right)^{-\gamma} dr^2 + \left(1 - \frac{b}{r}\right)^{1-\gamma} d\Omega_{(2)}^2,$$

the scalar field

$$\Phi = \frac{q}{b\sqrt{4\pi}} \ln \left(1 - \frac{b}{r}\right),$$

where

$$\gamma = \frac{2M}{b}, \quad b = 2\sqrt{M^2 + q^2}.$$

Here M and q , are the ADM mass and the scalar charge.

Thin accretion disk. Equations of motion for particles

Static and spherically symmetric spacetime metric:

$$ds^2 = g_{tt}dt^2 + g_{rr}dr^2 + g_{\theta\theta}d\theta^2 + g_{\phi\phi}d\phi^2,$$

Equations of motion. Effective potential. ($\theta = \pi/2$)

$$\frac{dt}{d\tau} = -\frac{\tilde{E}}{g_{tt}}, \quad \frac{d\phi}{d\tau} = -\frac{\tilde{L}}{g_{\phi\phi}}, \quad g_{rr} \left(\frac{dr}{d\tau} \right)^2 = V_{eff}(r)$$

$$V_{eff}(r) = -1 - \frac{\tilde{E}^2 g_{\phi\phi} + \tilde{L}^2 g_{tt}}{g_{tt} g_{\phi\phi}},$$

Stable circular orbits in the equatorial plane:

$$V_{eff}(r) = 0, \quad \frac{dV_{eff}(r)}{dr} = 0.$$

Thin accretion disk. Equations of motion for particles

Energy, angular momentum and angular velocity of the particles:

$$\tilde{E} = -\frac{g_{tt}}{\sqrt{-g_{tt} - g_{\phi\phi}\Omega^2}},$$

$$\tilde{L} = -\frac{g_{\phi\phi}\Omega}{\sqrt{-g_{tt} - g_{\phi\phi}\Omega^2}},$$

$$\Omega = \frac{d\phi}{dt} = \sqrt{-\frac{dg_{tt}}{dr} \bigg/ \frac{dg_{\phi\phi}}{dr}}.$$

Innermost stable circular orbit (ISCO):

$$V_{eff}(r) = 0, \quad \frac{dV_{eff}(r)}{dr} = 0, \quad \frac{d^2V_{eff}(r)}{dr^2} = 0.$$

$$\tilde{E}^2 \frac{d^2 g_{\phi\phi}}{dr^2} + \tilde{L}^2 \frac{d^2 g_{tt}}{dr^2} + \frac{d^2 g_{tt} g_{\phi\phi}}{dr^2} = 0$$

Spherical photon orbits. Shadow

Effective Potential for photons - V_{eff}

$$\left(\frac{dr}{d\lambda}\right)^2 + V_{eff} = 1$$

Unstable Spherical Photon Orbits:

$$V_{eff} = 1, \quad \frac{V_{eff}}{dr} = 0, \quad \frac{d^2 V_{eff}}{dr^2} \leq 0. \quad \rightarrow \quad \{\xi(r_0), \eta(r_0)\}$$

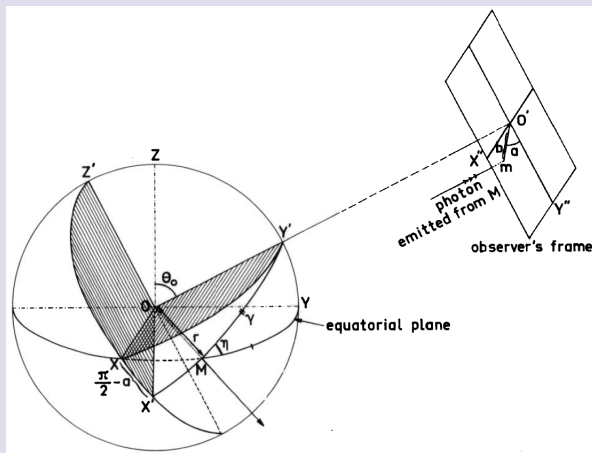
Integrals of Motion (ξ, η) - Celestial Coordinates (α, β) Mapping

$$\alpha = \lim_{r \rightarrow \infty} \left(-r^2 \sin \theta_0 \frac{d\varphi}{dr} \right) = -\frac{\xi}{\sin \theta_0},$$

$$\beta = \lim_{r \rightarrow \infty} r^2 \frac{d\theta}{dr} = \left(\eta - \frac{\xi^2}{\sin^2 \theta_0} \right)^{1/2},$$

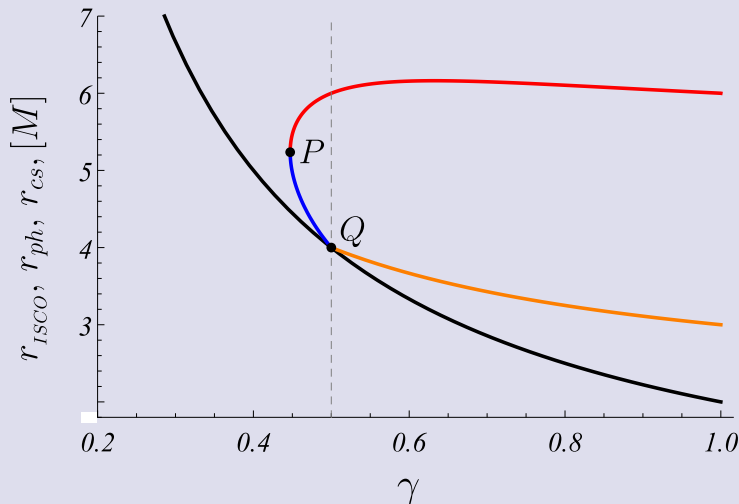
Coordinate system and observable quantities

Celestial Coordinates (α, β) or (X, Y) Mapping

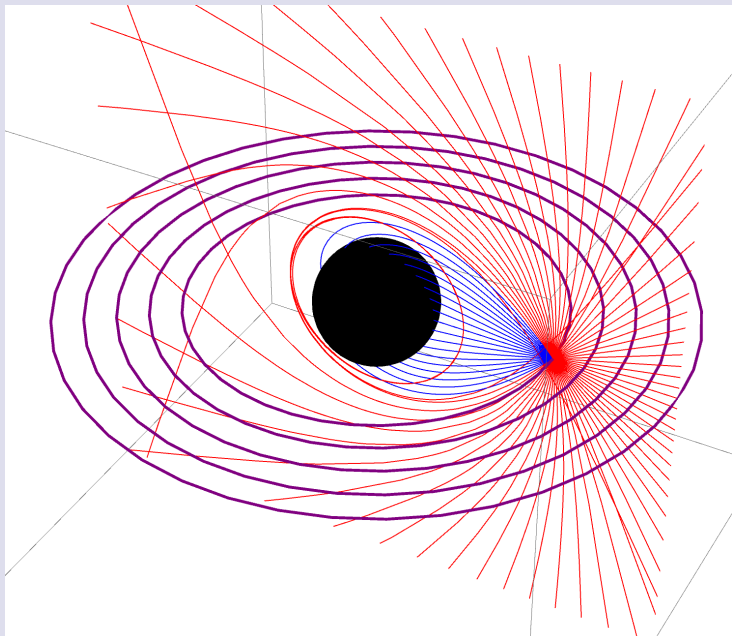


The image is taken from J.-P. Luminet: "Image of a spherical black hole with thin accretion disk *Astron. and Astrophys.* 75, 228-235 (1979).

Position of the ISCO, Photon Sphere and Curvature Singularity



Electromagnetic emission of the accretion disk. Ray orbits



The deflection of light

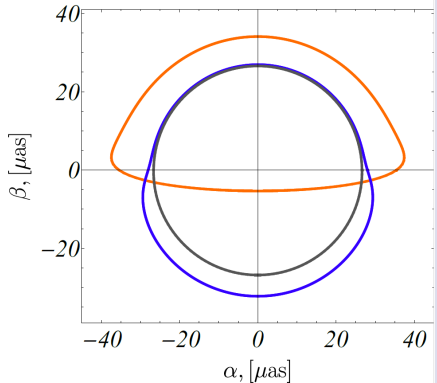
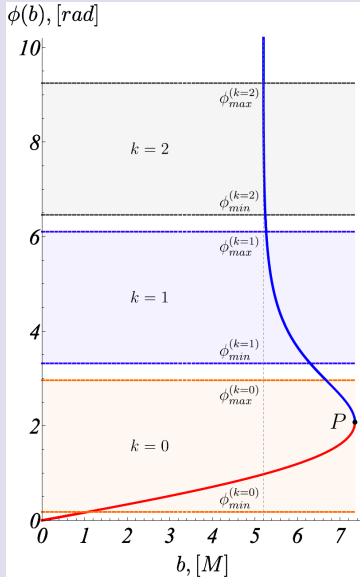
Deflection angle

$$\tilde{\alpha}(D) = \int_{r_S}^{r_{obs}} \frac{dr}{r^2(1 - \frac{b}{r})^{1-\gamma} \sqrt{\frac{1}{D^2} + \frac{1}{r^2}(1 - \frac{b}{r})^{2\gamma-1}}} - \pi$$

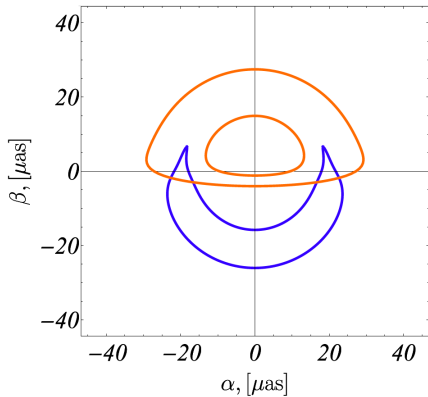
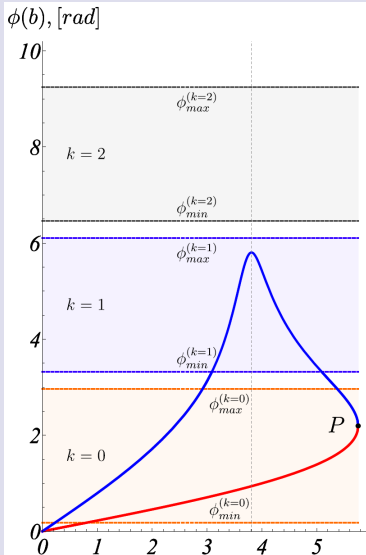
Variation of the azimuthal angle

$$\begin{aligned} \int_{r_S}^{r_{obs}} \frac{dr}{r^2(1 - \frac{b}{r})^{1-\gamma} \sqrt{\frac{1}{D^2} + \frac{1}{r^2}(1 - \frac{b}{r})^{2\gamma-1}}} \\ = k\pi - \arccos \frac{\sin \eta \tan \theta_o}{\sqrt{\sin^2 \eta \tan^2 \theta_o + 1}}, \end{aligned}$$

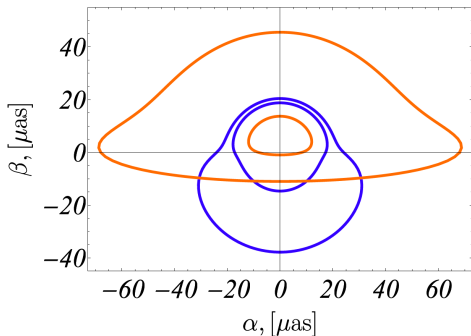
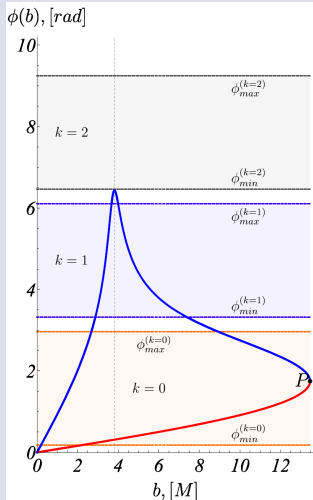
Variation of the azimuthal angle for Schwarzschild Black Hole and the Images. Influence of the Photon Sphere



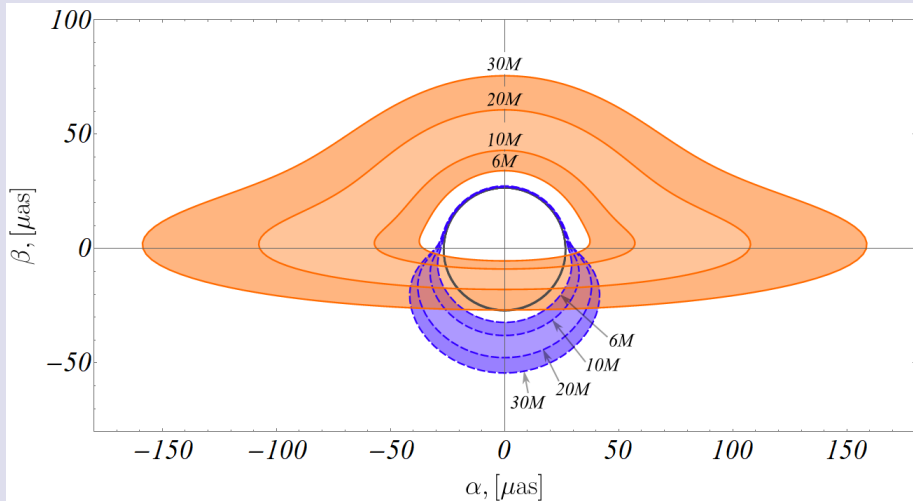
Variation of the azimuthal angle for JNW Naked Singularity Black Hole and the Images without photon sphere ($\gamma = 0.48$, $r = 5.896M$)



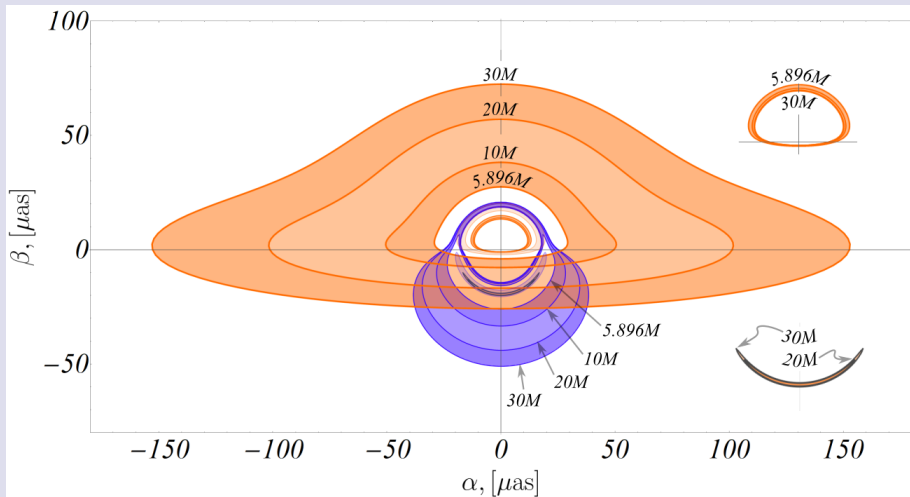
Variation of the azimuthal angle for JNW Naked Singularity Black Hole and the Images without photon sphere ($\gamma = 0.48$, $r = 13.56M$)



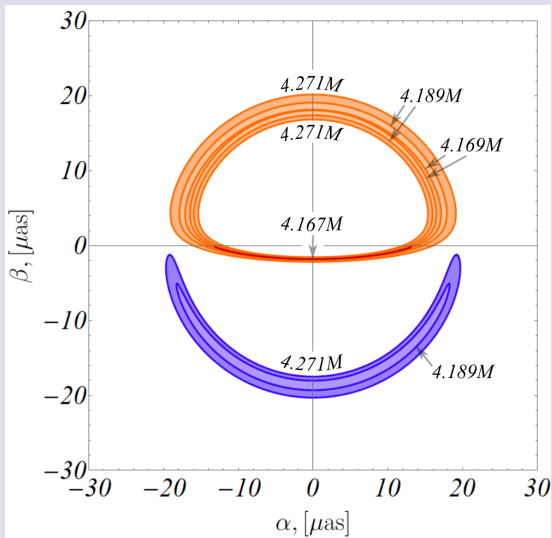
($\gamma = 1$). Isoradial curves for orbits with $r = \text{const}$, $\theta = 80^\circ$.



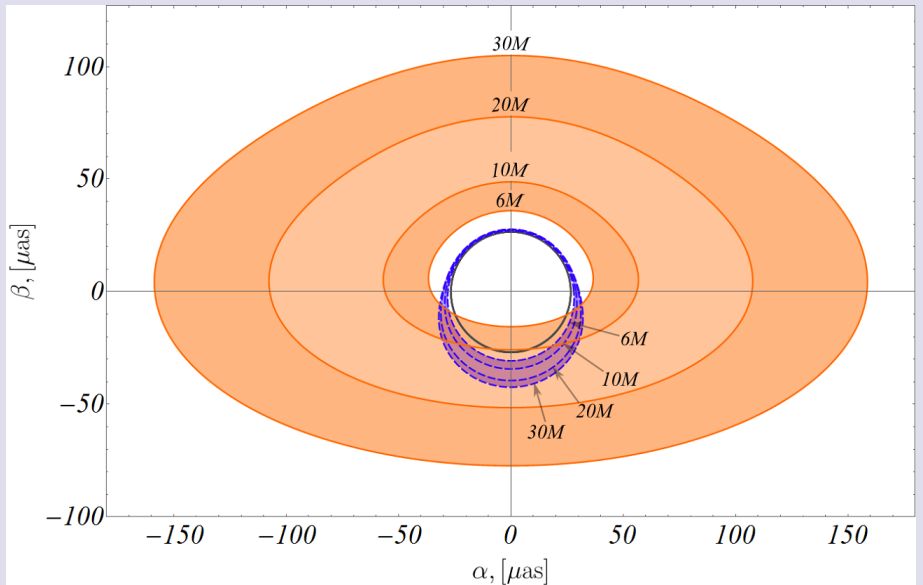
($\gamma = 0.48$). Isoradial curves for orbits with $r = \text{const}$, $\theta = 80^\circ$.



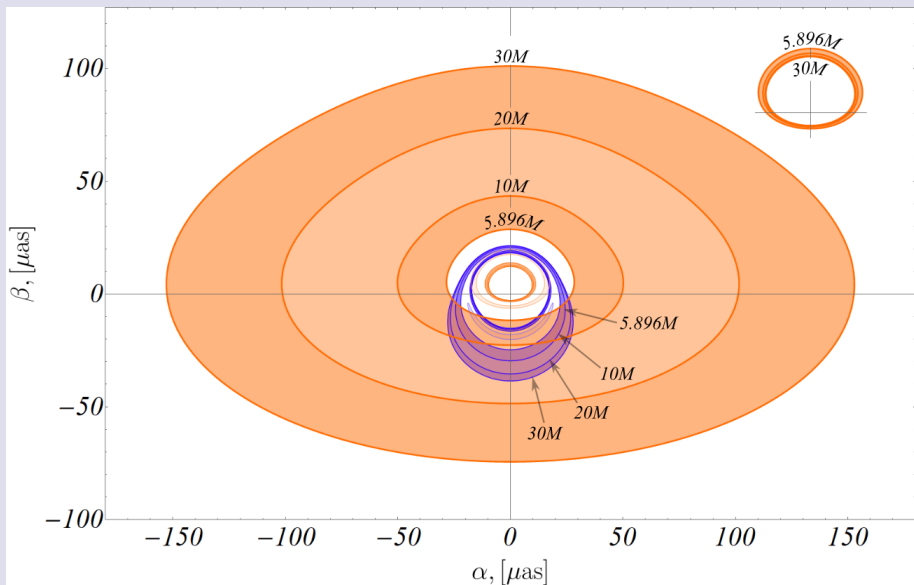
($\gamma = 0.48$). Isoradial curves for orbits with $r = \text{const}$, $\theta = 80^\circ$



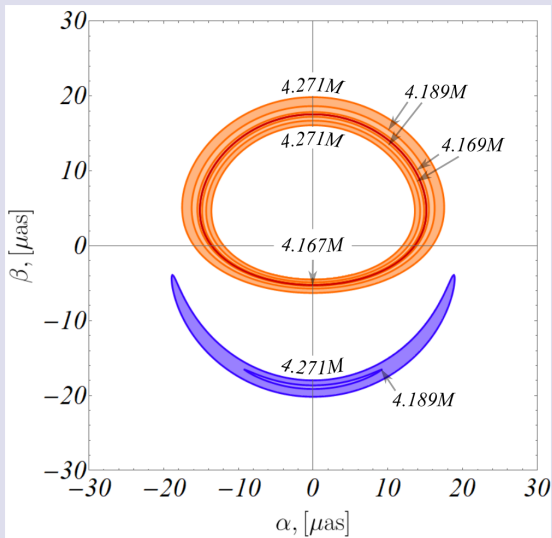
($\gamma = 1$). Isoradial curves for orbits with $r = \text{const}$, $\theta = 60^\circ$



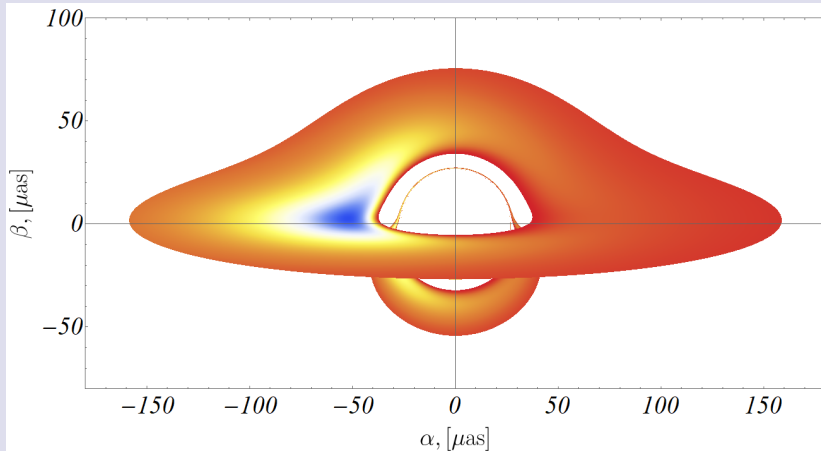
($\gamma = 0.48$). Isoradial curves for orbits with $r = \text{const}$, $\theta = 60^\circ$



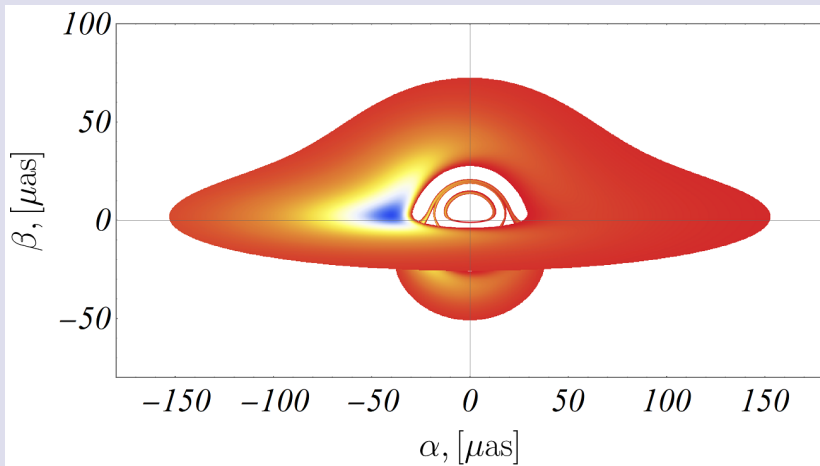
($\gamma = 0.48$). Isoradial curves for orbits with $r = \text{const}$, $\theta = 60^\circ$



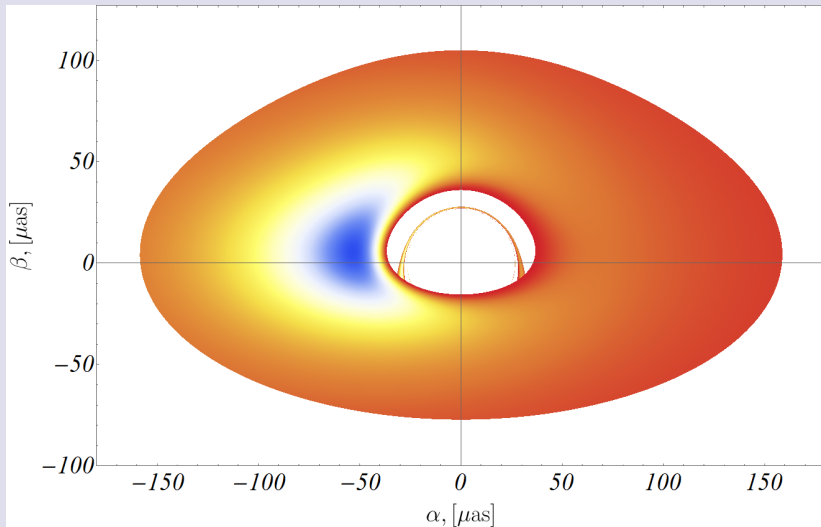
Ray tracing. Observable flux of the accretion disk ($\gamma = 1$)



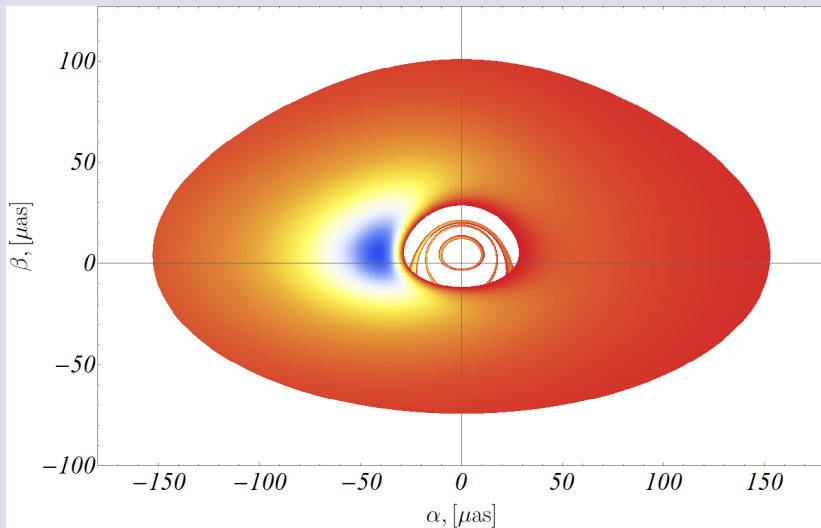
Ray tracing. Observable flux of the accretion disk ($\gamma = 0.48$)



Ray tracing. Observable flux of the accretion disk ($\gamma = 1$)



Ray tracing. Observable flux of the accretion disk ($\gamma = 0.48$)



- Image of the Janis-Newman-Winicour naked singularity with a thin accretion disk, [PHYSICAL REVIEW D 100, 024055 \(2019\)](#)
- Image Creation of a Thin Accretion disk Around Janis-Newman Naked Singularity, [\(2019\)](#)

Thank you for your attention!