Constraints on neutron stars equation of state from GW170817 event

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Introduction

- Neutron Stars (NS) are among the most interesting astrophysical objects in Nature.
- Conditions of extremely high matter densities prevail in their interiors, making them a natural laboratory for extreme physics. The precise form of the equation of state (EoS) of NSs remains unknown.
- Radius: $R \approx 10 - 15km$, Mass: $M \approx 1 - 2M_{\odot}$, Mean density: $\rho \approx 4 \times 10^{14} g/cm^3$, Frequency: $f \approx few\ Hz - 700Hz$, Magnetic field: $B \approx 10^{12} - 10^{18} Gauss$.

metric

The metric for a spherically symmetric configuration and for the static case is:

$$\mathrm{d}s^2 = e^{\lambda(r)}\mathrm{d}r^2 + r^2(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\phi^2) - e^{v(r)}\mathrm{d}t^2$$

The metric functions$\lambda(r), v(r)$ are related to the energy distribution $\mathcal{E}(r)$ and local pressure $P(r)$ as:

$$\frac{8\pi G}{c^4}\mathcal{E}(r) = \frac{1}{r^2}(1 - e^{-\lambda(r)}) + e^{-\lambda(r)}\frac{\lambda'(r)}{r},$$
$$\frac{8\pi G}{c^4}P(r) = -\frac{1}{r^2}(1 - e^{-\lambda(r)}) + e^{-\lambda(r)}\frac{v'(r)}{r},$$
$$P'(r) = -\frac{P(r) + \mathcal{E}(r)}{2}v'(r)$$

The equilibrium of the star's matter is given by the equations above, combined with the equation of state (EOS) $P = P(\mathcal{E})$.

Tolman-Oppenheimer-Volkov (TOV) equations

The system of TOV equations of the equilibrium is given by:

$$\frac{\mathrm{d}P}{\mathrm{d}r} = -\frac{G}{r^2}\left[\rho(r) + \frac{P(r)}{c^2}\right]\left[m(r) + \frac{4\pi r^3 P(r)}{c^2}\right]\left[1 - \frac{2Gm(r)}{rc^2}^{-1}\right]$$

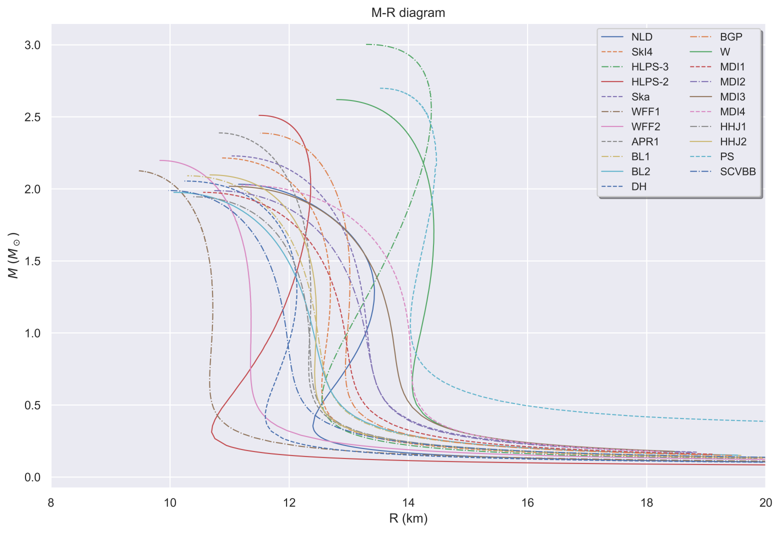
$$\frac{\mathrm{d}M(r)}{\mathrm{d}r} = 4\pi^2 r^2 \rho(r), \quad \rho(r) = \mathcal{E}(r)/c^2$$

For the numerical integration of the system we set:

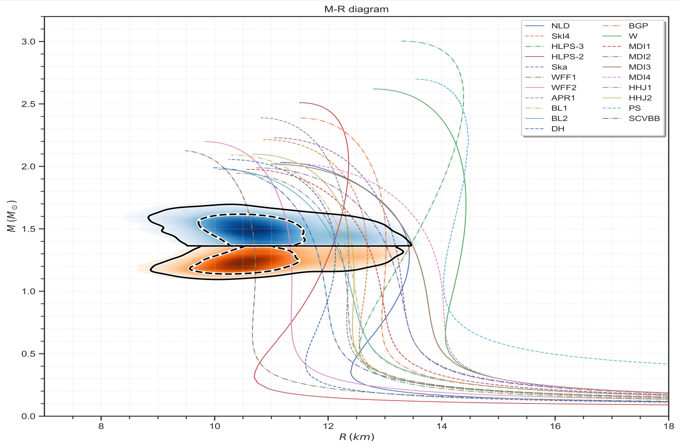
- $m(r) = \bar{m}(r)M_\odot,\ \mathcal{E}(r) = \bar{\mathcal{E}}(r)\mathcal{E}_0,\ P(r) = \bar{P}(r)\mathcal{E}_0$
- $\mathcal{E}_0 = 1\ MeV\ fm^{-3}$
- $\frac{GM_\odot}{c^2} = 1.474\ Km$
- $\frac{4\pi}{M_\odot c^2} = 0.7 \cdot 10^{-40} \frac{s^2}{Kg \cdot Km^2}$

M-R diagram

Results for a variety of EOS



Results for a variety of EOS using an insensitive relation[6]



- Marginalized posterior for the mass M and radius R of each binary using EOS-insensitive relation[6].
- The top blue (bottom orange) posterior corresponds to the heavier (lighter) NS component.
- At the 90% level, the radii of the two NSs are $R_1 = 10.8^{+2.0}_{-1.7}$ and $R_2 = 10.7^{+2.1}_{-1.5}$.

Tidal effects during inspiral of a BNS

- GW from the late phase of a BNS are among the most significant sources for a GW detector.
- Tidal field E_{ij} induces change of quadrapole moment Q_{ij} of the neutron star.

The quadrapole moment is given by[1]:

$$Q_{ij} = -\lambda(m)E_{ij}, \text{ with } \lambda(m) = \frac{2}{3}k_2(m)\frac{R^5}{G}$$

- The induced quadrupole moment depends on neutron star structure.
- Tidal deformability λ depends on radius R (the smaller the star, the harder to deform) and k_2.
- The reaction of the star to the tidal field is described by the parameter k_2, known as tidal love number.
- k_2 depends on the structure of the neutron star, i.e. its mass and equation of state.

Tidal love number

Mathematical formalism

The tidal love number k_2 is given by[1]:

$$k_2 = \frac{8\beta^5}{5}(1-2\beta)^2[2 - y_R + (y_R - 1)2\beta]$$
$$\times [2\beta(6 - 3y_R + 3\beta(5y_R - 8))$$
$$+ 4\beta^3(13 - 11y_R + \beta(3y_R - 2) + 2\beta^2(1 + y_R))$$
$$+ 3(1-2\beta)^2[2 - y_R + 2\beta(y_R - 1)] \ln(1-2\beta)]^{-1},$$

where $\beta = GM/Rc^2$ is the compactness parameter. The tidal love number depends also on the quantity y_R, which is determined by solving the following differential equation[1]:

$$r\frac{\mathrm{d}y(r)}{\mathrm{d}r} + y^2(r) + y(r)F(r) + r^2Q(r) = 0, \quad y(0) = 2,\ y_r \equiv y(R)$$

Tidal love number

$F(r)$ and $Q(r)$ are functionals of $\mathcal{E}(r)$, $P(r)$ and $M(r)$[1]:

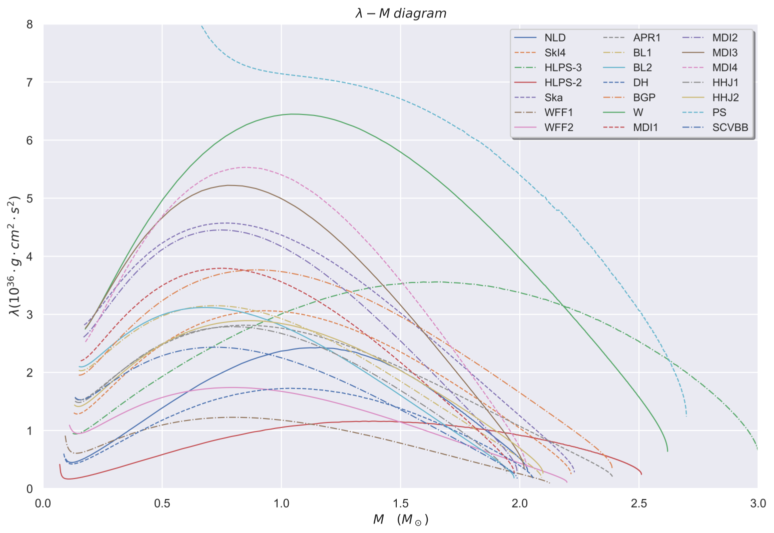
$$F(r) = \left[1 - \frac{4\pi r^2 G}{c^4}\left(\epsilon(r) - P(r)\right)\right]\left(1 - \frac{2M(r)G}{rc^2}\right)^{-1},$$

$$\begin{aligned} r^2 Q(r) = & \frac{4\pi r^2 G}{c^4}\left[5\epsilon(r) + 9P(r) + \frac{\epsilon(r) + P(r)}{\partial P(r)/\partial \epsilon(r)}\right]\left(1 - \frac{2M(r)G}{rc^2}\right)^{-1} \\ & - 6\left(1 - \frac{2M(r)G}{rc^2}\right)^{-1} \\ & - \frac{4M^2(r)G^2}{r^2c^4}\left(1 + \frac{4\pi r^3 P(r)}{M(r)c^2}\right)^2\left(1 - \frac{2M(r)G}{rc^2}\right)^{-2}. \end{aligned}$$

The numerical integration of these equations, combined with TOV equations, using as boundary conditions $y(0) = 2$, $P(0) = P_c$ and $M(0) = 0$ provide the mass M, the radius R of the neutron star (TOV solution), and the value $y_R = y(R)$. The compactness parameter b is determined, and therefore the tidal love number can be determined.

$\lambda - M$ diagram

Results for a variety of EOS



Inspiral

- During the last orbits before the neutron stars merging, the orbital phase evolution is affected by tidal deformability.
- At leading order,the tidal effects are imprinted in the gravitational-wave signal through the binary tidal deformability[5]:

$$\tilde{\Lambda} = \frac{16}{13}\frac{(12q+1)\Lambda_1 + (12+q)q^4\Lambda_2}{(1+q)^5},$$

where $q = m_2/m_1 \leq 1$ is the binary's mass ratio (m_1 corresponds to the heavier component mass) and $\Lambda_1 = \Lambda_1(m_1)$ and $\Lambda_2 = \Lambda_2(m_2)$ are the tidal deformabilities of the two NS. $M_{tot} = m_1 + m_2$ is the total mass of the binary system and $M_{chirp} = \frac{(m_1 m_2)^{3/5}}{(m_1+m_2)^{1/5}}$ is the chirp mass.

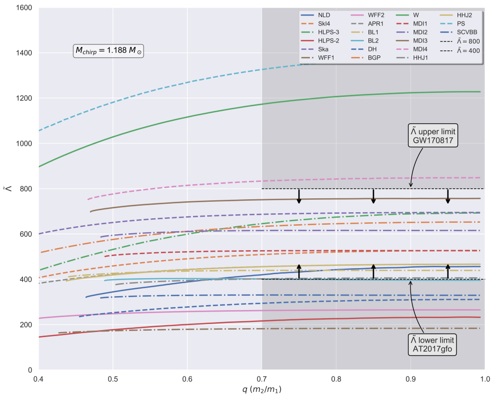
Constraints from GW170817

Constraints for the low spin case (*Abbott et al.,2017* [3])

	Low-spin priors ($\lvert\chi\rvert \leq 0.05$)
Primary mass m_1	1.36–$1.60\ M_\odot$
Secondary mass m_2	1.17–$1.36\ M_\odot$
Chirp mass $\mathcal{M}$	$1.188^{+0.004}_{-0.002} M_\odot$
Mass ratio m_2/m_1	0.7–1.0
Total mass $m_{\rm tot}$	$2.74^{+0.04}_{-0.01} M_\odot$
Radiated energy $E_{\rm rad}$	$> 0.025 M_\odot c^2$
Luminosity distance $D_{\rm L}$	40^{+8}_{-14} Mpc
Viewing angle Θ	$\leq 55°$
Using NGC 4993 location	$\leq 28°$
Combined dimensionless tidal deformability $\tilde{\Lambda}$	≤ 800
Dimensionless tidal deformability $\Lambda(1.4 M_\odot)$	≤ 800

- Fixed chirp mass of $M_{chirp} = 1.188\ M_\odot$.
- Binary's mass ratio: $0.7 < q < 1.0$
- Upper limit on $\tilde{\Lambda}$: $\tilde{\Lambda} \leq 800$.

$\tilde{\Lambda} - q$ for a variety of EoS



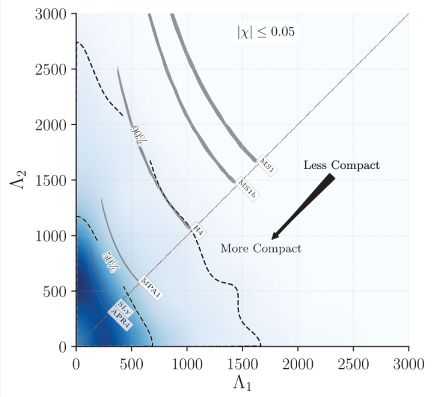
- $\tilde{\Lambda} = \tilde{\Lambda}(q, M_{chirp} = 1.188\ M_\odot; EOS)$.
- Binary's mass ratio: $0.4 < q < 1.0$
- Upper limit on $\tilde{\Lambda}$: $\tilde{\Lambda} \leq 800(GW170817)$ [3]
- Lower limit on $\tilde{\Lambda}$: $\tilde{\Lambda} \geq 400$ from EM data (AT2017gfo) [7,8]

- $\tilde{\Lambda} = \Lambda_1 = \Lambda_2$ when $m_1 = m_2$.
- The figure shows the posterior probability densities for Λ_1,Λ_2 with 90% and 50% credible region contours
- Under the assumption that $\Lambda \simeq \alpha\beta^{-6}$ and $R_1 \simeq R_2 \simeq \hat{R}$[5]:

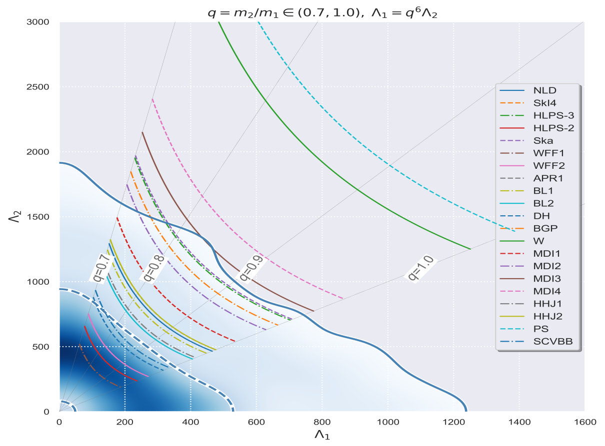
$$\Lambda_1(\tilde{\Lambda}, q) = \frac{13}{16}\tilde{\Lambda}\frac{q^2(1+q)^4}{12q^2 - 11q + 12}$$

$$\Lambda_2(\tilde{\Lambda}, q) = q^{-6}\Lambda_1$$

$\Lambda_1 - \Lambda_2$ diagram for the low spin case (*Abbott et al.,2017* [3])



$\Lambda_1 - \Lambda_2$ space for a variety of EoS



- For $m_1 = m_2 = 1.36\ M_\odot \Rightarrow \tilde{\Lambda} = \Lambda_1 = \Lambda_2$, unique value for each EoS.
- Posterior probability densities for Λ_1,Λ_2 using the PhenomPNRT model's data). [4,6]
- 90% and 50% credible region contours using PhenomPNRT waveform model (blue contours) [4,6]

Remarks

- GW from BNS coalescence provide strong constraints on the EoS of dense matter.
- Observations of massive NS ($1.97 \pm 0.04 M_\odot$ from PSR J1617-2230 and $2.01 \pm 0.04 M_\odot$ from PSR J0348-0432) prefer stiff EoSs [7].
- The upper limit on $\tilde{\Lambda}$ obtained from GW170817 exclude stiff EoSs which predict ther values bigger than 800, such as the EoS: W, PS, MDI4.
- The lower bound on $\tilde{\Lambda}$ derived from EM observations and UV/optical analysis of GW170817 signal, exclude soft EoSs ($\tilde{\Lambda} \geq 400$).
- Further upgrade of terrestrial GW detectors will lead to stronger constraints on the bounds.

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