Effects on the Equation of State through the uniform rotation of neutron stars

P.S. Koliogiannis and Ch.C. Moustakidis

Aristotle University of Thessaloniki
Department of Theoretical Physics

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Neutron Stars in the GW Era

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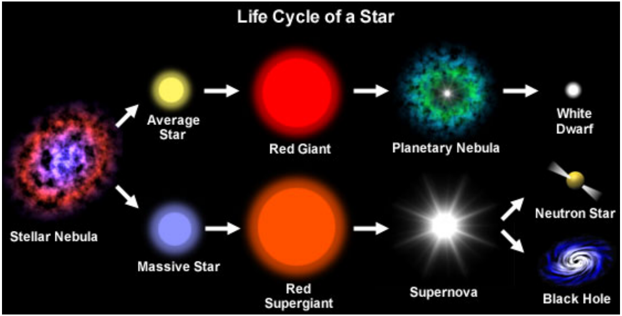
Stellar Remnants - Neutron Stars

The endpoints of stellar evolution can take one of the three forms

- White Dwarf
- Neutron Star
- Black Hole



Stellar Remnants - Neutron Stars

The endpoints of stellar evolution can take one of the three forms

- White Dwarf
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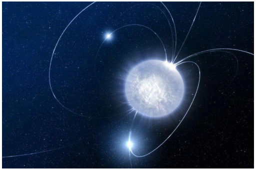
Neutron Star

- Mass: 1.4-2.5 $M_\odot$
- Eq. Radius: 10-15 km
- Mean density: 4×10^{14} gr/cm^3
- Frequency: up to 2.2 kHz



The fastest observed rotating neutron star is at 716 Hz

Stellar Remnants - Neutron Stars

Why we study Neutron Stars

- They are the most compact stars known to exist in the universe
- They have densities equal to that of the early universe
- Gravity is similar to that of a black hole
- They have the most extreme magnetic fields known in the universe (up to $10^{16}G$)
- They present a unique interplay among astrophysics, gravitational physics and nuclear physics

From Non-rotating to Rotating Neutron Stars

Non-rotating Configuration

- Space-time metric :
  $ds^2 = e^{\nu} dt^2 - e^{\lambda} dr^2 - dr^2 - r^2 \left(d\theta^2 + \sin^2 \theta d\phi^2 \right)$
- Perfect fluid : $T_{\mu\nu} = (\epsilon + p) \, u_\mu u_\nu + p g_{\mu\nu}$
- TOV system $\rightarrow m(r)$, $p(r)$, $\epsilon(r)$, etc.

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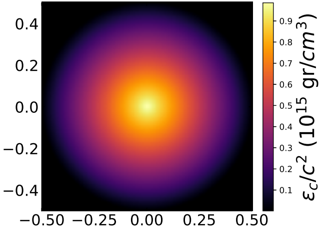
Rotating Configuration

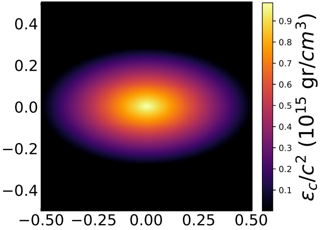
- Space-time metric :
  $ds^2 = e^{\nu} (1 + 2h)\, dt^2 - e^{\lambda} \left[1 + \frac{2m}{r - 2M_0} \right] dr^2 -$
  $r^2 (1 + 2k) \left[d\theta^2 + \sin^2\theta \left(d\phi - \omega dt \right)^2 \right] + O\left(\Omega^3 \right)$
- Perfect fluid : $T_{\mu\nu} = (\epsilon + p)\, u_\mu u_\nu + p g_{\mu\nu}$
- TOV system $\rightarrow m(r)$, $p(r)$, $\epsilon(r)$, $\omega(r)$, $I(r)$, ϵ etc.

From Non-rotating to Rotating Neutron Stars

Differences between the non-rotating and rotating configuration

- $M_{\max}^{\rm st} < M_{\max}^{\rm rot}$
- $R_{\rm eq,max}^{\rm st} < R_{\rm eq,max}^{\rm rot}$
- $\epsilon_c^{\rm st} > \epsilon_c^{\rm rot}$
- The existence of the angular velocity, the moment of inertia, the eccentricity and the Kerr parameter (only in rotating configuration)





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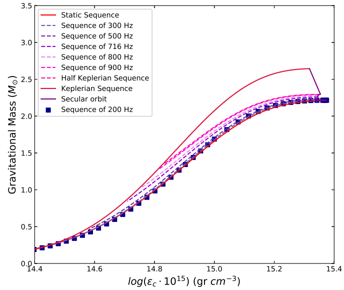

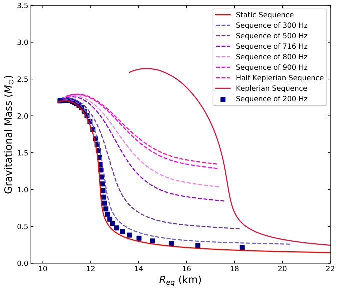

APR-1 Equation of State

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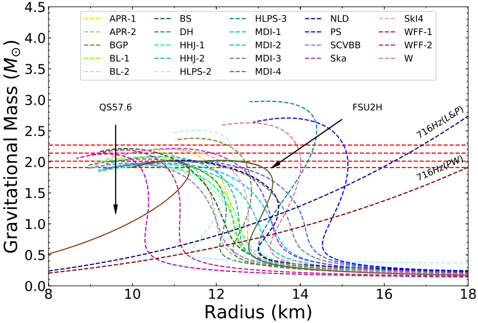
The knowledge of the maximum mass and the maximum frequency can help us:

- Identify a compact object as a Black Hole
- Constrain the high density part of the Equation of State

From Non-rotating to Rotating Neutron Stars

- We employ a total of 23 realistic EoSs for neutron star matter, based on various theoretical nuclear models
- Hyperons: FSU2H and quark stars: QS57.6
- Mass: $1.908M_{\odot}$, $2.01M_{\odot}$, $2.14M_{\odot}$ and $2.27M_{\odot}$
- Frequency: 716 Hz[1]



[1] J. M. Lattimer and M. Prakash, Science 304, 536 (2004), ISSN 0036-8075, URL https://science.sciencemag.org/content/304/5670/536

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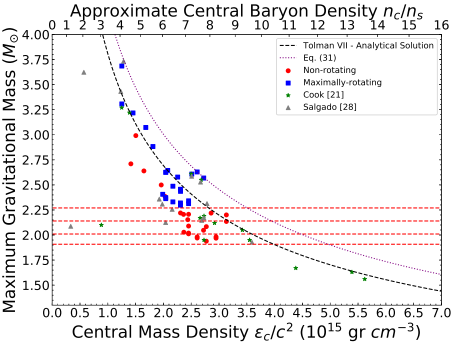

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[2] M. Salgado, S. Bonazzola, E. Gourgoulhon and P. Haensel, Astron. Astrophys. Suppl. Ser. 108, 455-459 (1994)

[3] Gregory B. Cook, Stuart L. Shapiro and Saul A. Teukolsky, The Astrophysical Journal, Vol. 424, 823-845 (1994)

Effects of the Keplerian Sequence - Maximum frequency

- Determines the maximum rotation rate
- Depends on the gravitational mass of the star $\rightarrow$ EoS dependent
- $f_{\rm k} = \mathcal{C}_\alpha \left(\frac{M^\alpha_{\rm max}}{M_\odot}\right)^{1/2} \left(\frac{10km}{R^\alpha_{\rm eq,max}}\right)^{3/2} = \mathcal{C}_\alpha x^\alpha_{\rm max}$

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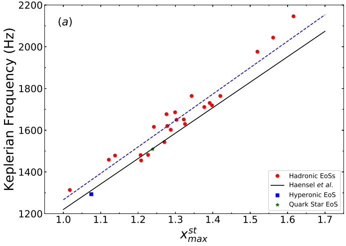

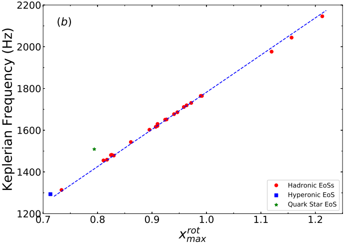

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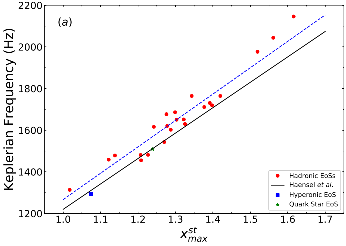

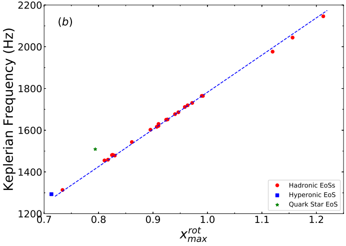

For $f_k = 716$ Hz: If $M_{\mathrm{max}} \leq 2.14 M_{\odot}$ then $R \leq 18.8498$ km

Effects of the Keplerian Sequence - Moment of Inertia

- Quantifies how fast an object can spin with a given angular momentum
- $I = J/\Omega$

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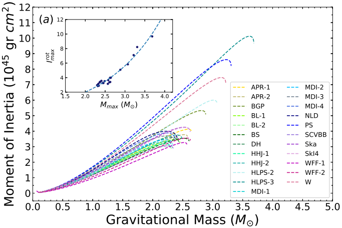
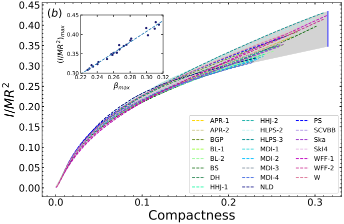
Moment of inertia



$$I^{\rm rot}_{\rm max} = -1.568 + 0.883 \exp\left[0.7\left(\frac{M_{\rm max}}{M_\odot}\right)\right]$$

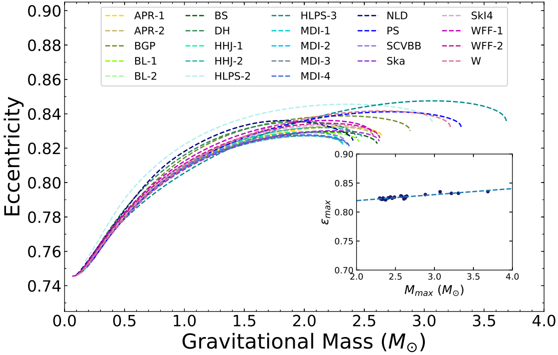
Dimensionless moment of inertia



$$\left(I/MR^2\right)_{\rm max} = -0.006 + 1.379\beta_{\rm max}$$

Effects of the Keplerian Sequence - Eccentricity

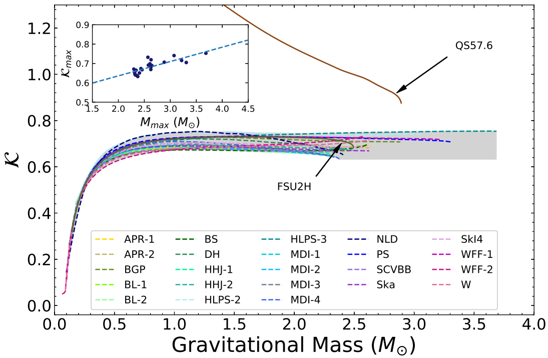
- Rapid rotation deforms the models of equilibrium
- $\epsilon = \sqrt{1 - \left(\frac{r_p}{r_e}\right)^2}$
- $\epsilon = 0.799 + 0.01 M_{\max}$



Effects of the Keplerian Sequence - Kerr parameter

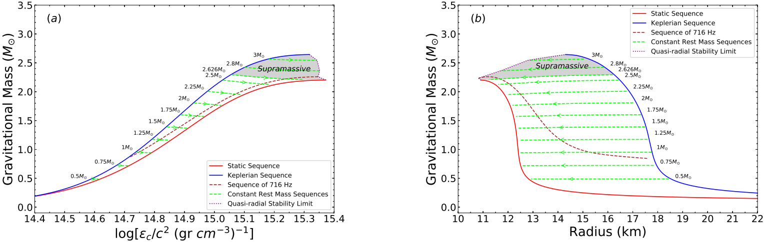
- $j = cJ/(GM^2)$
- $(\alpha/M)_{\max} \approx 0.75$
- $(\alpha/M)_{\max} = 0.074 M_{\max} + 0.488$

$(\alpha/M)^{B.H.}_{max} \approx 0.998$



Rest Mass Sequences

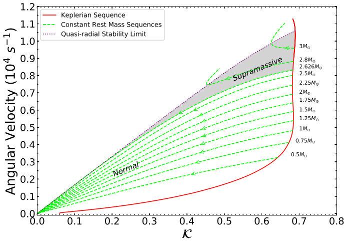
- Time evolutionary sequences
- Normal & Supramassive

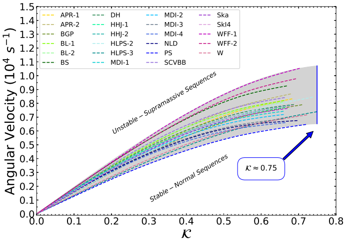


APR-1 Equation of State

Rest Mass Sequences

- Prior to collapse the star spins up
- Gravitation collapse
- Last stable rest mass

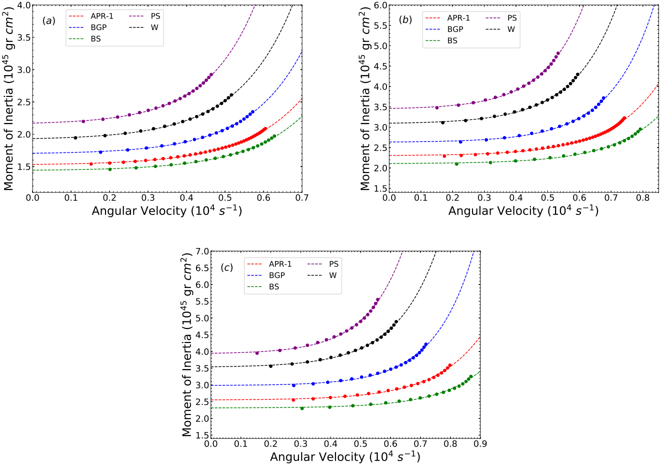




APR-1 Equation of State

Braking index

- Slow-down torque on neutron stars
- $\dot{\Omega} \equiv \frac{d\Omega}{dt} = -\mathcal{J}\Omega^n \Rightarrow n(\Omega) = 3 - \frac{3\Omega I' + \Omega^2 I''}{2I + \Omega I'} = 3 - \lambda$



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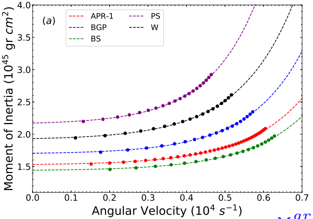
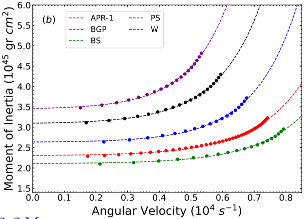
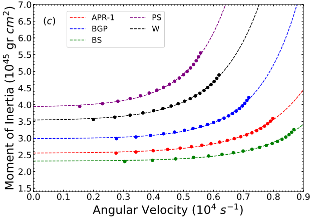
$M^{gr}_{max} = 1.45 M_\odot$



$M^{gr}_{max} = 2 M_\odot$



$M^{gr}_{max} = 2.2 M_\odot$



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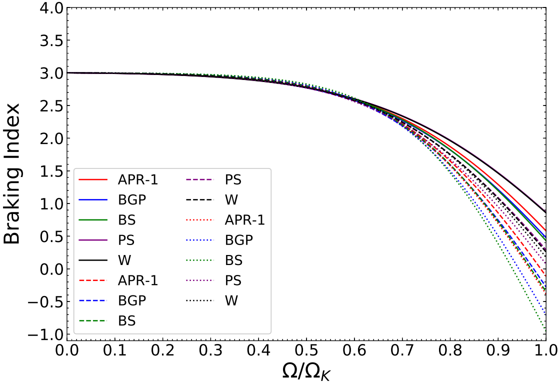

$-$: $M^{gr}_{max} = 1.45 M_\odot$, $--$: $M^{gr}_{max} = 2 M_\odot$ and $\cdots$: $M^{gr}_{max} = 2.2 M_\odot$

Conclusions

- The $M_{\max}^{\rm rot}$ and $f_{\rm k}$ can constrain the EoS at high densities
- The $M_{\max}^{\rm rot}$ can provide us with the ultimate density of cold baryonic matter
- There are 3 universal relations connecting the moment of inertia, the eccentricity and the Kerr parameter with the $M_{\max}^{\rm rot}$
- The calculation of Kerr parameter can help us to identify a compact object as a black hole and possibly imply the existence of universal limiting value of the neutron star compactness
- The Kerr parameter determines the final fate of the collapse of a rotating compact star
- Supramassive curves may provide an observable precursor to gravitation collapse to a black hole

Conclusions

- Last Stable Rest Mass sequences may help to constrain the EoS in a narrow region
- From 70% - 100% of the Keplerian angular velocity, braking index may provide us with useful insights on the constitution of the dense nuclear matter
- If the millisecond pulsars acquire angular momentum by accretion, a rapidly-rotating one may eventually be found in a binary system with measured masses.
- The remnant formed in the immediate aftermath of the GW170817 merger although is believed to have been differentially rotating and not uniformly, it contains sufficient angular momentum to be near its mass-shedding limit.

Thank you for your attention!

All calculations for the rotating neutron stars were made with RNS code of Stergioulas and Friedman (1995)