Preserving Kerr symmetries in deformed space-times

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Georgios O. Papadopoulos[1] Kostas D. Kokkotas[2]

[1]National & Kapodistrian University of Athens
&
Aristotelian University of Thessaloniki
Greece

[2]Theoretical Astrophysics, IAAT, Eberhard-Karls University of Tübingen,
Tübingen, Germany

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- provide an opportunity to test the strong gravity limit of any GT where no post-Newtonian approach applies
- can give insights into not only the foundations of quantum GTs but also to the other three fundamental interactions
- are related to other phenomena like the (strong) gravitational waves

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A fundamental question:

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Indeed, there is no invariant description of a BH, regardless the space-time signature and number of dimensions. Never the less, there is a physical motivation for one to adopt as a prototype of a BH the space-time geometry described by the Kerr-Newman line element which contains 3 essential constants: mass (M), angular momemtum (J), and charge (Q) and a few characteristic symmetries (i.e., Petrov type D and stationary axisymmetric geometry) cf. no hair theorem in General Relativity (GR)

The observations/measurements mainly concern the M and J parameters (assuming an electrically neutral system). The accuracy of the experiments can not eliminate the possibility for deviations from the standard (i.e., Kerr) BH. The experiments can be based on the study of the geometry of BH horizon by probing it with some field (like photons), or on the study of quasi-normal modes emanating from a system of BHs. The reason of existence of such deviations can be

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in any case, the first check will be the validity of the no-hair theorem

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Thus far while the spaces to be found in the literature do not admit (in general) a Killing tensor of order 2, there are efforts to make them to allow a separation of variables for the Klein-Gordon and Hamilton-Jacobi equations so it is natural to ask for a criterion on choosing an alternative definition for a BH based solely on physical criteria

Metric	functions	K.T.	H-J Sep.	K-G Sep.
J	4	yes	yes	no
KSZ	3	yes	yes	yes
PK	5 (+5)	yes	yes	no
S	3	yes	yes	yes
PK2	6 (+6)	Conformal	for null only	yes

where

J: Tim Johannsen, *Regular black hole metric with three constants of motion*, Phys. Rev. D **88**, 044002 (2013)

KSZ: R.A. Konoplya, Z. Stuchlík, and A. Zhidenko, *Axisymmetric black holes allowing for separation of variables in the Klein-Gordon and Hamilton-Jacobi equations*, Phys. Rev. D **97**, 084044 (2018)

PK2: Papadopoulos and Kokkotas, to appear

S: Rajibul Shaikh, *Black hole shadow in a general rotating spacetime obtained through Newman-Janis algorithm*, Phys. Rev. D **100**, 024028 (2019)

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The starting point would be two necessary axioms:

1. stationarity, axial symmetry and asymptotical flatness must all be preserved

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1. stationarity, axial symmetry and asymptotical flatness must all be preserved
2. one must be able not only to probe the BH with test particles but also to get analytical solutions to the equations. In other words: (only) the Hamilton-Jaboci equation for a test particle

$$(\nabla_a S) g^{ab} (\nabla_b S) = \text{const.} \in \mathbb{R} \tag{1}$$

(where S is the action of the particle) must be separable

In an extremely important work, Benenti & Francaviglia (Ann. Inst. Henri Poincaré **34** 45, (1981)) have proven that the separability axiom is closely related to a third (quadratic in phase-space momenta) integral of motion; the Carter constant. This integral of motion reflects a higher symmetry: given a local coordinate system (r, x, ϕ, t) the most general metric satisfying the second axiom assumes the form

$$g^{ab} = \frac{1}{A_1 + B_1} \begin{pmatrix} A_2 & 0 & 0 & 0 \\ 0 & B_2 & 0 & 0 \\ 0 & 0 & A_3 + B_3 & A_4 + B_4 \\ 0 & 0 & A_4 + B_4 & A_5 + B_5 \end{pmatrix} \tag{2}$$

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This metric admits a 2nd order Killing tensor (which is a similar expression in terms of As and Bs, but too entangled to fit on here) of which the defining property is

$$\nabla_{(a} K_{bc)} = 0, \quad K_{[ab]} = 0 \tag{3}$$

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For instance the Kerr solution admits a Yano tensor which remains invariant under the transformation

$$M \rightarrow M + H/2r \tag{5}$$

which gives the modified Kerr BH!

Now, one can formulate the following

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> *Any alternative geometry meant to describe the physics of a black hole, deviating smoothly from the Kerr black hole, must be*
>
> - *susceptible either to an algebraically general Yano tensor of rank two, in which the two distinct eigenvalues supposed to be non constant, or to a non trivial Killing tensor of rank two along with two non null, and commuting, Killing vector fields*
> - *asymptotically flat*

A calculation shows that the main examples to be found in the literature obey all the conditions of the Proposition.

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- try to identify compatible GTs cf. previous table