Testing GR with neutron stars

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Neutron Stars in the GW Era

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Motivation
Neutron Stars, Astrophysics, and Spacetime

Neutron Star's Structure in GR
General structure
Multipole moments and their spin dependance
Multipole moments' universal relations (3-hair relations)
Some more universal relations (I-Love-Q and other)

Neutron Stars in other theories of gravity
The degeneracy problem
Scalar-Tensor theory with a massless scalar field
Moments' universal relations in ST
Other universal relations in alternative theories
Moments beyond the massless case

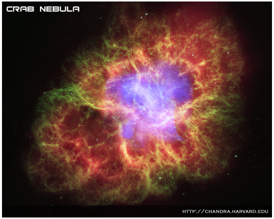
Astrophysical observables and NS moments
QPOs and geodesic motion frequencies
Frequencies and multipole moments
Conclusions

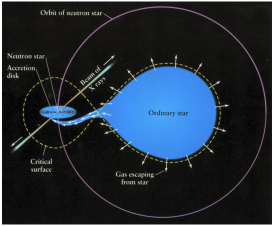
Neutron stars are the results of stellar evolution.
We can see them in stellar remnants.
A typical example is the Crab nebula that hosts the Crab pulsar[a].

[a] APOD 2006 October 26

Very often we find rapidly rotating pulsars at the end of stellar evolution. The fastest rotating known pulsar (PSR J1748-2446ad) spins at 716Hz and it is part of a binary system[a].
Low mass X-ray binaries are systems that are comprised by a compact object (NS or BH) and a regular star companion. The main source of the X-rays is the accretion disk that forms around the compact object.

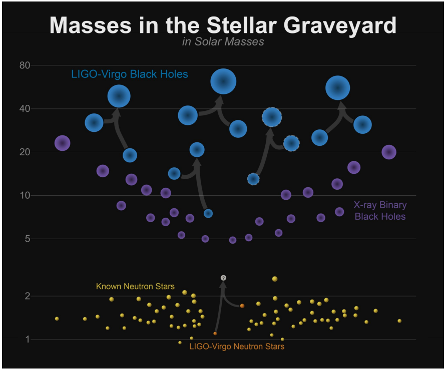
[a] J. W. T. Hessels et al., Science **311** 1901 (2006)

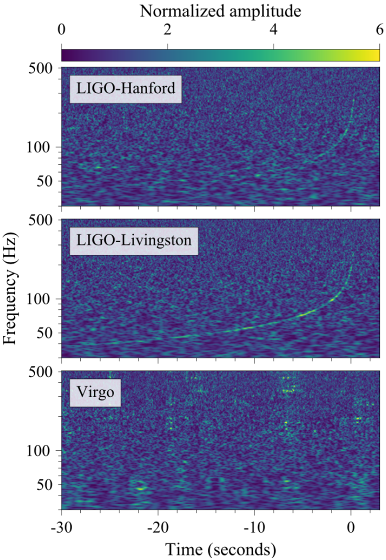




Interesting astrophysics takes place around NSs that depends on the background spacetime. Matter in their interior is at very high densities, where the equation of state is unknown. NSs have strong enough gravitational fields that can test our theories of gravity.

And additionally with the detection of GW170817 we have "heard", through gravitational waves, the inspiral and collision of a binary neutron star system.[a]

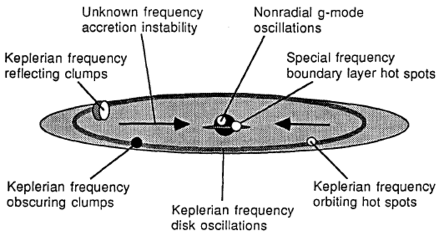




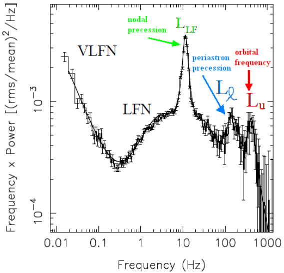
[a] B. P. Abbott et al.* (LIGO Scientific Collaboration and Virgo Collaboration), PRL **119**, 161101 (2017)

In low-mass X-ray binaries we can have observables related to geodesic motion. An example of observables related to orbits around neutron stars are the quasi-periodic oscillations (QPOs) of the spectrum[1] of an accretion disc.

Mechanisms for producing QPOs[2] from orbital motion



Typical X-Ray spectrum[3]



Effects: orbiting hot spots, oscillations on the disc, precessing rings or misaligned precessing discs, and so on. These could result in a modulated emission or they could be eclipsing the emission from the central object.

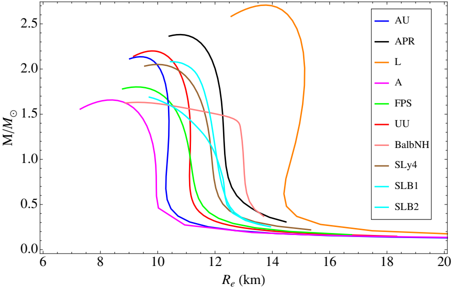
[1] Stella & Vietri, 1998, ApJ, 492, L59.

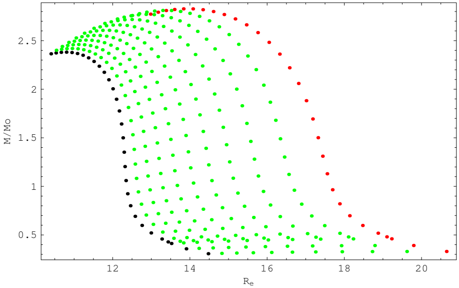
[2] F.K. Lamb, Advances in Space Research, 8 (1988) 421.

[3] Boutloukos et al., 2006, ApJ, 653, 1435-1444.

Results from numerical models:

Solving the GR field equations we can calculate models of rotating neutron stars for a given equation of state.





Non-rotating models for various EoSs (left) and rotating models for APR EoS (right). The models with the fastest rotation have a spin parameter, $j = J/M^2$, around 0.7 and a ratio of the polar radius over the equatorial radius, r_p/r_e, around 0.56.

The numerical integration gives the various physical characteristics of the NS, the metric functions, as well as the relativistic multipole moments, i.e., M, $S_1 \equiv J$, $M_2 \equiv Q$, $S_3 \equiv J_3$ and M_4. [a] These moments characterise both NS and spacetime.

[a] G.P. & T. A. Apostolatos, Phys. Rev. Lett. **108** 231104 (2012); K. Yagi, K. Kyutoku, G. P., N. Yunes, & T.A. Apostolatos, Phys.Rev. D **89** 124013 (2014).

Neutron star multipole moments in GR

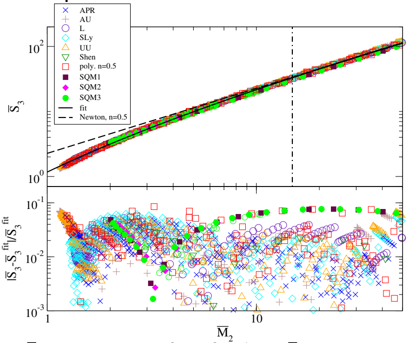
Black Hole-like behaviour of the moments:[4]

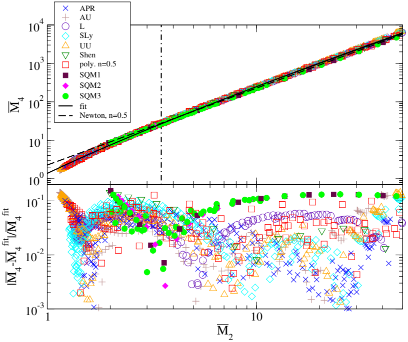
Kerr moments	Neutron star moments
$$\begin{aligned} M_0 &= M, \\ J_1 \equiv J &= jM^2, \\ M_2 \equiv Q &= -j^2 M^3, \\ J_3 \equiv S_3 &= -j^3 M^4, \\ M_4 &= j^4 M^5, \\ &\vdots \\ M_{2n} &= (-1)^n j^{2n} M^{2n+1}, \\ J_{2n+1} \equiv S_{2n+1} &= (-1)^n j^{2n+1} M^{2n+2} \end{aligned}$$	$$\begin{aligned} M_0 &= M, \\ J_1 &= jM^2, \\ M_2 &= -\alpha(EoS, \rho_c) j^2 M^3, \\ J_3 &= -\beta(EoS, \rho_c) j^3 M^4, \\ M_4 &= \gamma(EoS, \rho_c) j^4 M^5, \\ &\vdots \\ M_{2n} &= ?, \\ J_{2n+1} &= ? \end{aligned}$$

where $j = J/M^2$.

[4] W.G. Laarakkers and E. Poisson, Astrophys. J. **512** 282 (1999); G.P. and T. A. Apostolatos, Phys. Rev. Lett. **108** 231104 (2012); K. Yagi, K. Kyutoku, G. P., N. Yunes, and T.A. Apostolatos, Phys.Rev. D **89** 124013 (2014).

EoS independent behaviour of the moments[5] :



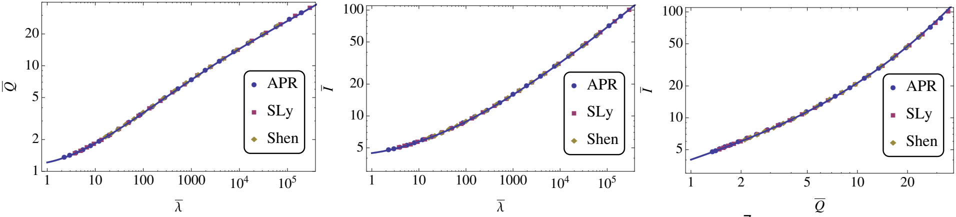


$\bar{M}_{2n} = |M_{2n}/(j^{2n}M^{2n+1})|$, $\bar{S}_{2n+1} = |S_{2n+1}/(j^{2n+1}M^{2n+2})|$

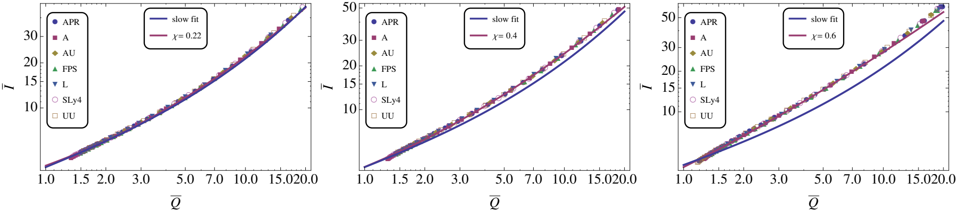
All these are properties that characterise the spacetime around neutron stars and are therefore relevant for doing astrophysics and are related to astrophysical observables.

[5] G.P. and T. A. Apostolatos, Phys.Rev.Lett. **112** 121101 (2014); K. Yagi, K. Kyutoku, G. P., N. Yunes, and T.A. Apostolatos, Phys.Rev. D **89** 124013 (2014).

Slow rotation Q-Love, I-Love and $I-Q$ relations[6] ($\bar{I} \equiv I/M^3$, $\bar{\lambda} \equiv \lambda/M^5$)



Slow and rapid rotation $I-Q$ relations ($\chi \equiv j$)[7]



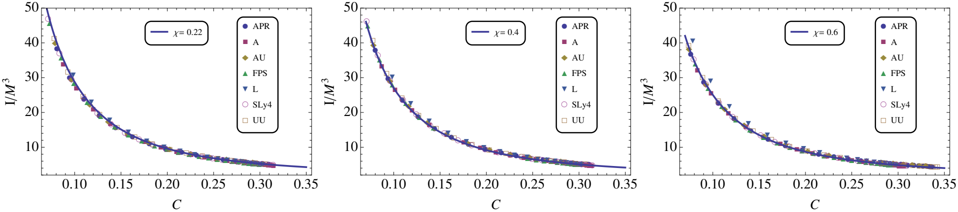
[6] K. Yagi and N. Yunes, Science 341, 365 (2013); Phys. Rev. D 88, 023009 (2013).

[7] Doneva et al., ApJ **781** L6 (2013); G.P. and T. A. Apostolatos, Phys.Rev.Lett. **112** 121101 (2014)

$I - \mathcal{C}$ relations:

First studied by Lattimer et al. and inspired by analytic models such as the Tolman VII model ($\rho = \rho_c[1 - (r/R)^2]$).

$I - \mathcal{C}$ relations for different rotation rates[8]



$$I/M^3 = (1.471+0.448\chi) - \frac{0.0802+0.27289\chi}{\mathcal{C}} + \frac{0.438-0.0346\chi}{\mathcal{C}^2} - \frac{0.01694+0.0056\chi}{\mathcal{C}^3} + \frac{(3.316+1.57\chi)\times 10^{-4}}{\mathcal{C}^4},$$

where $\mathcal{C} = M/R$ is the compactness.

[8] C. Breu and L. Rezzolla, MNRAS 459, 646 (2016); K. V. Staykov, D. D. Doneva, and S. S. Yazadjiev, Phys. Rev. D 93, 084010 (2016); Doneva D.D., P.G. (2018) Universal Relations and Alternative Gravity Theories. In: Rezzolla L., Pizzochero P., Jones D., Rea N., Vidana I. (eds) The Physics and Astrophysics of Neutron Stars. Astrophysics and Space Science Library, vol 457. Springer, Cham.

Astro-seismology universal relations:[9]

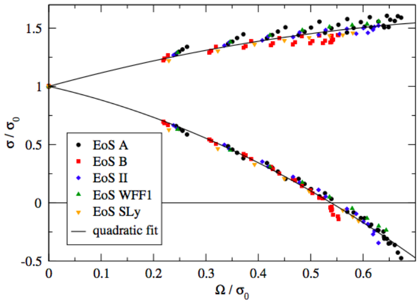


FIG. 1. Universal relations for the $l = |m| = 2$ f-mode frequencies from our polytropic models and two exemplary sequences based on realistic EoSs.

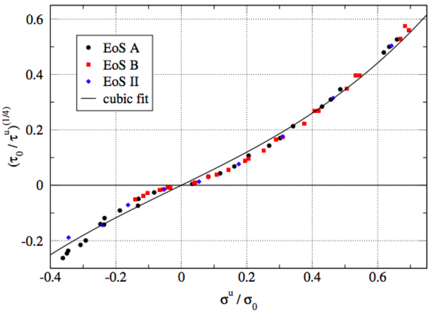
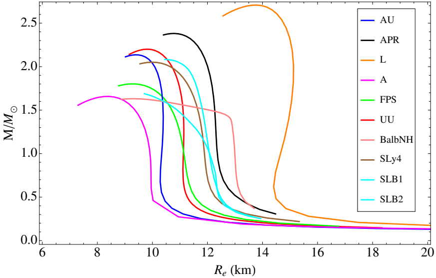


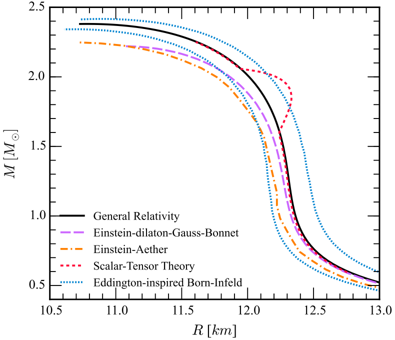
FIG. 2. Data points and universal relation for the damping time of the counter-rotating $l = m = 2$ f-mode.

Full result for rapidly rotating models without any approximations.

9 Christian J. Krüger, Kostas D. Kokkotas, arXiv:1910.08370 [gr-qc]

There is a degeneracy problem between different EoSs within GR and modifications to GR[10] :





Universal relations could be used to work around this problem.

[10] Kostas Glampedakis, GP, Hector O. Silva, Emanuele Berti, 2015 Phys. Rev. D 92, 024056.

In the case of Scalar-Tensor theories with a massless scalar field,

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-\tilde{g}}\left(\tilde{R} - 2\tilde{\nabla}^{\mu}\phi\tilde{\nabla}_{\mu}\phi\right) + S_m(g_{\mu\nu},\psi),$$

the field equations in the Einstein frame take the form,

$$\tilde{R}_{ab} = 2\partial_a\phi\partial_b\phi + 8\pi G\left(T_{ab} - \frac{1}{2}g_{ab}T\right), \quad \tilde{g}^{ab}\tilde{\nabla}_a\tilde{\nabla}_b\phi = -4\pi\alpha(\phi)T$$

These equations can be solved as in GR in order to construct neutron stars.[11] On the other hand, the vacuum field equations can admit an Ernst formulation as in GR,[12]

$$(Re(\mathcal{E}))\nabla^2\mathcal{E} = \nabla\mathcal{E}\cdot\nabla\mathcal{E},$$

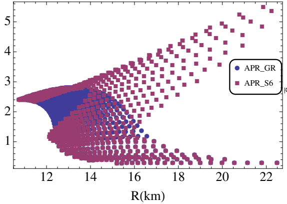
with the addition of a Laplace equation for the scalar field $\nabla^2\phi = 0$.
One can extend the definition of multipole moments in this case as well, where the moments (mass, spin, scalar) are defined in the Einstein frame. The actual physics though is done in the Jordan (physical) frame, where the metric is given by the conformal transformation $g_{\mu\nu} = A^2(\phi)\tilde{g}_{\mu\nu}$. [13]

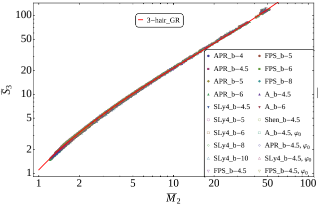
[11] D.D. Doneva, S.S. Yazadjiev, N. Stergioulas, K.D. Kokkotas, Phys. Rev. D 88, 084060 (2013)

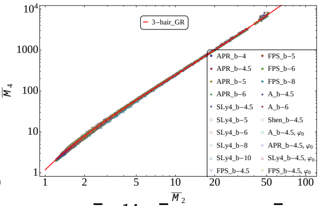
[12] GP, T.P. Sotiriou, Phys. Rev. D91, 044011 (2015)

[13] GP, T.P. Sotiriou, MNRAS 454, 4066 (2015)

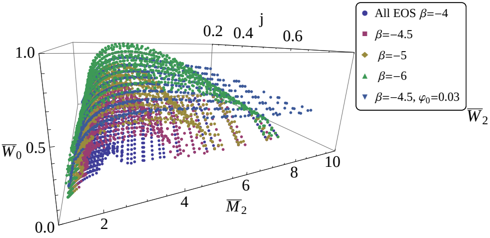
ST models against GR models and S_3^{ST}, M_4^{ST} vs Q^{ST} relations for various EoSs,

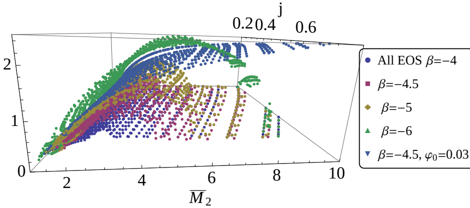






Scalar field normalised moments plotted against the spin j and $\bar{M}_2$,[14] $\bar{W}_a = f_\beta(j, \bar{M}_2)$

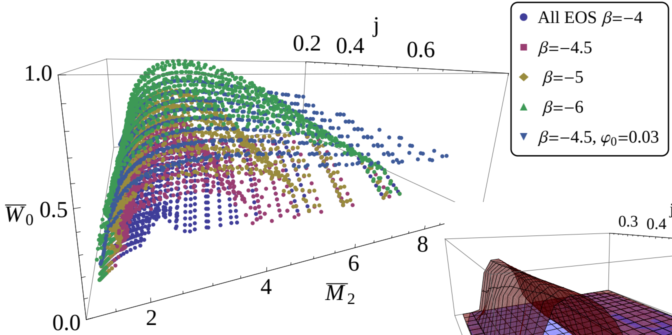


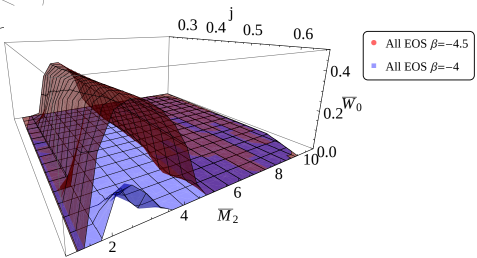


[14] D.D. Doneva, G.P., T.P. Sotiriou, S.S. Yazadjiev, and K.D. Kokkotas, Phys. Rev. D 99, 104014 (2019).

Scalar field normalised monopole plotted against the spin j and $\bar{M}_2$,[15]

$\bar{W}_0 = f_\beta(j, \bar{M}_2)$





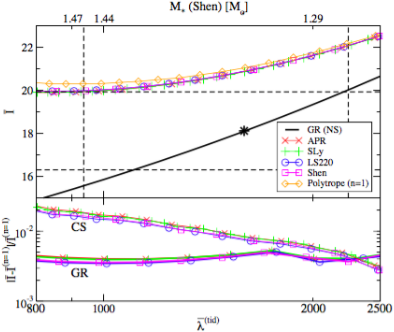
15 D.D. Doneva, G.P., T.P. Sotiriou, S.S. Yazadjiev, and K.D. Kokkotas, Phys. Rev. D 99, 104014 (2019).

I-Love-Q relations:

Such relations have been examined in dCS gravity, in massless and massive scalar-tensor theories (ST), in Einstein-dilaton-Gauss-Bonnet (EdGB) gravity, in Eddington-inspired Born-Infeld (EiBI) theory and in $f(R)$ theories.

$*$ The deviations from GR are almost negligible in the cases of the massless ST, the EdGB and the EiBI theory.

$*$ Larger differences on the other hand are observed for dCS gravity, massive ST and $f(R)$ theories.

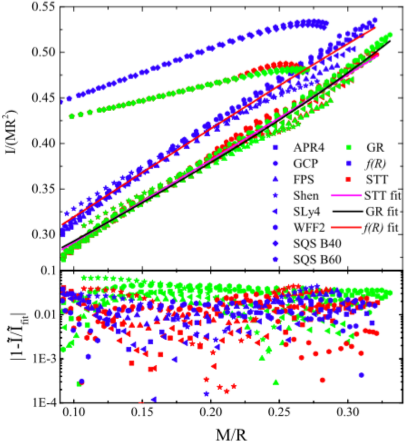


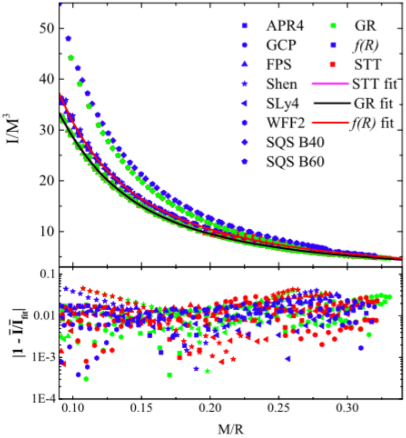
For example here we see how dCS compares to GR.[a]

[a] K. Yagi and N. Yunes, Phys. Rev. D 88, 023009 (2013).

$I-\mathcal{C}$ relations:

Slow rotation models for a massless ST theory, an $f(R)=R+\alpha R^2$ theory and GR (in ST $\beta=-4.5$).[16]





16 K. V. Staykov, D. D. Doneva, and S. S. Yazadjiev, Phys. Rev. D 93, 084010 (2016).

In the case of Scalar-Tensor theories with a massive scalar field,

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-\tilde{g}}\left(\tilde{R} - 2\tilde{\nabla}^{\mu}\phi\tilde{\nabla}_{\mu}\phi - 4V(\phi)\right) + S_m(g_{\mu\nu},\psi)\,,$$

the field equations in the Einstein frame take the form,

$$\tilde{R}_{ab} = 2\partial_a\phi\partial_b\phi + 8\pi G\left(T_{ab} - \frac{1}{2}g_{ab}T\right) + 2V(\phi)g_{ab}$$

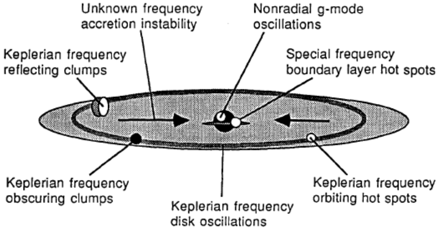
$$\tilde{g}^{ab}\tilde{\nabla}_a\tilde{\nabla}_b\phi = -4\pi\alpha(\phi)T + \frac{dV(\phi)}{d\phi}$$

These equations can be solved as in GR in order to construct neutron stars.[17]

In this case, one cannot have a construction as in the massless case in order to define multipole moments as far as the scalar field is concerned. The mass and angular momentum moments though, far enough from the star, will be the same as in GR.

The question then is, how would the presence of the potential affect the properties and behaviour of the multipole moments in these theories. This is something to be explored in the near future.

[17] D.D. Doneva, S.S. Yazadjiev, N. Stergioulas, K.D. Kokkotas, Phys. Rev. D 88, 084060 (2013)



Circular equatorial orbits: If we define $\Omega \equiv \frac{d\phi}{dt}$, the energy, angular momentum and orbital frequency for the circular orbits take the form,

$$\tilde{E} \equiv \frac{E}{m} = \frac{-g_{tt}-g_{t\phi}\Omega}{\sqrt{-g_{tt}-2g_{t\phi}\Omega-g_{\phi\phi}\Omega^2}},$$

$$\tilde{L} \equiv \frac{L}{m} = \frac{g_{t\phi}+g_{\phi\phi}\Omega}{\sqrt{-g_{tt}-2g_{t\phi}\Omega-g_{\phi\phi}\Omega^2}},$$

$$\Omega = \frac{-g_{t\phi,\rho}+\sqrt{(g_{t\phi,\rho})^2-g_{tt,\rho}g_{\phi\phi,\rho}}}{g_{\phi\phi,\rho}}$$

For more general orbits: Equations of motion can take the general form,

$$-g_{\rho\rho}\left(\frac{d\rho}{d\tau}\right)^2 - g_{zz}\left(\frac{dz}{d\tau}\right)^2 = 1 - \frac{\tilde{E}^2 g_{\phi\phi} + 2\tilde{E}\tilde{L}g_{t\phi} + \tilde{L}^2 g_{tt}}{(g_{t\phi})^2 - g_{tt}g_{\phi\phi}} = V_{eff}.$$

We can study the precession properties from the properties of the effective potential.

$$-g_{\rho\rho}\left(\frac{d(\delta\rho)}{d\tau}\right)^2 - g_{zz}\left(\frac{d(\delta z)}{d\tau}\right)^2 = \frac{1}{2}\frac{\partial^2 V_{eff}}{\partial\rho^2}(\delta\rho)^2 + \frac{1}{2}\frac{\partial^2 V_{eff}}{\partial z^2}(\delta z)^2,$$

This equation describes two harmonic oscillators with epicyclic frequencies, $\bar{\kappa}_\rho^2 = \frac{g^{\rho\rho}}{2}\frac{\partial^2 V_{eff}}{\partial\rho^2}\Big|_c$, $\bar{\kappa}_z^2 = \frac{g^{zz}}{2}\frac{\partial^2 V_{eff}}{\partial z^2}\Big|_c$. The differences of these frequencies (corrected for redshift) from the orbital frequency, $\Omega_a = \Omega - \kappa_a$, define the precession frequencies.

The precession frequencies are related to the spacetime multipole moments (Ryan, 1995),

in GR:

$$\frac{\Omega_\rho}{\Omega} = 3U^2 - 4\frac{J_1}{M^2}U^3 + \left(\frac{9}{2} - \frac{3M_2}{2M^3}\right)U^4 - 10\frac{J_1}{M^2}U^5 + \left(\frac{27}{2} - 2\frac{J_1^2}{M^4} - \frac{21M_2}{2M^3}\right)U^6 + \ldots$$

$$\frac{\Omega_z}{\Omega} = 2\frac{J_1}{M^2}U^3 + \frac{3M_2}{2M^3}U^4 + \left(7\frac{J_1^2}{M^4} + 3\frac{M_2}{M^3}\right)U^6 + \left(11\frac{J_1 M_2}{M^5} - 6\frac{S_3}{M^4}\right)U^7 + \ldots$$

where $U = (M\Omega)^{1/3}$.

The Orbital frequency gives the Keplerian mass: $\Omega = (M/r^3)^{1/2}(1 + O(r^{-1/2}))$.

in Scalar-Tensor theory:[a]

$$\frac{\Omega_\rho}{\Omega} = \left(3 - \frac{W_0\left(\beta_0 W_0 - 8\alpha_0 \bar{M}\right)}{2\bar{M}^2}\right)U^2 - \frac{4J_1}{\bar{M}^2}U^3 + \left[\left(\frac{9}{2} - \frac{3M_2}{2\bar{M}^3}\right) + (\beta - 1)\frac{W_0^2}{\bar{M}^2} - \frac{13\beta^2 W_0^4}{24\bar{M}^4}\right]U^4 + \ldots$$

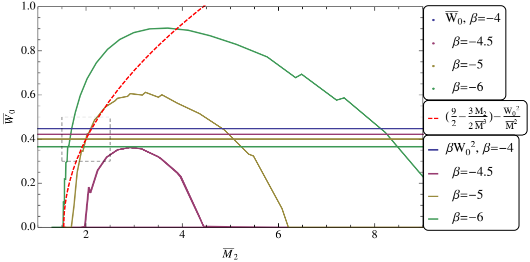
$$\frac{\Omega_z}{\Omega} = \frac{2J_1}{\bar{M}^2}U^3 + \frac{3(M_2 - \alpha_0 W_2)}{2\bar{M}^3}U^4 - \frac{2J_1 W_0\left(\beta_0 W_0 - \alpha_0 \bar{M}\right)}{\bar{M}^4}U^5 + \ldots$$

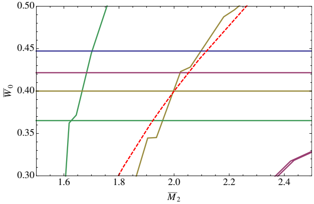
where $U = (\bar{M}\Omega)^{1/3}$. The calculations are done in the Jordan frame. Again the orbital frequency gives the Keplerian mass: $\Omega = (\bar{M}/r^3)^{1/2}(1 + O(r^{-1/2}))$, but this time the Keplerian mass is $\bar{M} = M - W_0\alpha_0$. W_0 is the scalar charge, W_2 is the scalar quadrupole and $\alpha \equiv (d\ln A)/d\phi, \beta \equiv d\alpha/d\phi$. These observables could in principle distinguish between GR and Scalar-Tensor theory.

[a] GP, T.P. Sotiriou, MNRAS 454, 4066 (2015)

Measurement example:

Radial precession frequency: $(\Omega_r/\Omega) = \sum C_a \Omega^{a/3}$, where we can imagine measuring the coefficients up to $C_5^{\rm STT}$, using QPOs. These together with $\bar{W}_0 = f_\beta(j, \bar{M}_2)$ can give us β and W_0.





$$45 C_3^{\rm STT} \bar{M}^{2/3} = 10 C_2^{\rm STT} C_3^{\rm STT} + 6 C_5^{\rm STT}, \quad j = -\frac{C_3^{\rm STT}}{4\bar{M}}$$

$$\beta \left(W_0/\bar{M}\right)^2 = 2\left(3 - C_2^{\rm STT} \bar{M}^{-2/3}\right),$$

$$\left[\left(\frac{9}{2} - \frac{3M_2}{2\bar{M}^3}\right) - \frac{W_0^2}{\bar{M}^2}\right] = \bar{C}_4.$$

- Neutron stars exhibit some black hole-like behaviour with respect to their moments structure, but the moments are different from black hole moments, so the geometry is essentially different from the geometry of Kerr black holes.
- There are several NS properties that show universal behaviour (EoS independent).
- These universal properties are present in several alternative theories of gravity as well, such as in ST theories.
- Some of these relations could be used to distinguish between some alternative theories.
- The multipole moments determine the orbital dynamics and are of relevance to the study of accretion discs and quasi-periodic oscillations (QPOs).
- These geodesic properties could also distinguish between different theories of gravity such as GR and Scalar-Tensor theory.
- There is a lot of work to be done (in particular on the astrophysics side).

Thank you