

EQUATION OF STATE CONSTRAINS THROUGH BINARY NEUTRON STAR MERGERS

NIKOLAOS STERGIOULAS

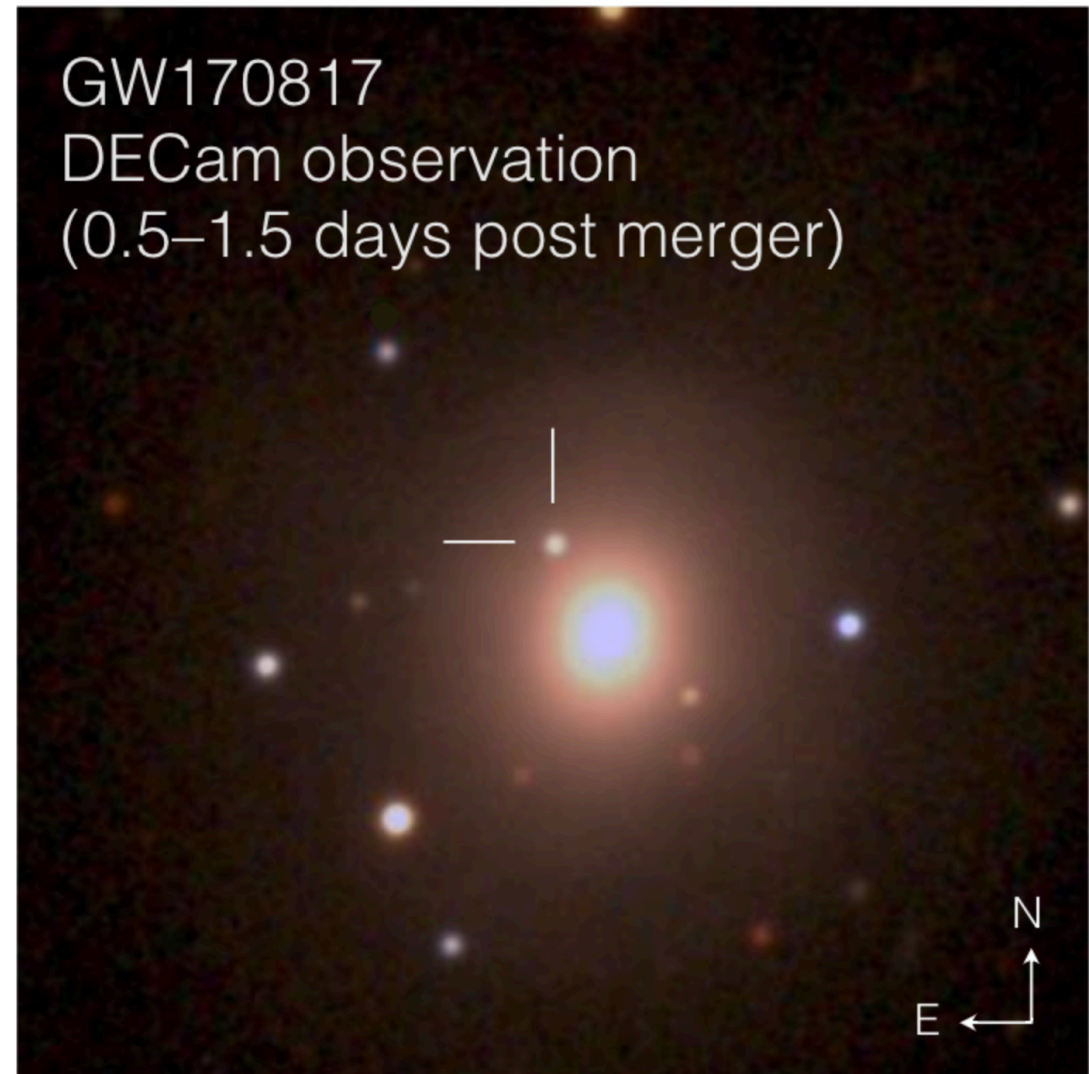
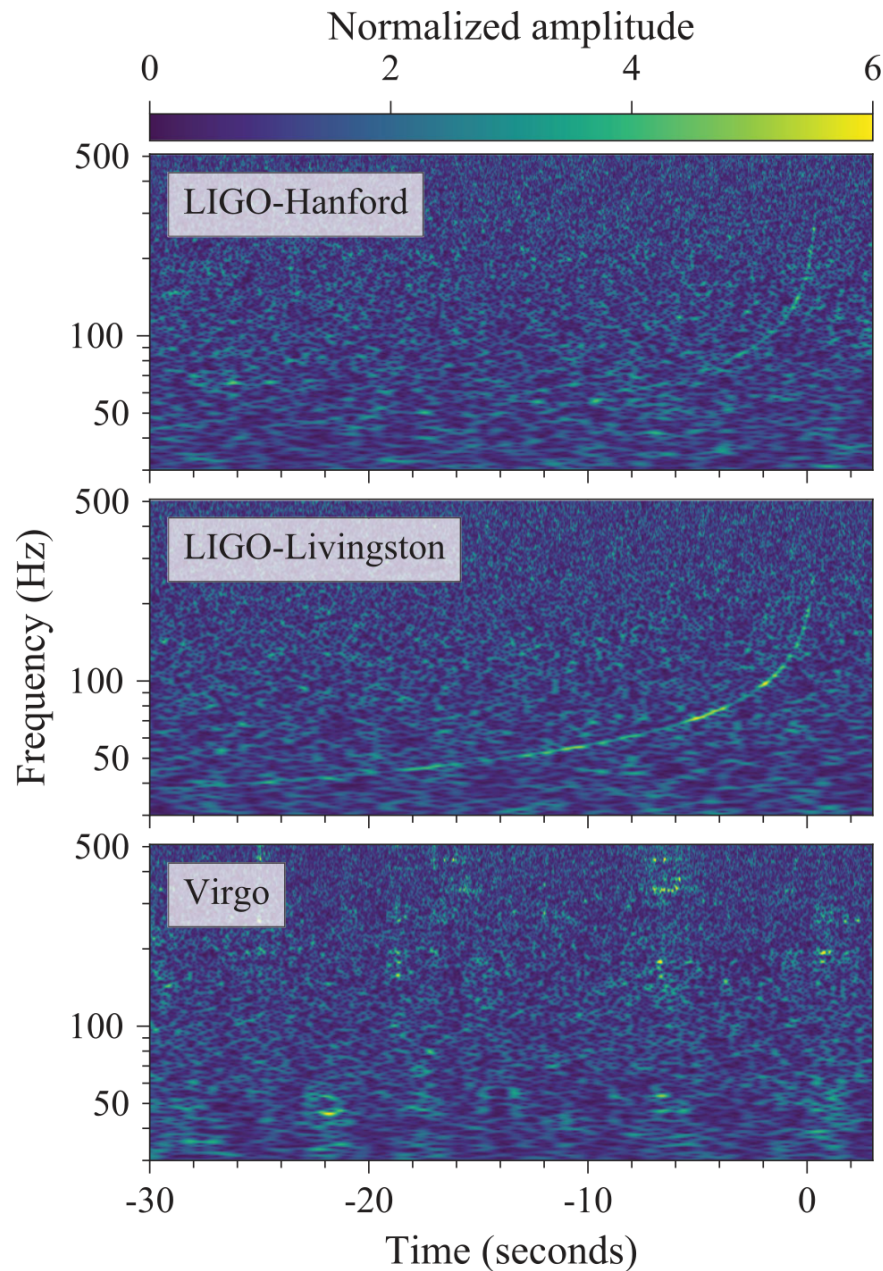
DEPARTMENT OF PHYSICS
ARISTOTLE UNIVERSITY OF THESSALONIKI



Tuebingen, November 14, 2019

GW170817 Binary Neutron Star (BNS) merger

GW + EM follow up



Rence BNS merger candidates

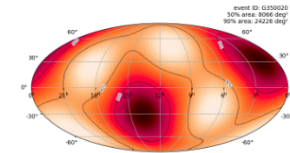
So far in O3 LVC run:

[S190910h](#)

BNS (61%), Terrestrial
(39%)

Sept. 10, 2019
08:29:58 UTC

[GCN Circulars](#)
[Notices](#) | [VOE](#)



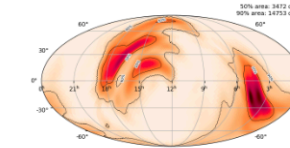
1.1312 per year

[S190901ap](#)

BNS (86%), Terrestrial
(14%)

Sept. 1, 2019
23:31:01 UTC

[GCN Circulars](#)
[Notices](#) | [VOE](#)



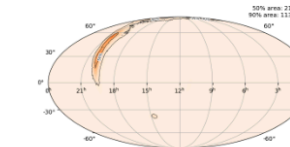
1 per 4.5093
years

[S190426c](#)

BNS (49%), MassGap
(24%), Terrestrial
(14%), NSBH (13%)

April 26, 2019
15:21:55 UTC

[GCN Circulars](#)
[Notices](#) | [VOE](#)



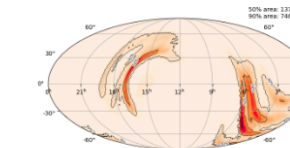
1 per 1.6276
years

[S190425z](#)

BNS (> 99%)

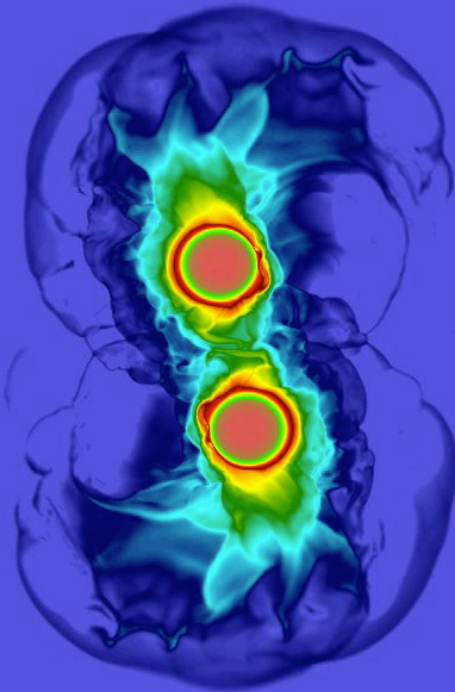
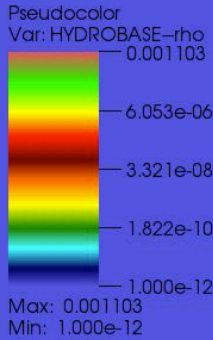
April 25, 2019
08:18:05 UTC

[GCN Circulars](#)
[Notices](#) | [VOE](#)



1 per 69834
years

DB: rho.xy_merged.h5
Cycle: 21504 Time:252

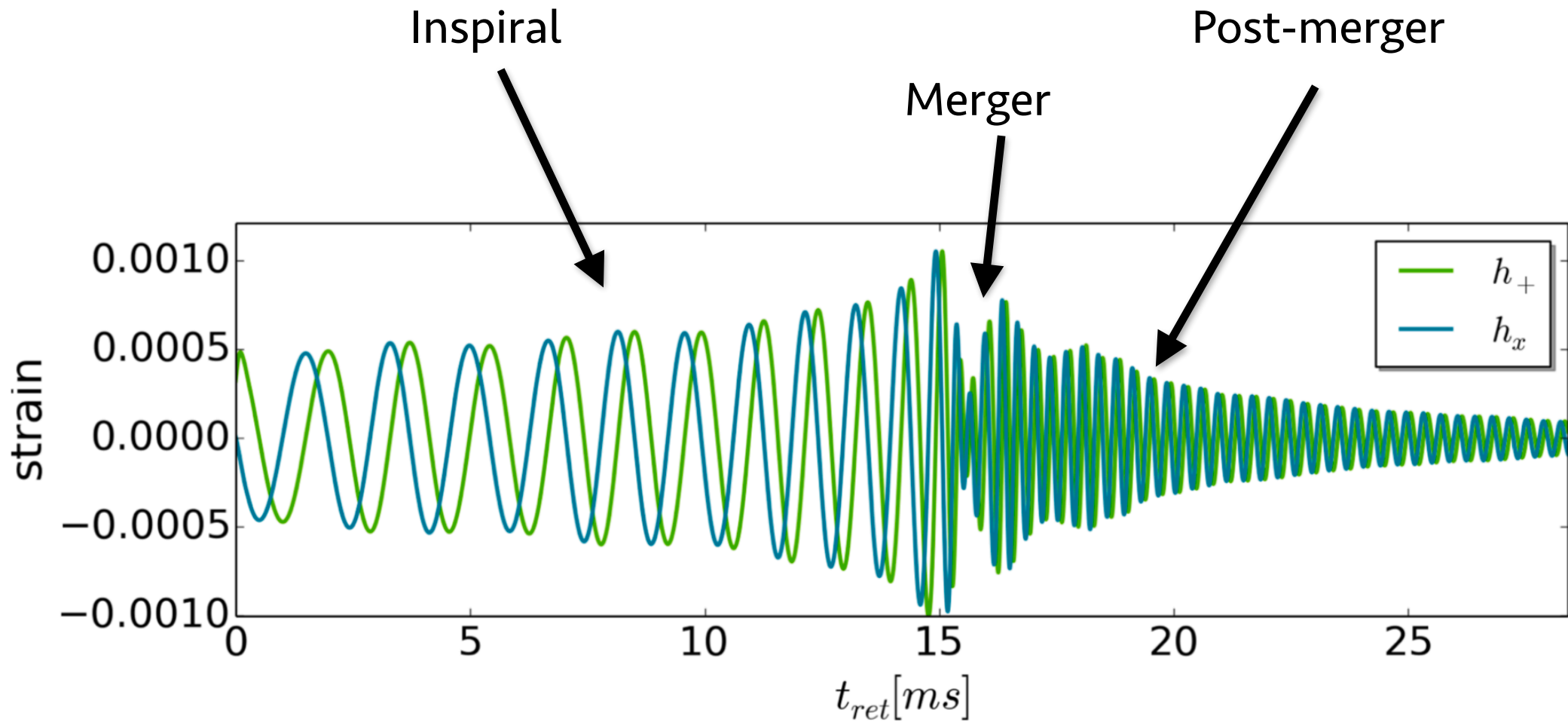


Simulation: Th. Soultanis & NS
Visualization: K. Zagkouris

user: c3po
Sat Mar 16 00:22:11 2019

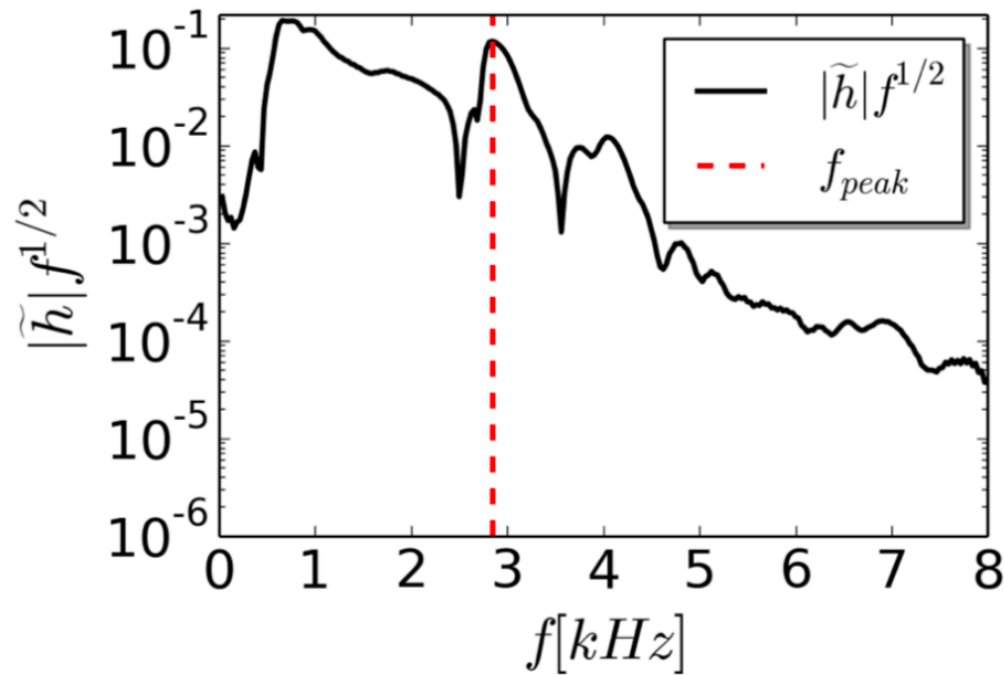
Gravitational Waves

Soultanis & NS (2018)



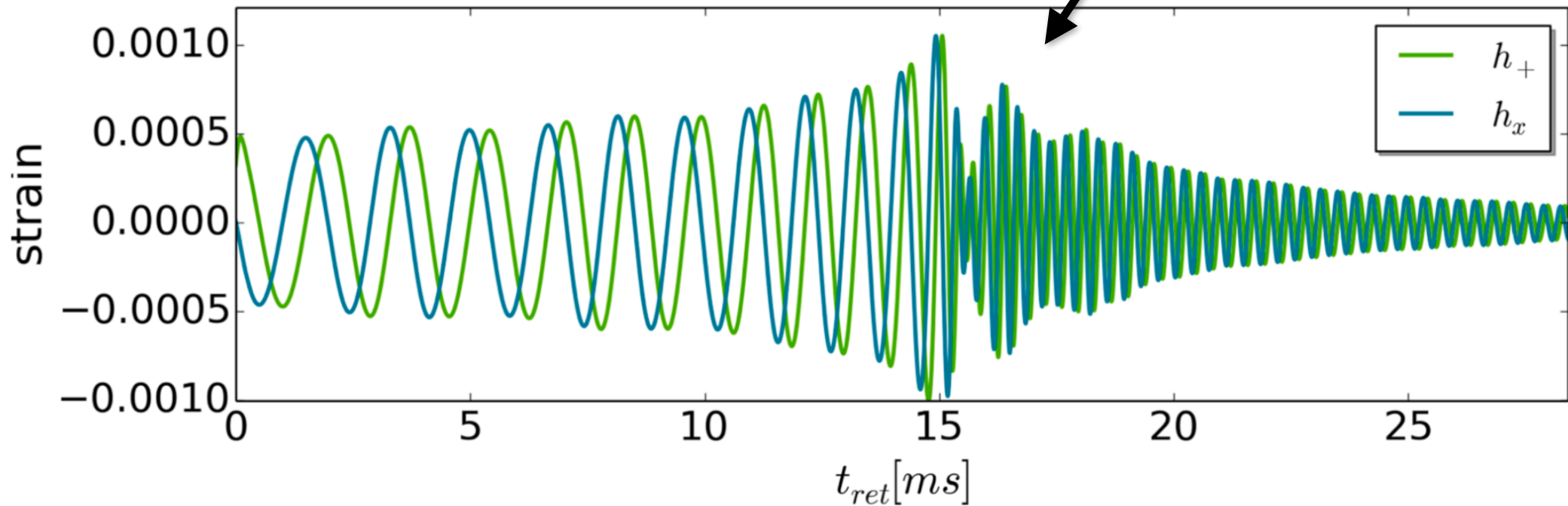
Gravitational Waves

Soultanis & NS (2018)



GW spectrum

GW strain



Cold Neutron Star Equation of State (EOS)

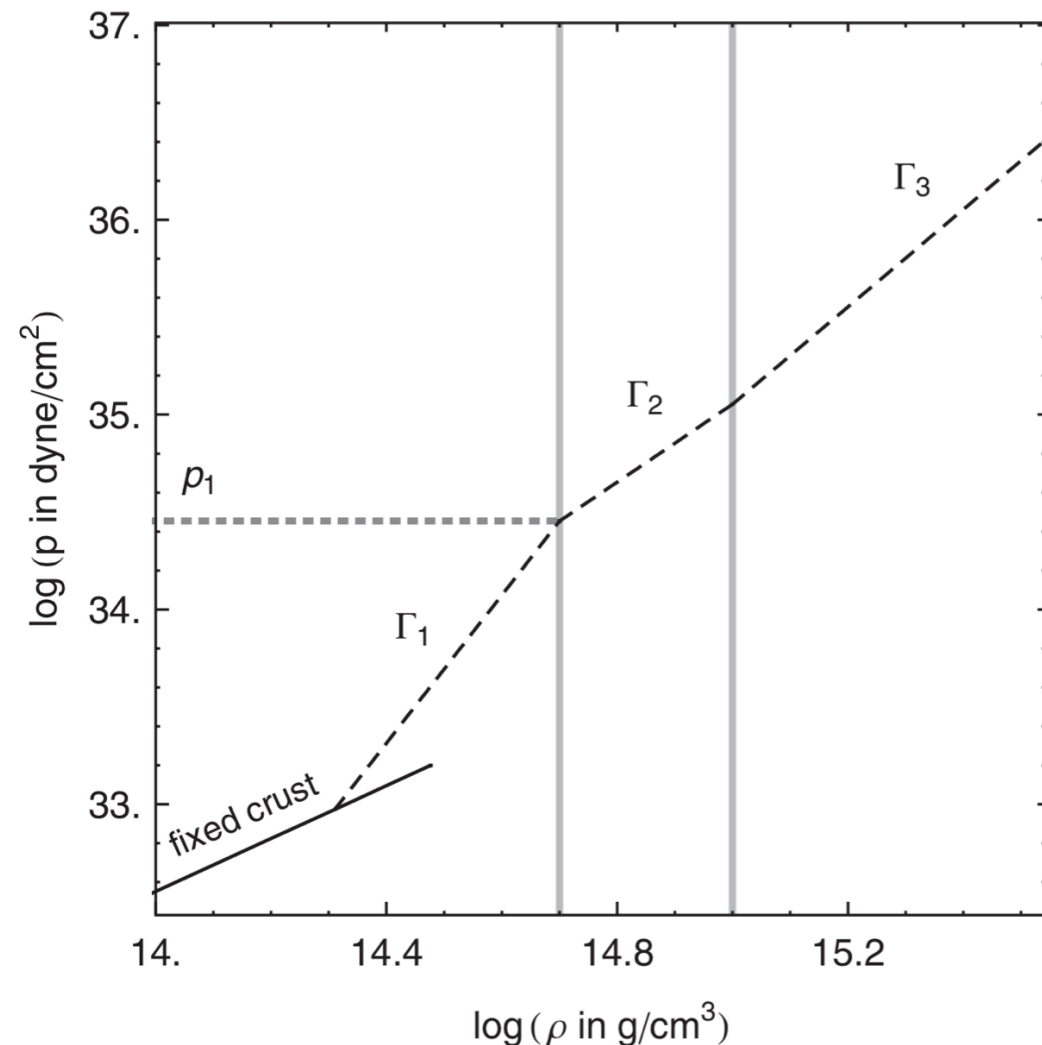
The interior of isolated, cold neutron stars is described in different approximations:

- 1) **Microphysical** (tabulated) EOS (needs high-order interpolation)
- 2) **Piecewise Polytropic** approximation
- 3) **Spectral** approximation
(in terms of basis functions)

Problems of Piecewise Polytropes:

- 1) *sound speed is not continuous*
- 2) shock waves and rarefactions waves may not be described correctly in a time evolution

Spectral representation does not have the above problems



Phase Transitions

Both piecewise and spectral are problematic when there are phase transitions.

If hadrons undergo **deconfinement** at high densities
-> quark-matter core

Several **effective models** to describe phase transitions:

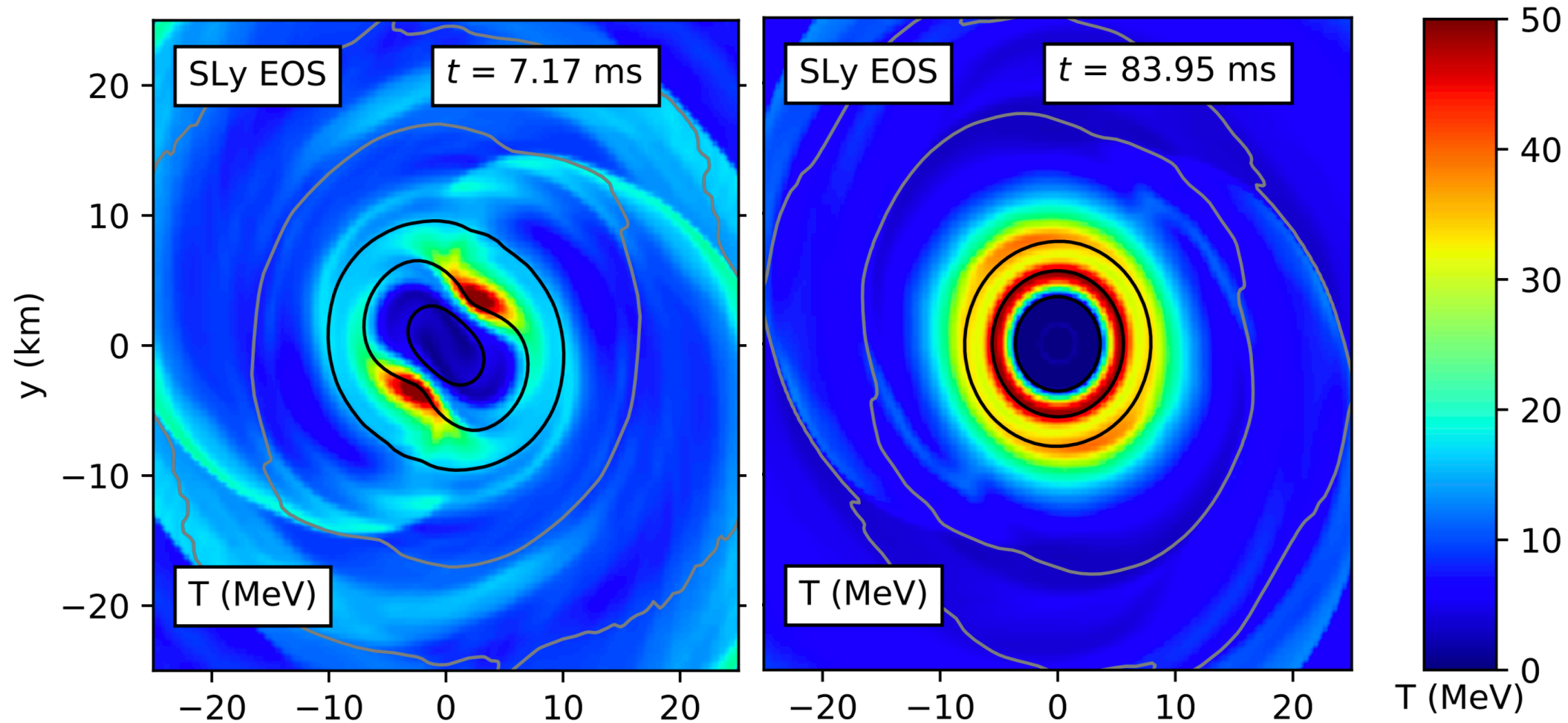
- 1) simple MIT bag model (massless quarks confined in a bag of finite dimension)
- 2) more complicated Nambu–Jona-Lasinio model
- 3) assume a density-independent speed of sound (constant-speed-of-sound parameterisation)

More generally, assume that:

- EoS is known at low densities
- pressure approaches that of deconfined quark matter at high densities

Thermal Structure of BNS Merger Remnant

Parts of the remnant reach temperatures of several tens MeV.



De Pietri et al. (2019)

Hot Neutron Star Equation of State (EOS)

Description of **shock heating** in the BNS remnant requires high-T EOS

- 1) Hot Microphysical, $P = P(\rho, s, Y_e, \dots)$
- 2) Cold EOS + thermal part e.g. $P_{\text{thermal}} = (\gamma-1)\rho\varepsilon$ (ideal fluid)
- 3) Approximate *Taylor expansions* of temperature dependence:
 - a) of the energy per nucleon E
 - b) of the proton fraction $x_p = \rho_p/(\rho_n + \rho_p)$ from pure neutron matter to symmetric neutron matter

EOS Constraints through BNS mergers

- 1) GW @ inspiral (already achieved with GW170817)
- 2) GW @ post-merger (expected with A+ and 3G detectors)
- 3) EM emission
- 4) GW + EM (multimessenger)
- 5) all of the above + laboratory experiments

Tidal Deformability

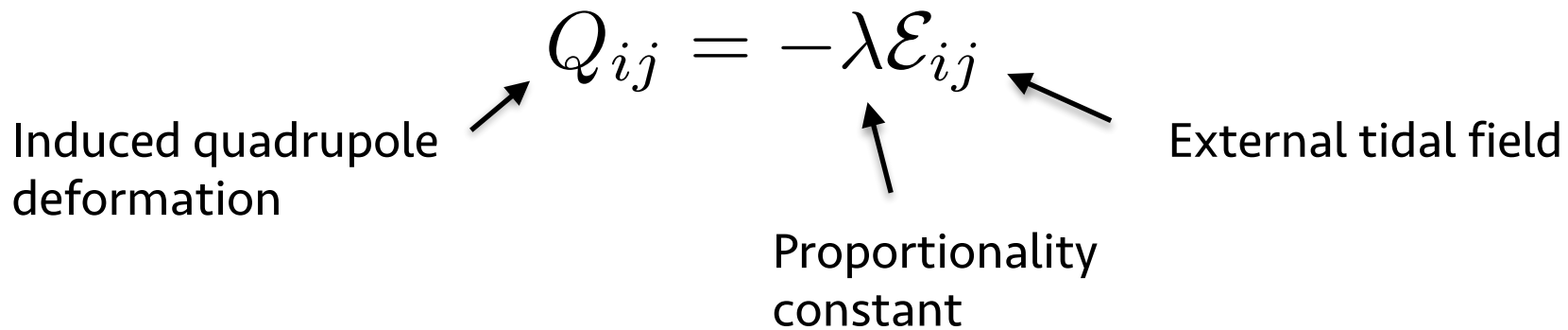
Tidal deformation of NS in the field of a companion:

$$Q_{ij} = -\lambda \mathcal{E}_{ij}$$

Induced quadrupole deformation

Proportionality constant

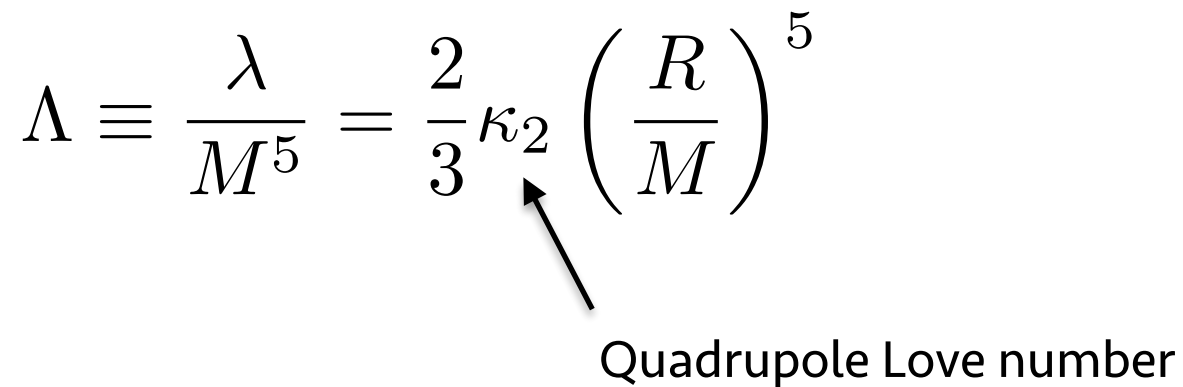
External tidal field

A diagram illustrating the equation for induced quadrupole deformation. The equation is $Q_{ij} = -\lambda \mathcal{E}_{ij}$. Three arrows point from descriptive text to parts of the equation: one from 'Induced quadrupole deformation' to Q_{ij} , one from 'Proportionality constant' to λ , and one from 'External tidal field' to $\mathcal{E}_{ij}$.

Dimensionless tidal deformability

$$\Lambda \equiv \frac{\lambda}{M^5} = \frac{2}{3} \kappa_2 \left(\frac{R}{M} \right)^5$$

Quadrupole Love number

A diagram showing the equation for dimensionless tidal deformability. The equation is $\Lambda \equiv \frac{\lambda}{M^5} = \frac{2}{3} \kappa_2 \left(\frac{R}{M} \right)^5$. An arrow points from the text 'Quadrupole Love number' to the term κ_2 in the equation.

Calculating the Tidal Deformability

Assume a spherically symmetric, static background spacetime

$$ds_0^2 = -e^{\nu(r)} dt^2 + e^{\gamma(r)} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2)$$

Then

$$\begin{aligned} \Lambda = & \frac{16}{15} (1 - 2C)^2 [2 + 2C(y(R) - 1) - y(R)] \\ & \{ 2C[6 - 3y(R) + 3C(5y(R) - 8)] \\ & + 4C^3 [13 - 11y(R) + C(3y(R) - 2) + 2C^2(1 + y(R))] \\ & + 3(1 - 2C)^2 [2 - y(R) + 2C(y(R) - 1)] \ln(1 - 2C) \}^{-1} \end{aligned}$$

where

$$C \equiv M/R$$

and

$$\frac{dy}{dr} = \frac{4(m + 4\pi r^3 p)^2}{r(r - 2m)^2} + \frac{6}{r - 2m} - \frac{y^2}{r} - \frac{r + 4\pi r^3(p - \rho)}{r(r - 2m)} y - \frac{4\pi r^2}{r - 2m} \left\{ 5\rho + 9p + \frac{\rho + p}{(dp/d\rho)} \right\}$$

Effect of Λ on GWs

Binary system with masses m_A and m_B

the waveform is affected simultaneously by Λ_A and Λ_B

However, to lowest post-Newtonian order, the waveform is to a good approximation influenced by a *single parameter*, the mass-weighted **effective tidal deformability**

$$\tilde{\Lambda} = \frac{16}{13} \frac{(1 + 12q)\Lambda_A + (12 + q)q^4\Lambda_B}{(1 + q)^5}$$

where

$$q \equiv m_B/m_A (\leq 1)$$

is the mass ratio of the binary.

From Λ to Radius

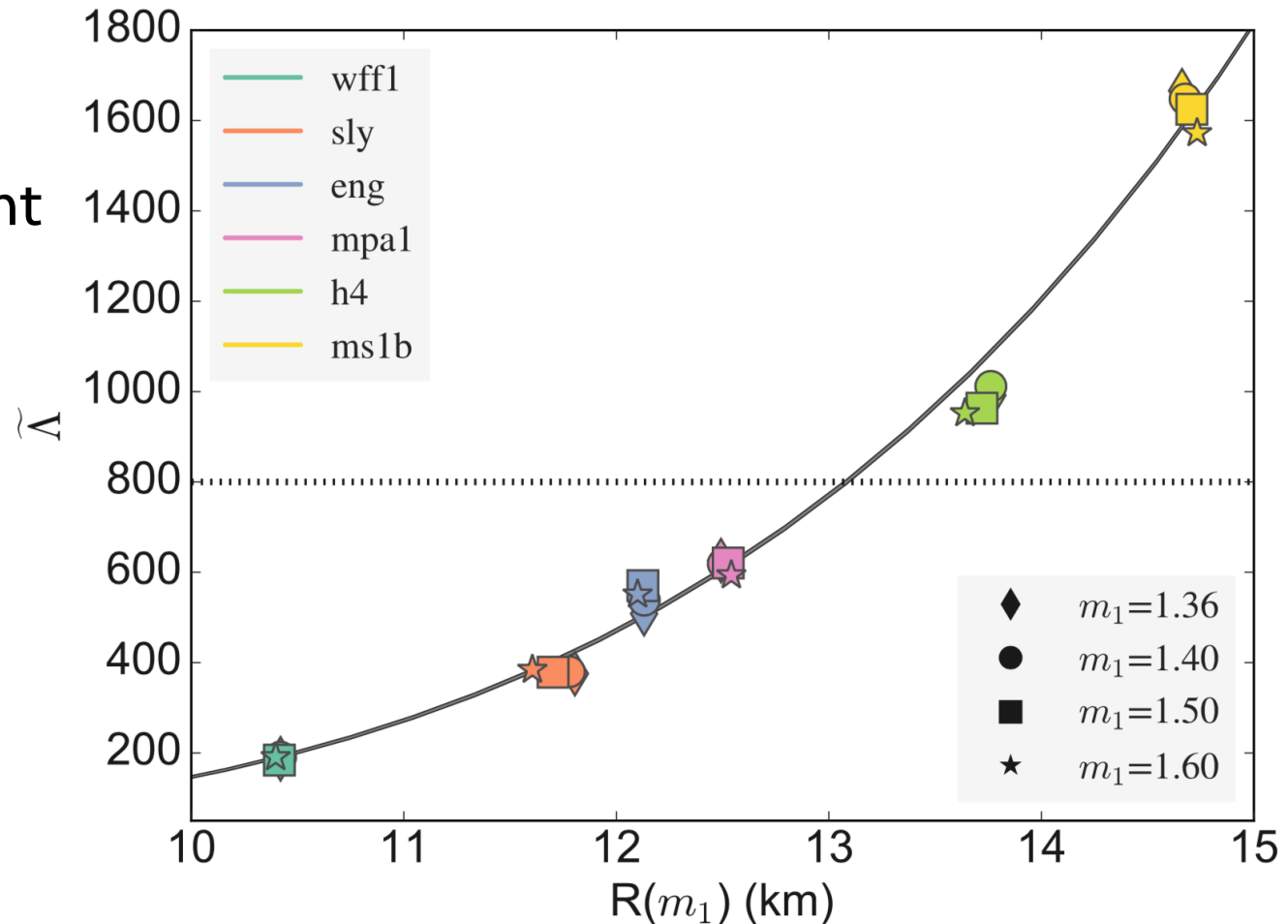
One of the main observables of the inspiral GW waveform is the **chirp mass**

$$\mathcal{M}_c = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}} = m_1 \frac{q^{3/5}}{(1+q)^{1/5}}$$

For GW170817 $\mathcal{M}_c = 1.188^{+0.004}_{-0.002} M_\odot$

For fixed $\mathcal{M}_c \rightarrow$
nearly mass-independent
empirical relation

Raithel, Oezel
& Psaltis (2018)



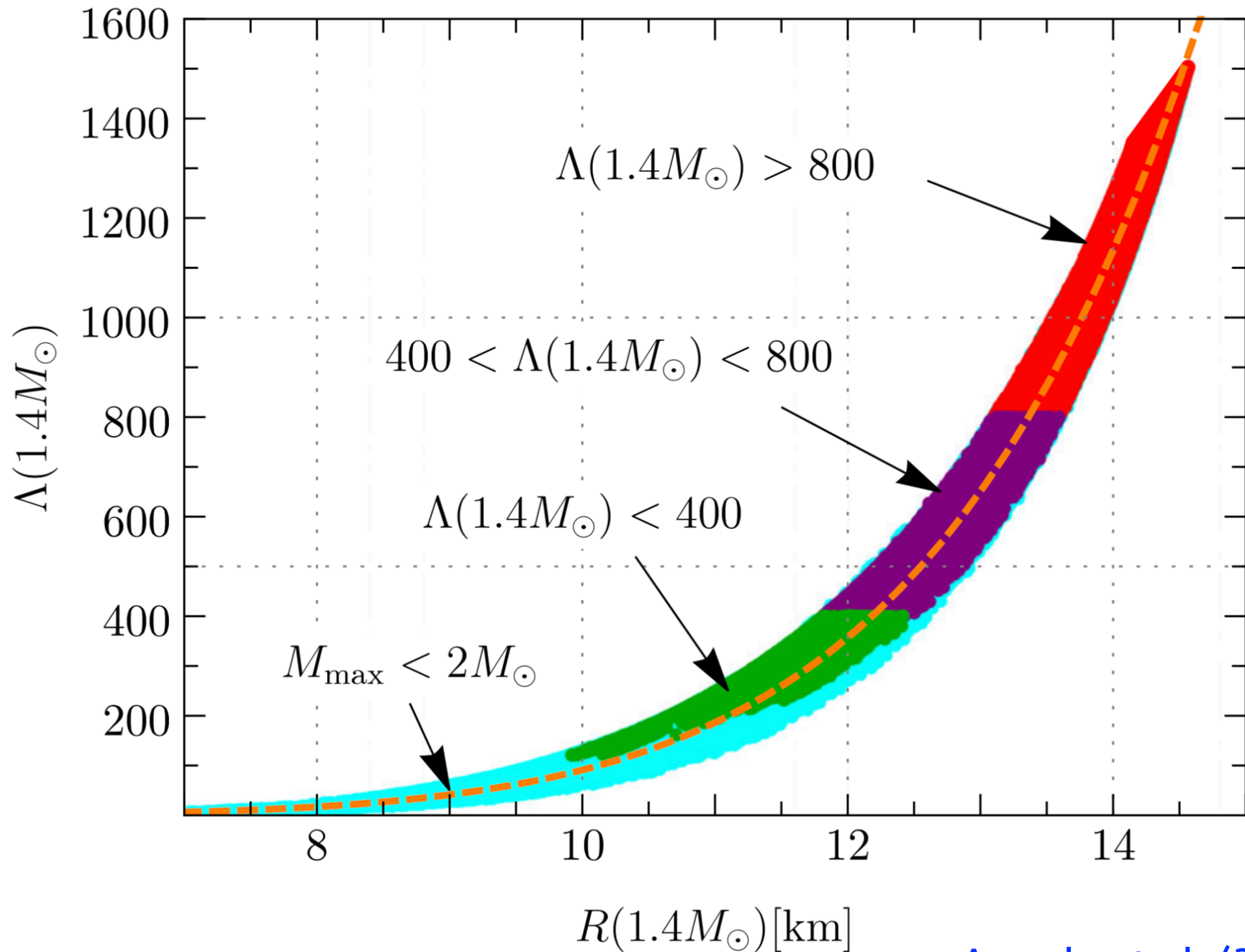
Λ through GW @ Inspiral

For NS binaries with individual masses around $1.4M_{\odot}$, the dimensionless tidal deformability Λ could be realistically determined with about **10% accuracy** by combining information from about 20 – 100 sources.

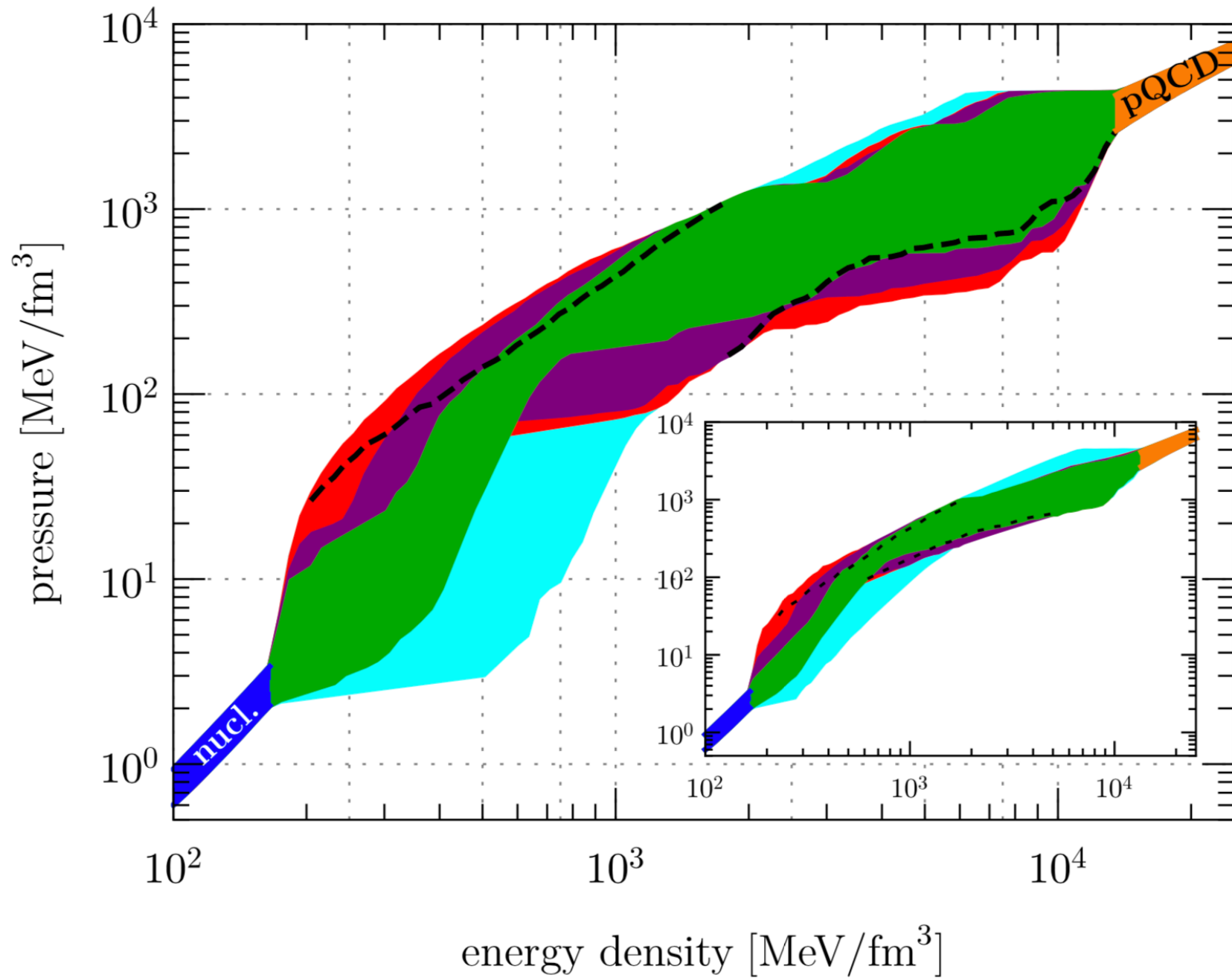
Additional effects (may become relevant for next-generation detectors)

- 1) tidal excitations of *resonant modes*
- 2) gravitomagnetic excitations of resonant modes
- 3) resonant shattering of the NS crust by tides
- 4) nonlinear tidal effects

Empirical Relation for Λ @ Specific Mass



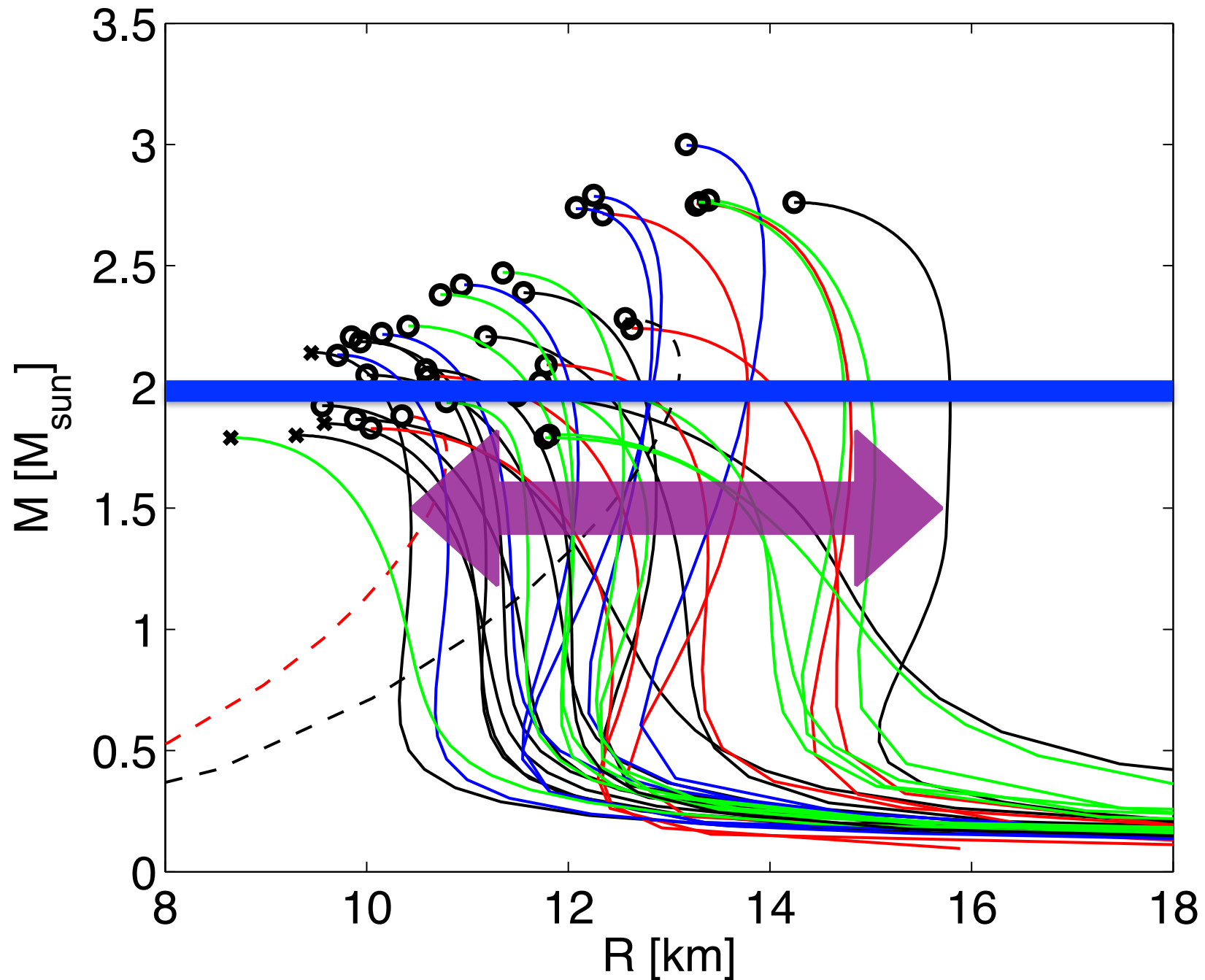
EOS Constraints



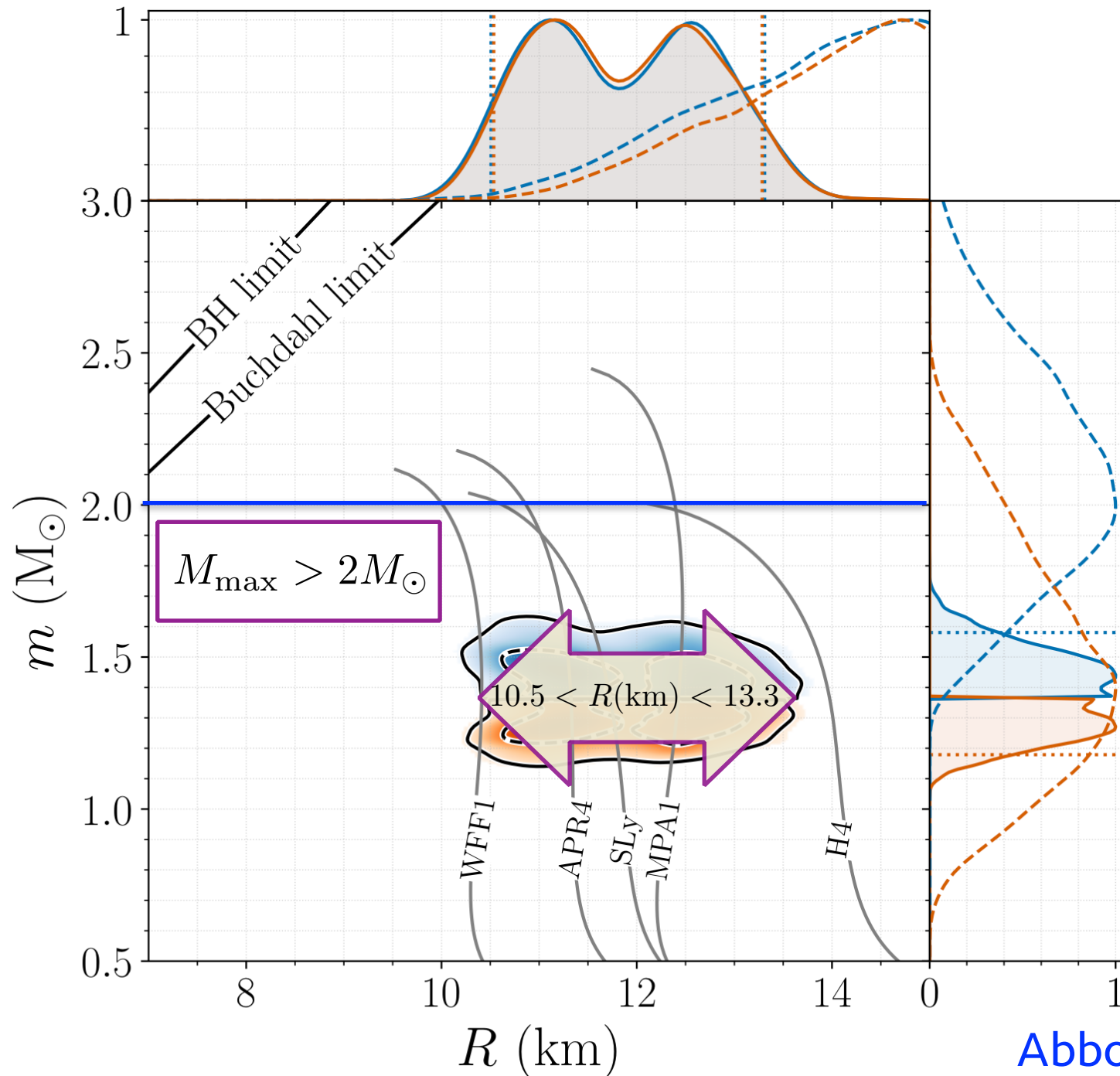
Annala et al. (2018)

Radius uncertainties before GW170817

Bauswein, Janka, Hebeler & Schwenk (2012)



Constraints from the inspiral of GW170817



Abbot et al. (2018)

Various Radius Constraints based on GW170817

Montana et al. (2018)

Reference	R_i [km]
<i>Without a phase transition</i>	
Bauswein et al. [42]	$10.68^{+0.15}_{-0.03} \leq R_{1.6}$
Most et al. [51]	$12.00 \leq R_{1.4} \leq 13.45$
Burgio et al. [54]	$11.8 \leq R_{1.5} \leq 13.1$
Tews et al. [55]	$11.3 \leq R_{1.4} \leq 13.6$
De et al. [56]	$8.9 \leq R_{1.4} \leq 13.2$
LIGO/Virgo [57]	$10.5 \leq R_{1.4} \leq 13.3$
<i>With a phase transition</i>	
Annala et al. [46]	$R_{1.4} \leq 13.6$
Most et al. [51]	$8.53 \leq R_{1.4} \leq 13.74$
Burgio et al. [54]	$R_{1.5} = 10.7$
Tews et al. [55]	$9.0 \leq R_{1.4} \leq 13.6$
<i>This work</i>	
NS	$R_{1.4} = 13.11$
HS Model-2	$12.9 \leq R_{1.4} \leq 13.11$
HS _T Model-1	$10.1 \leq R_{1.4} \leq 12.9$
HS _T Model-2	$10.4 \leq R_{1.4} \leq 11.9$

Various Radius Constraints based on GW170817

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A Minimal Set of Assumptions

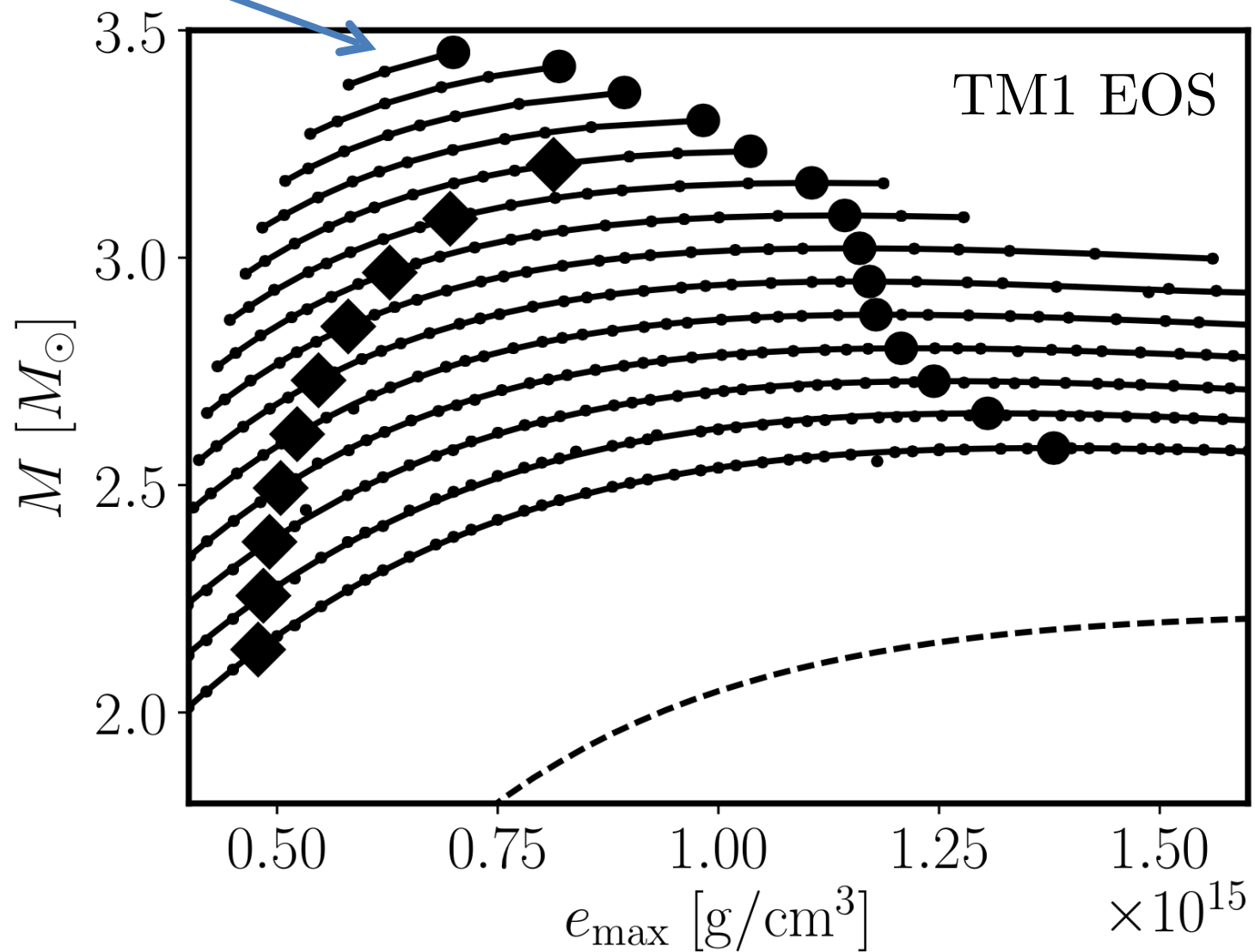
Bauswein, Just, Janka, NS (2017)
ApJ Letters

- 1) GR is valid.
- 2) A **hypermassive neutron star** (HMNS) was formed as a result of the GW170817 merger event, lasting for at least 10ms.
- 3) $M_{\text{max}} > 2M_{\odot}$
- 4) Causality holds ($v_s < c$)
- 5) $M_{\text{tot}}^{\text{GW170817}} = 2.74_{-0.01}^{+0.04} M_{\odot}$

Differentially Rotating Models

(Bauswein & NS 2017)

hypermassive stars

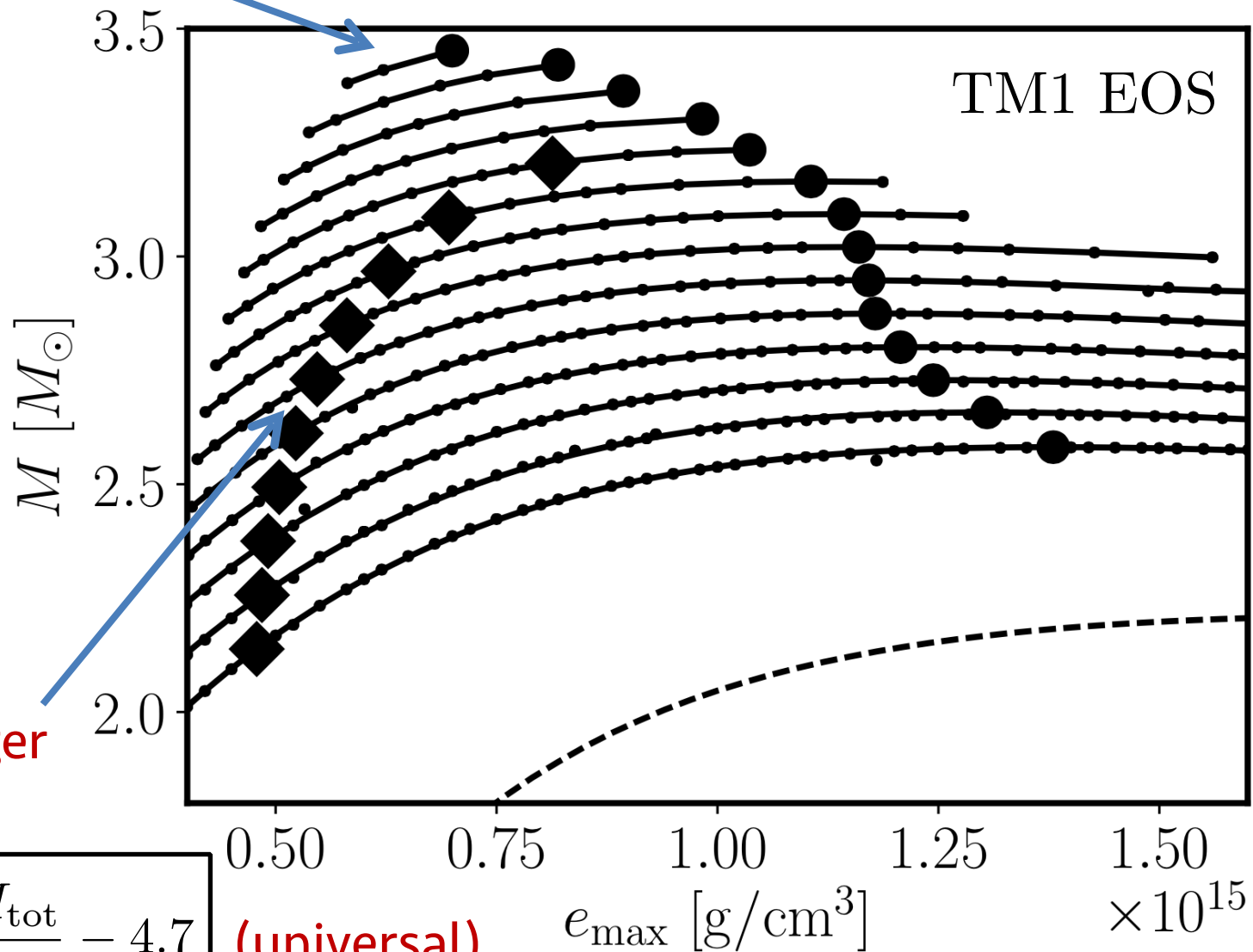


Differentially Rotating Models

(Bauswein & NS 2017)

hypermassive stars

BNS merger
remnants



(universal)

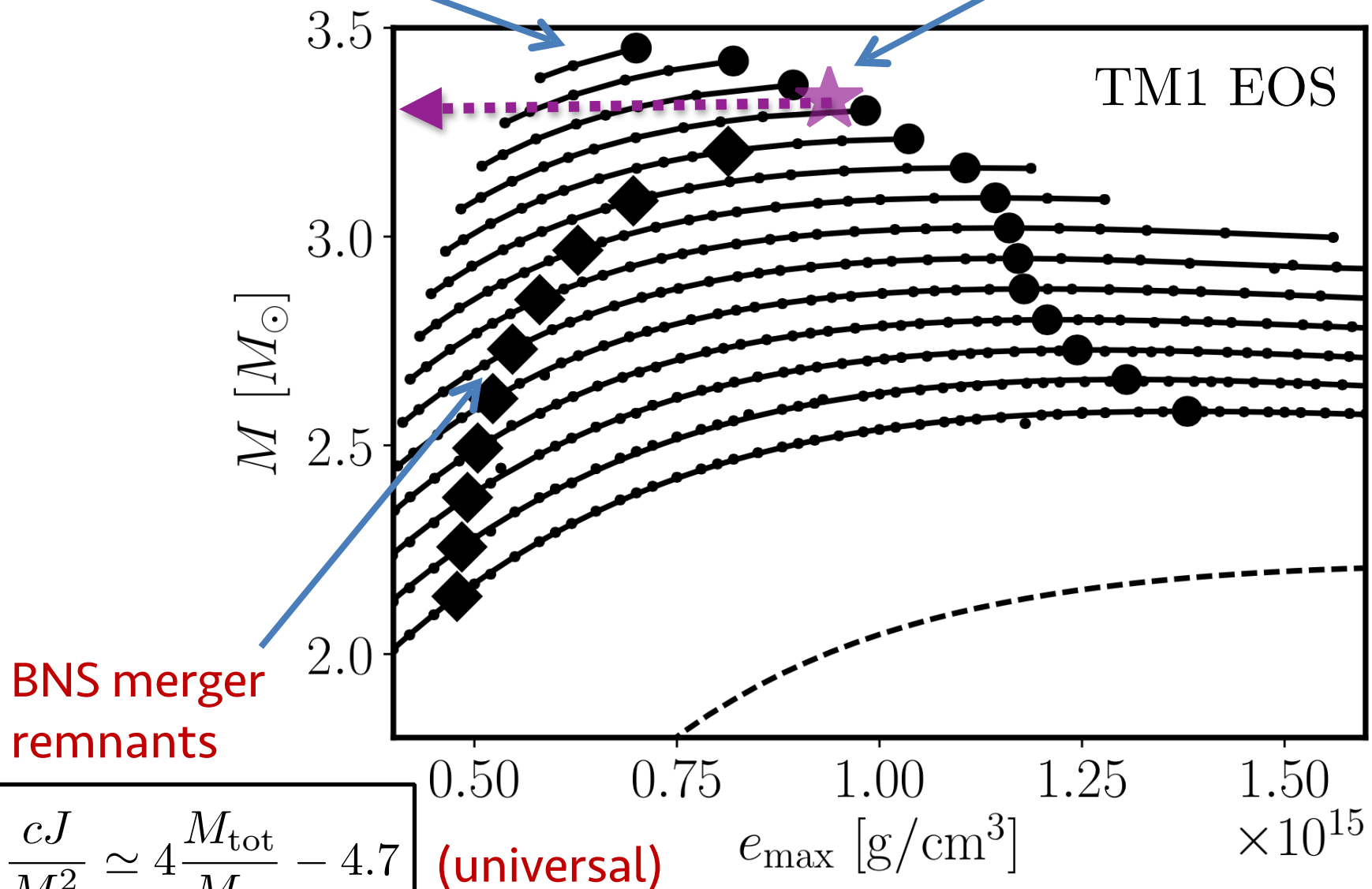
$$\frac{cJ}{M_{\odot}^2} \simeq 4 \frac{M_{\text{tot}}}{M_{\odot}} - 4.7$$

Differentially Rotating Models

(Bauswein & NS 2017)

hypermassive stars

Threshold mass



$$\frac{cJ}{M_\odot^2} \simeq 4 \frac{M_{\text{tot}}}{M_\odot} - 4.7$$

Differentially Rotating Models

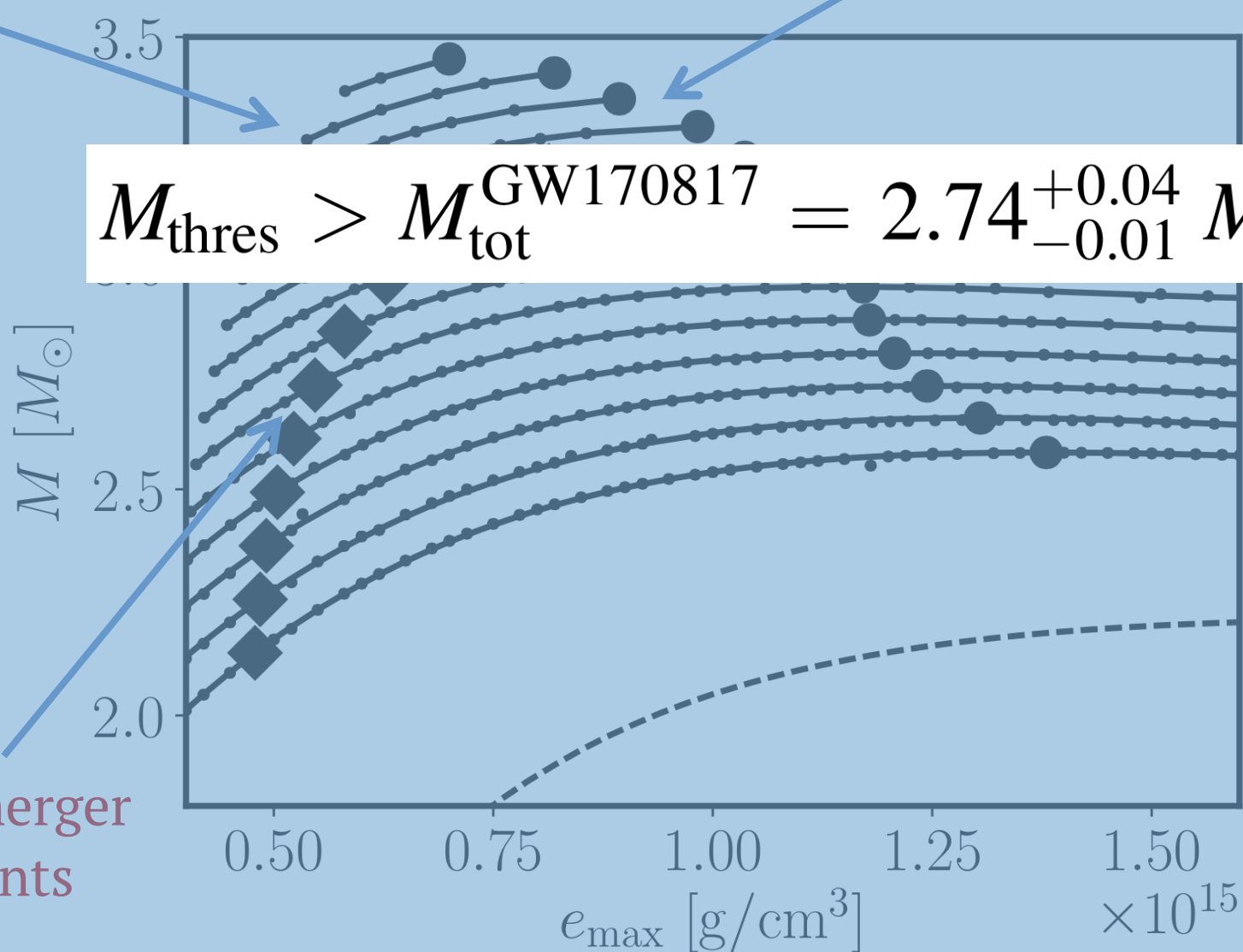
(Bauswein & NS 2017)

hypermassive stars

Threshold mass

$$M_{\text{thres}} > M_{\text{tot}}^{\text{GW170817}} = 2.74_{-0.01}^{+0.04} M_{\odot}$$

BNS merger
remnants

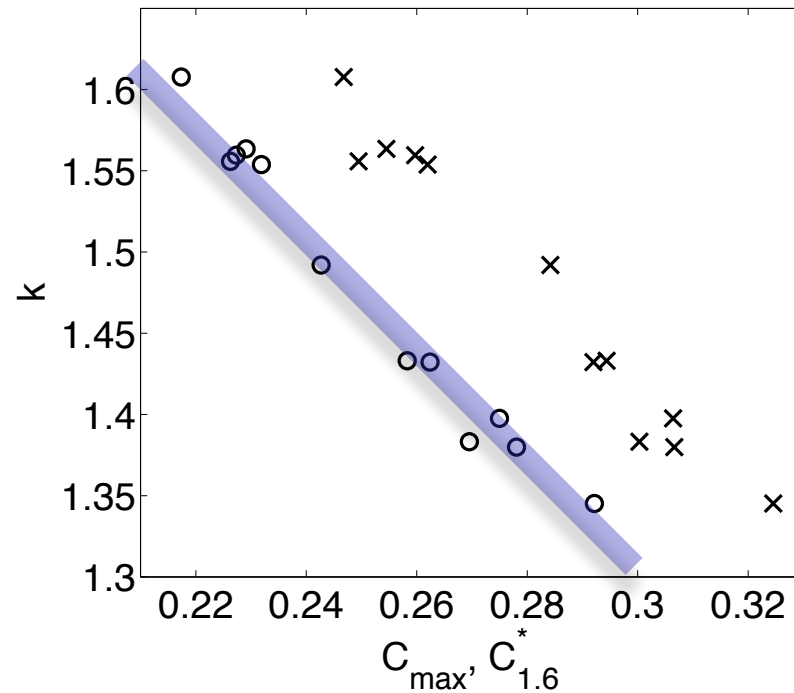


M_thres vs. M_max correlation

Bauswein, Baumgarte, Janka PRL (2013)

The threshold mass is related to the maximum TOV mass as

$$M_{\text{thres}} = k(C_{\text{max}}) \cdot M_{\text{max}} \quad C_{\text{max}} = \frac{M_{\text{max}}}{R_{\text{max}}}$$

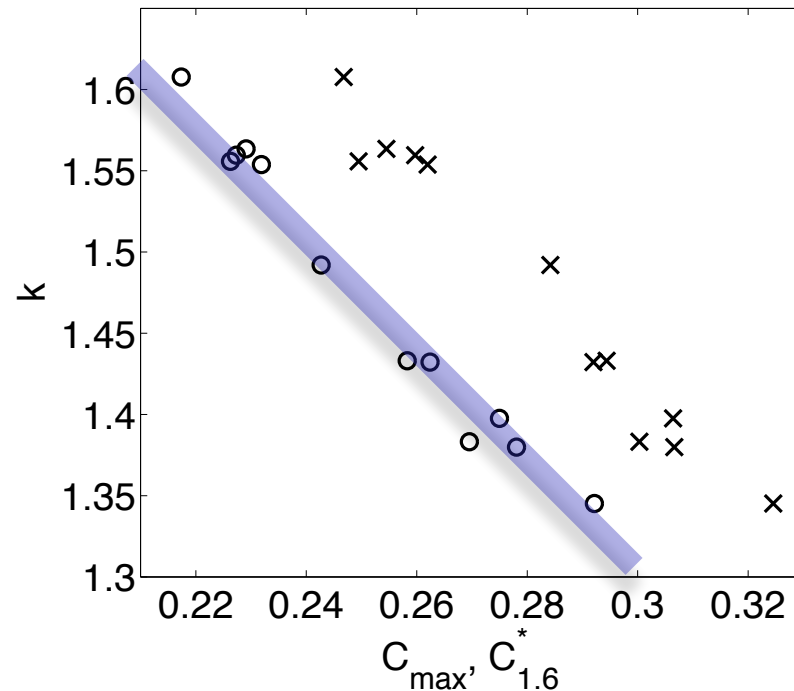


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This leads to the relation

$$M_{\text{thres}} = \left(-3.606 \frac{GM_{\text{max}}}{c^2 R_{1.6}} + 2.38 \right) \cdot M_{\text{max}} > 2.74 M_{\odot}$$

Maximally Compact EOS

Maximal compactness $\Leftrightarrow$ Maximally stiff core + maximally soft crust

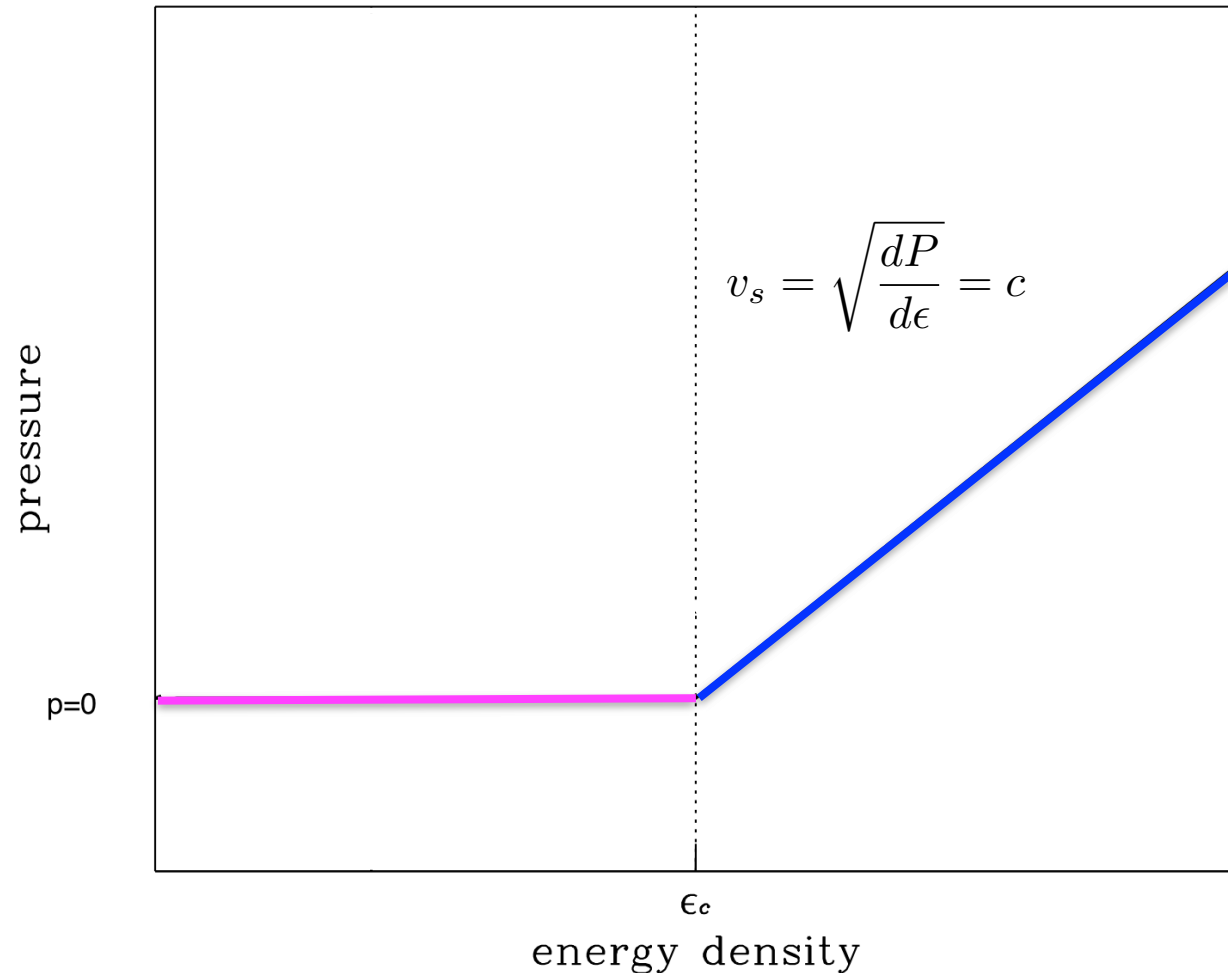
Koranda, NS & Friedman (1997)

Core is at the causal limit

$$v_s = c$$

$$M_{\max} \leq \frac{1}{2.82} \frac{c^2 R_{\max}}{G}$$

(Lattimer & Prakash)



Maximally Compact EOS

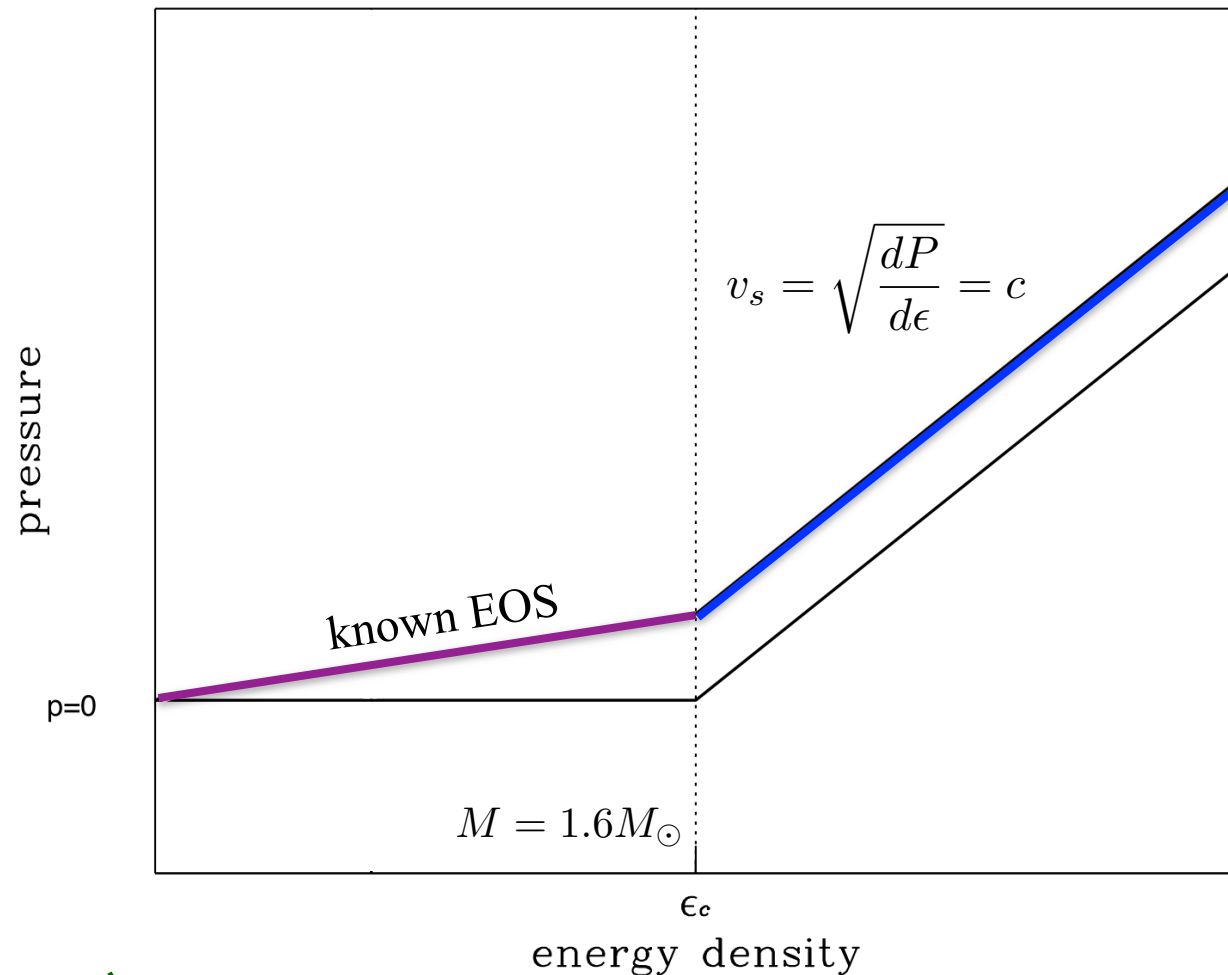
Maximal compactness $\Leftrightarrow$ Maximally stiff core + maximally soft crust

Assume causal limit
above

$$M > 1.6M_{\odot}$$

for each EOS

$$M_{\max} \leq \frac{1}{3.10} \frac{c^2 R_{1.6}}{G}$$

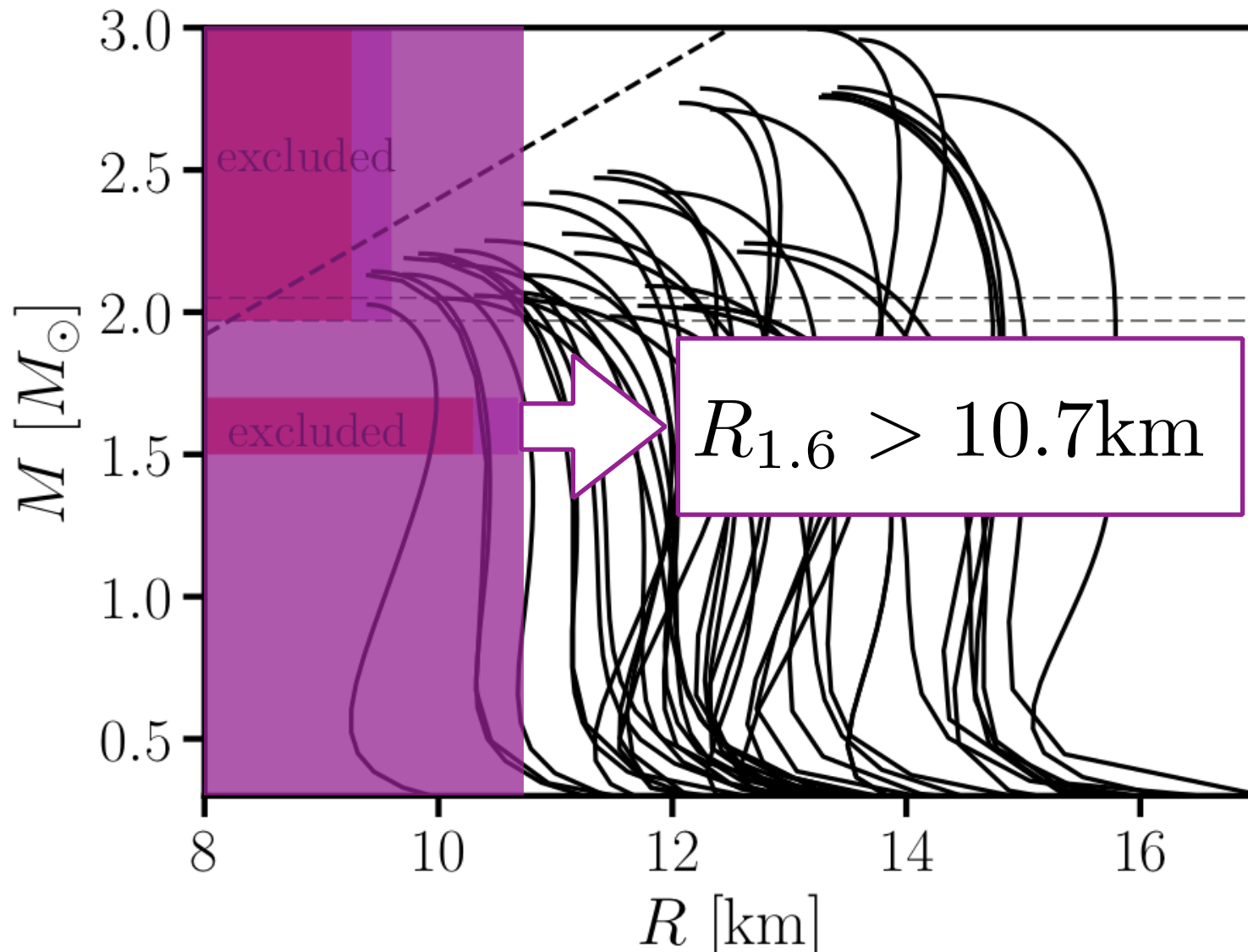


Bauswein, Just, Janka & NS (2017)

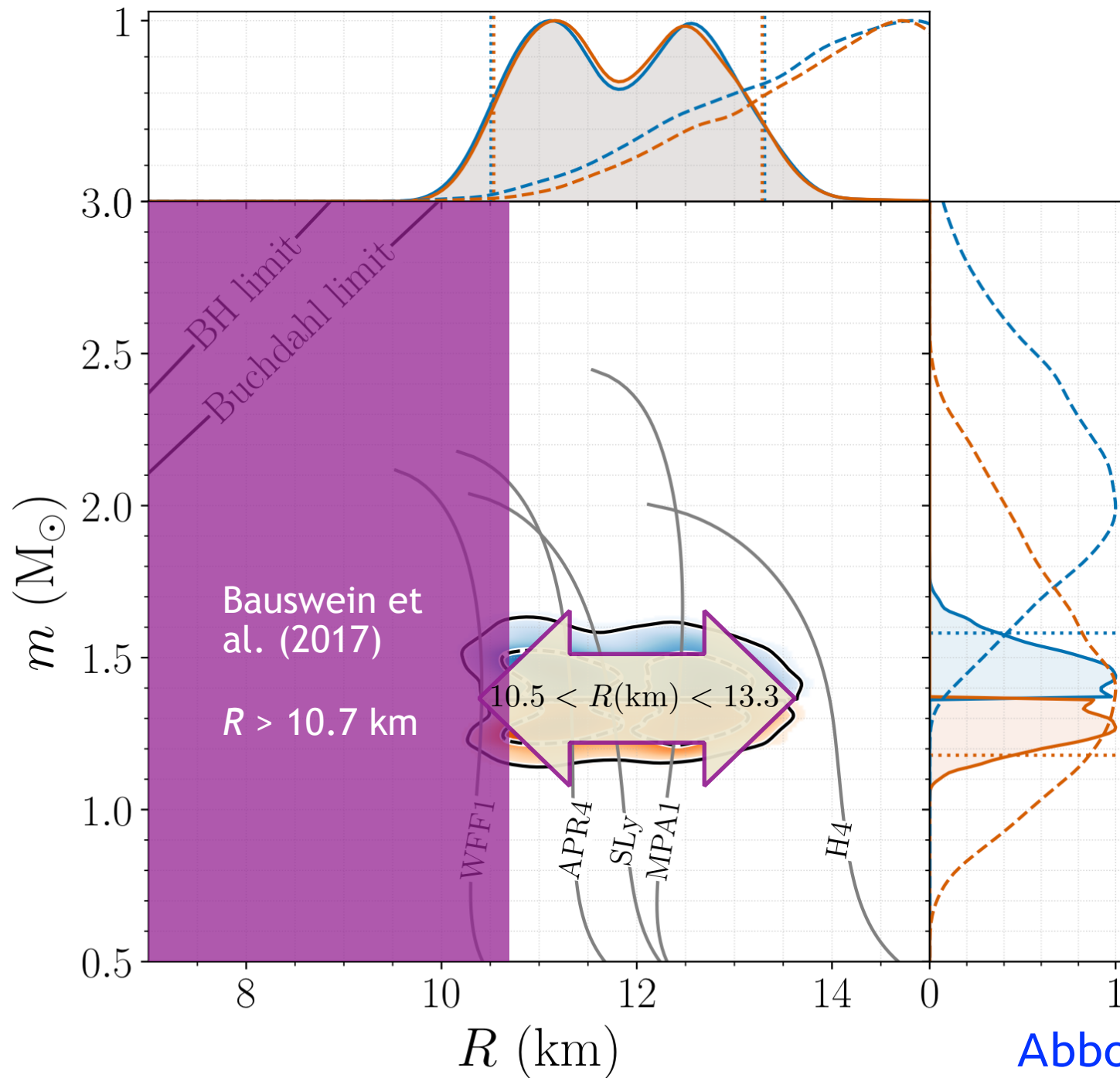
Radius Constraints from Minimal Assumptions

Bauswein, Just, Janka & NS (2017)

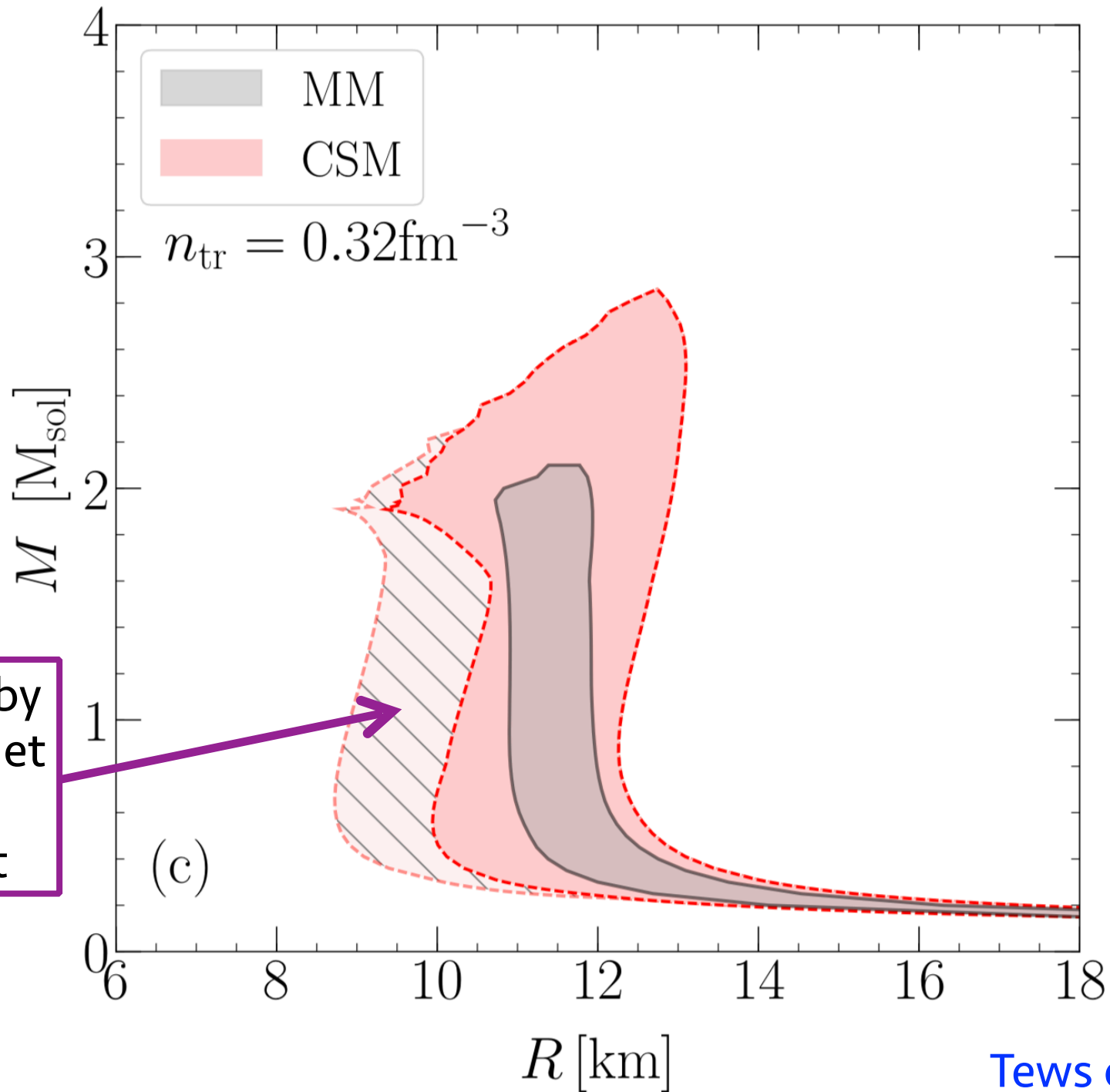
The combination of the minimal set of assumptions leads to a strict constraint on the radius:



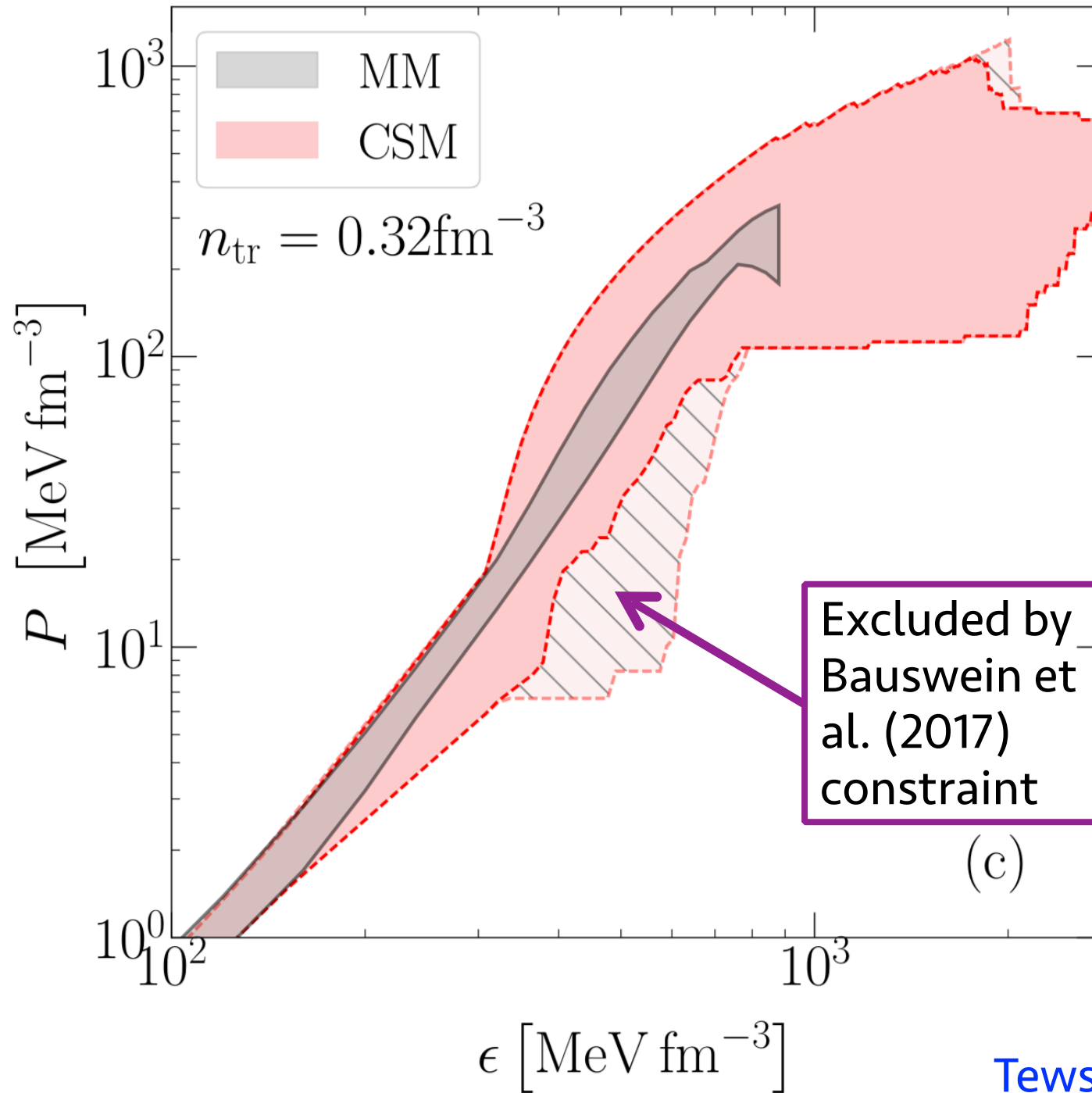
Comparison to Abbot et al. (2008)



Constraints on M-R Relation



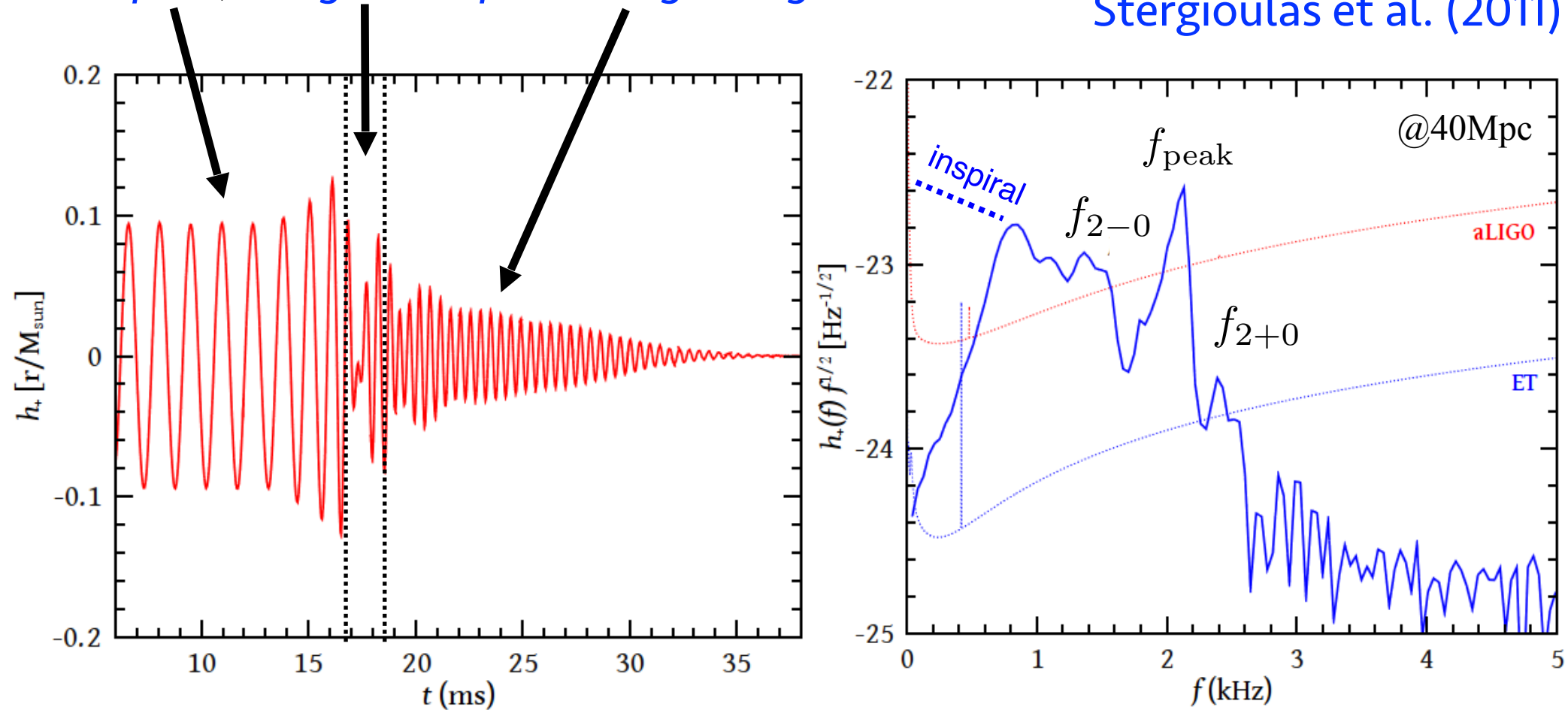
Constraints on EOS



Post-Merger Gravitational Waves

The GW signal can be divided into three distinct phases:
inspiral, *merger* and *post-merger ringdown*.

Stergioulas et al. (2011)



$$f_{\text{peak}} = f_2$$

is due to the fundamental $l=m=2$ *f-mode* oscillation

$$f_{2-0} = f_2 - f_0$$

$$f_{2+0} = f_2 + f_0$$

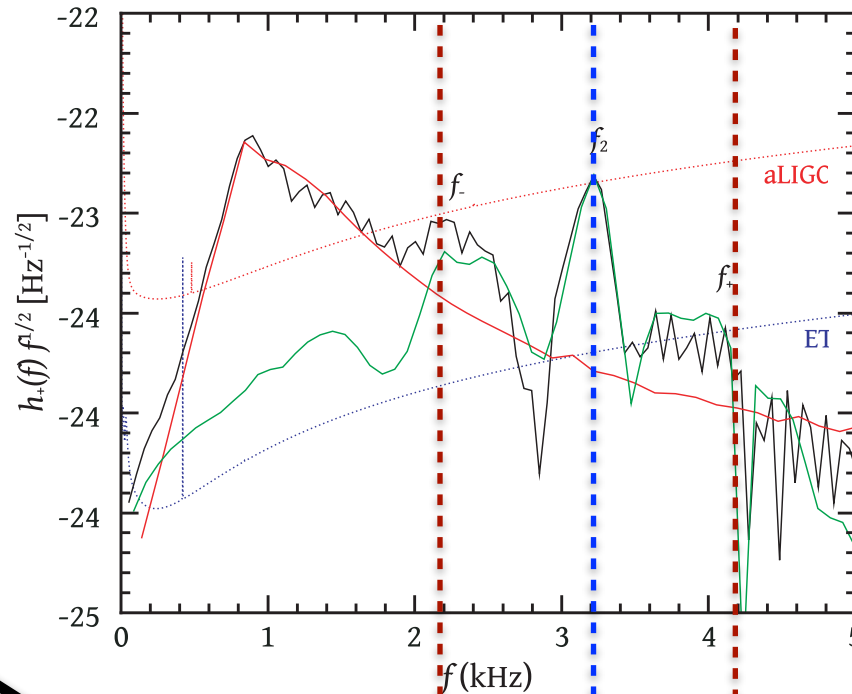
are *quasi-linear combination tones* -> (Tartini tones)

Sorge (1745), Tartini (1754)

Post-Merger GW Oscillations

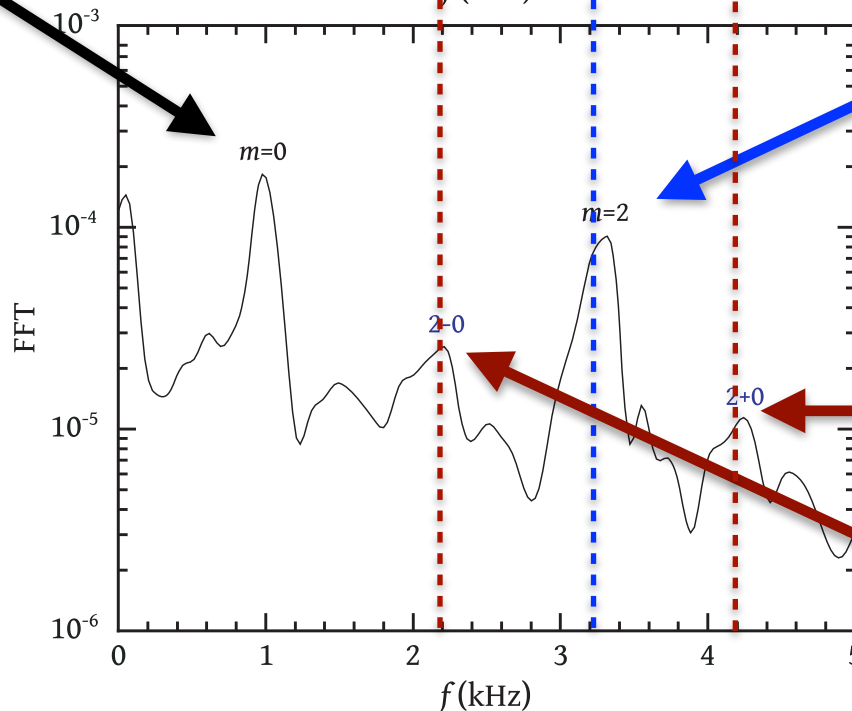
Stergioulas et al. (2011)

GRAVITATIONAL
WAVES



$m=0$ radial
mode

HYDRODYNAMICS

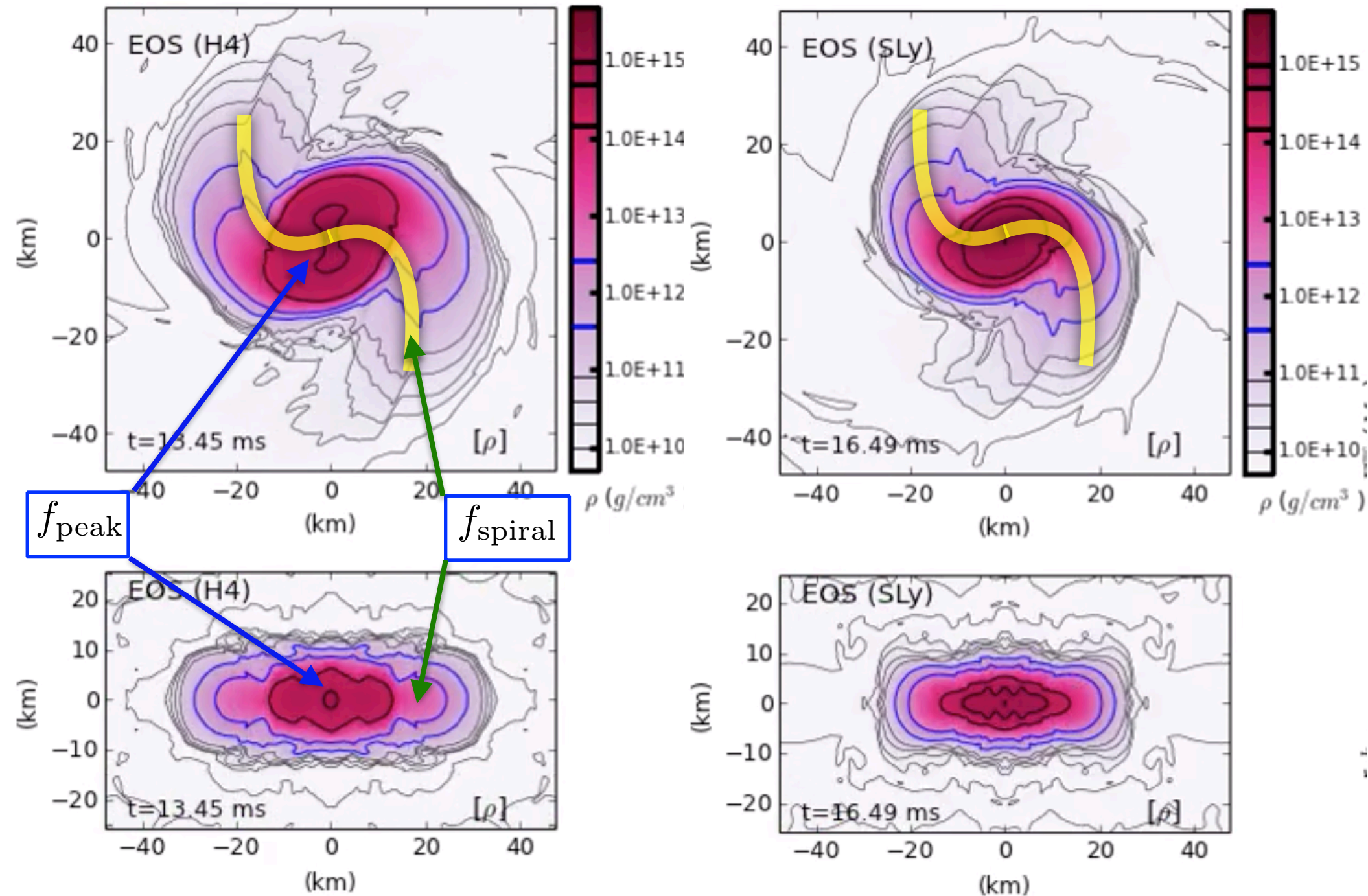


$m=2$ quadrupole
mode

“2-0” and “2+0”
quasi-linear
combination
frequency

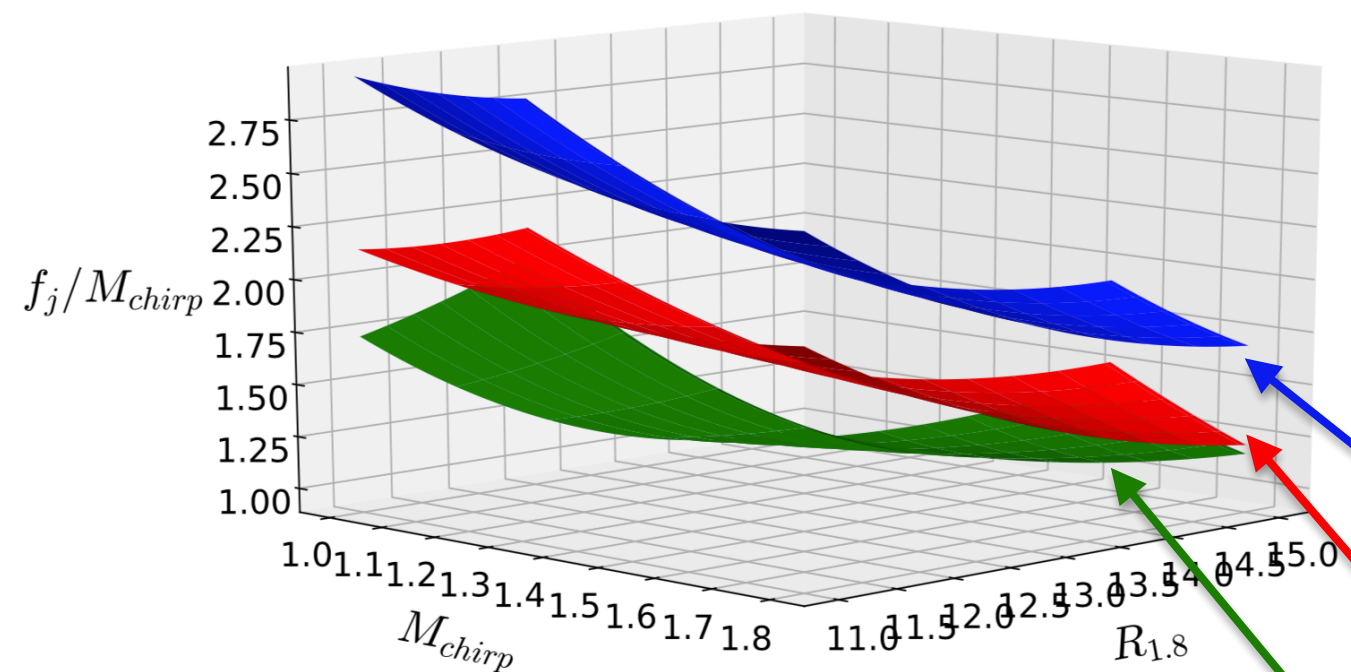
Spiral Deformation

Bauswein & NS (2015)

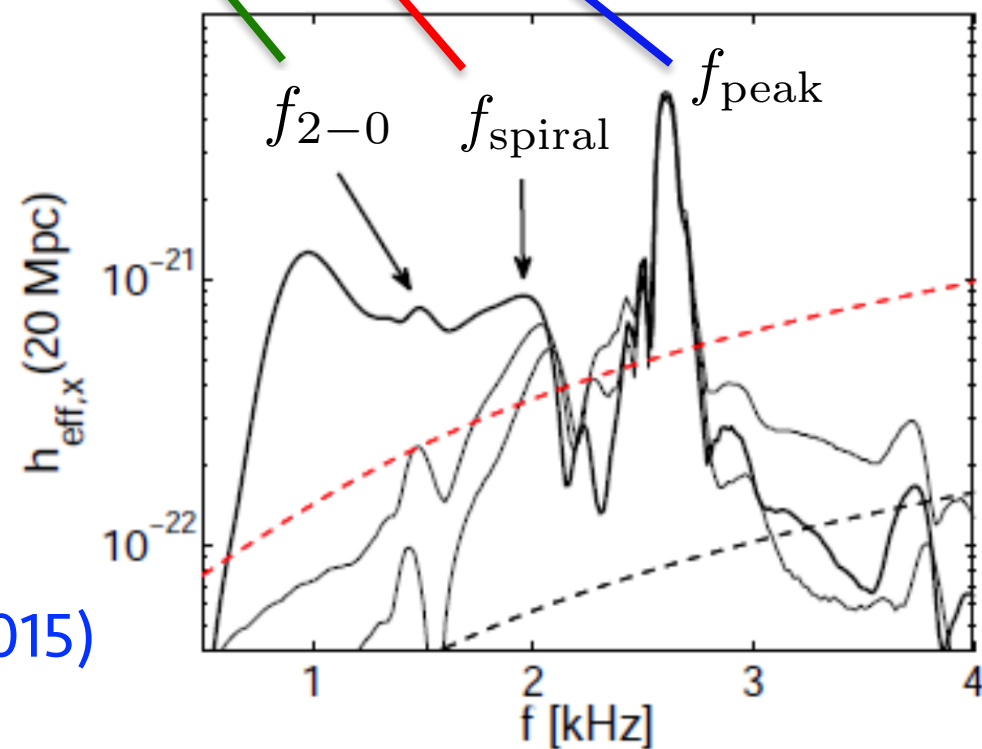


Three Detectable Post-Merger Frequencies

Vreinaris, NS &
Bauswein (2019)

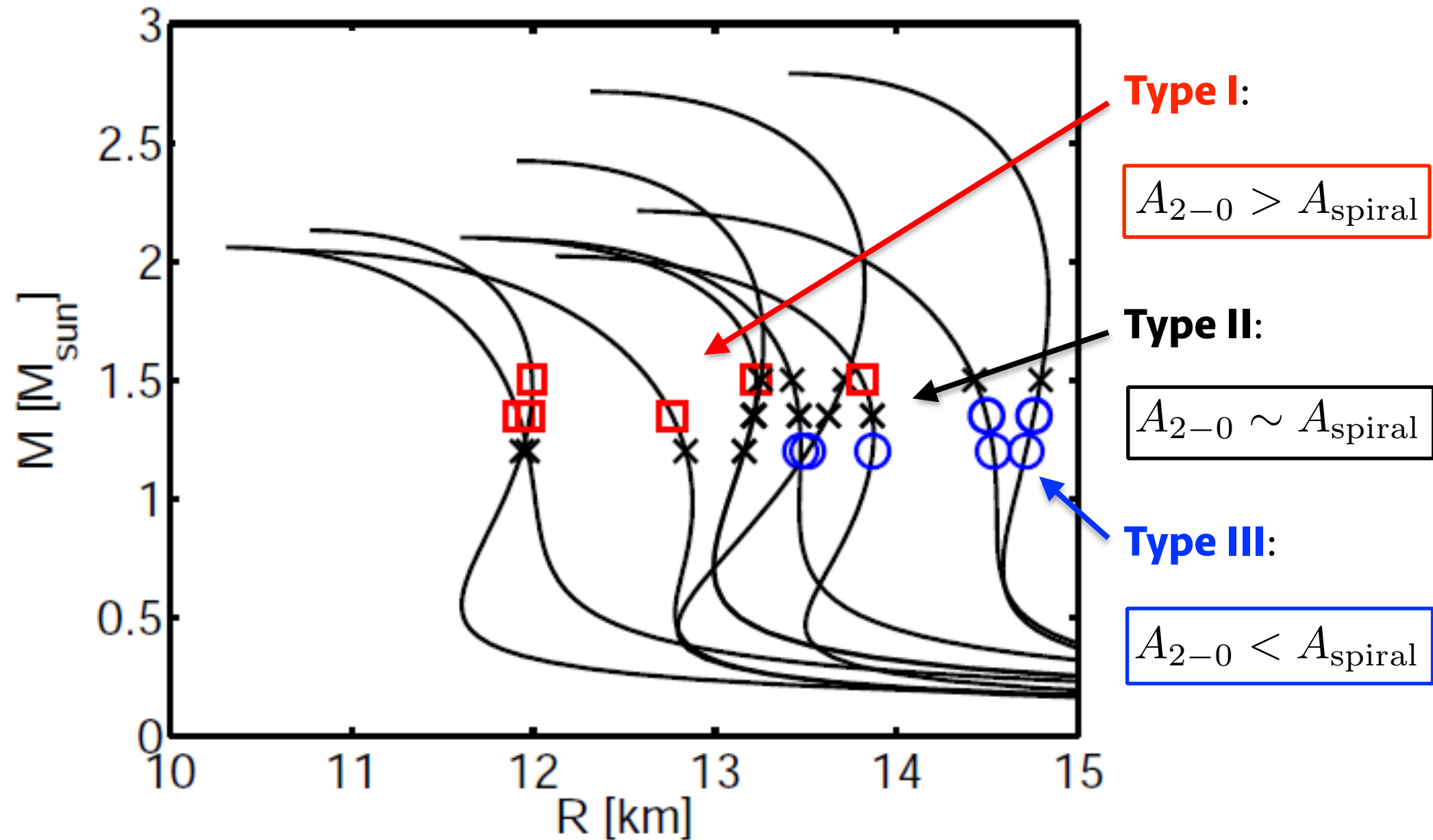


Bauswein & NS (2015)



Spectral Classification Scheme

Bauswein & NS (2015)

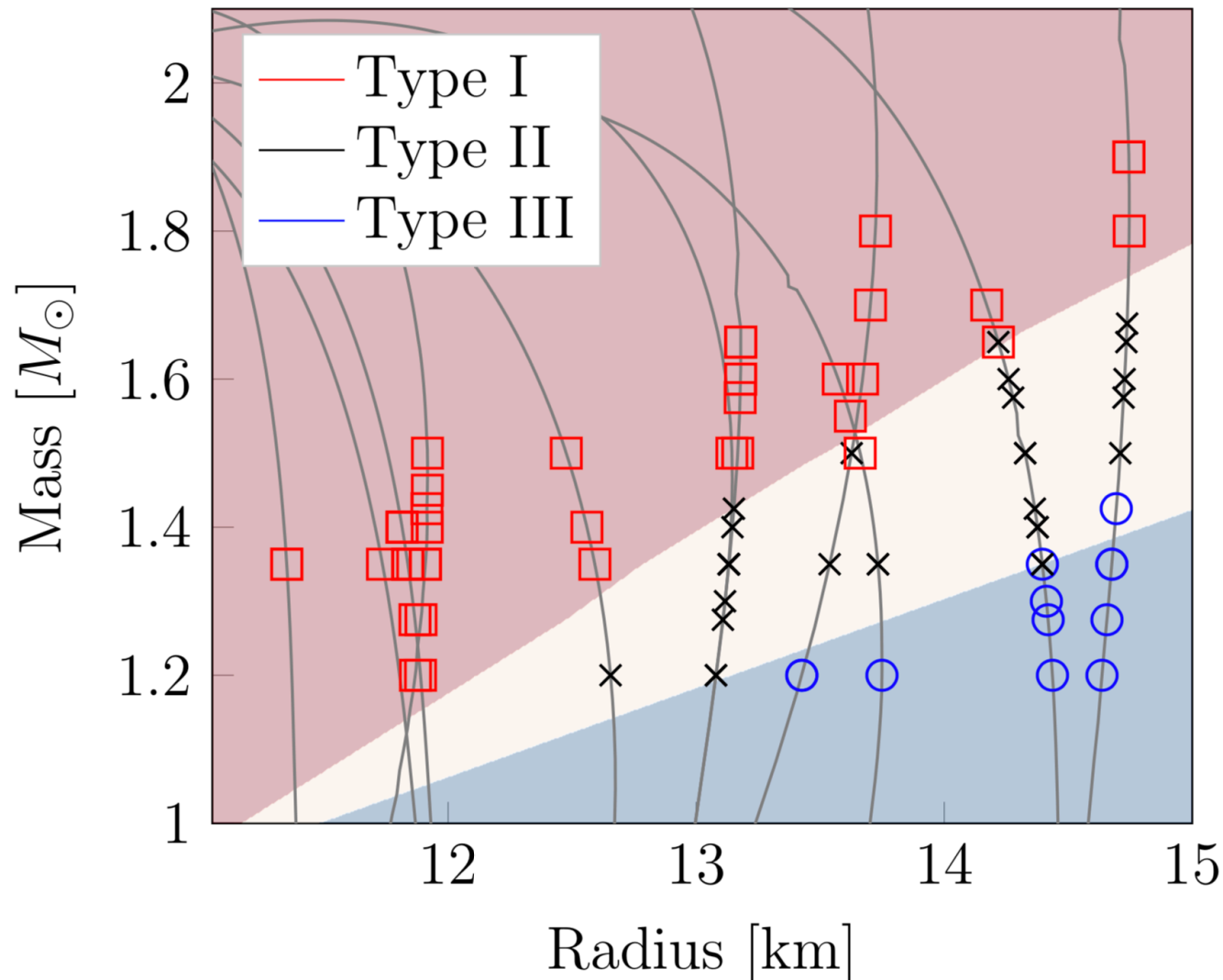


Spectral Classification using Machine Learning

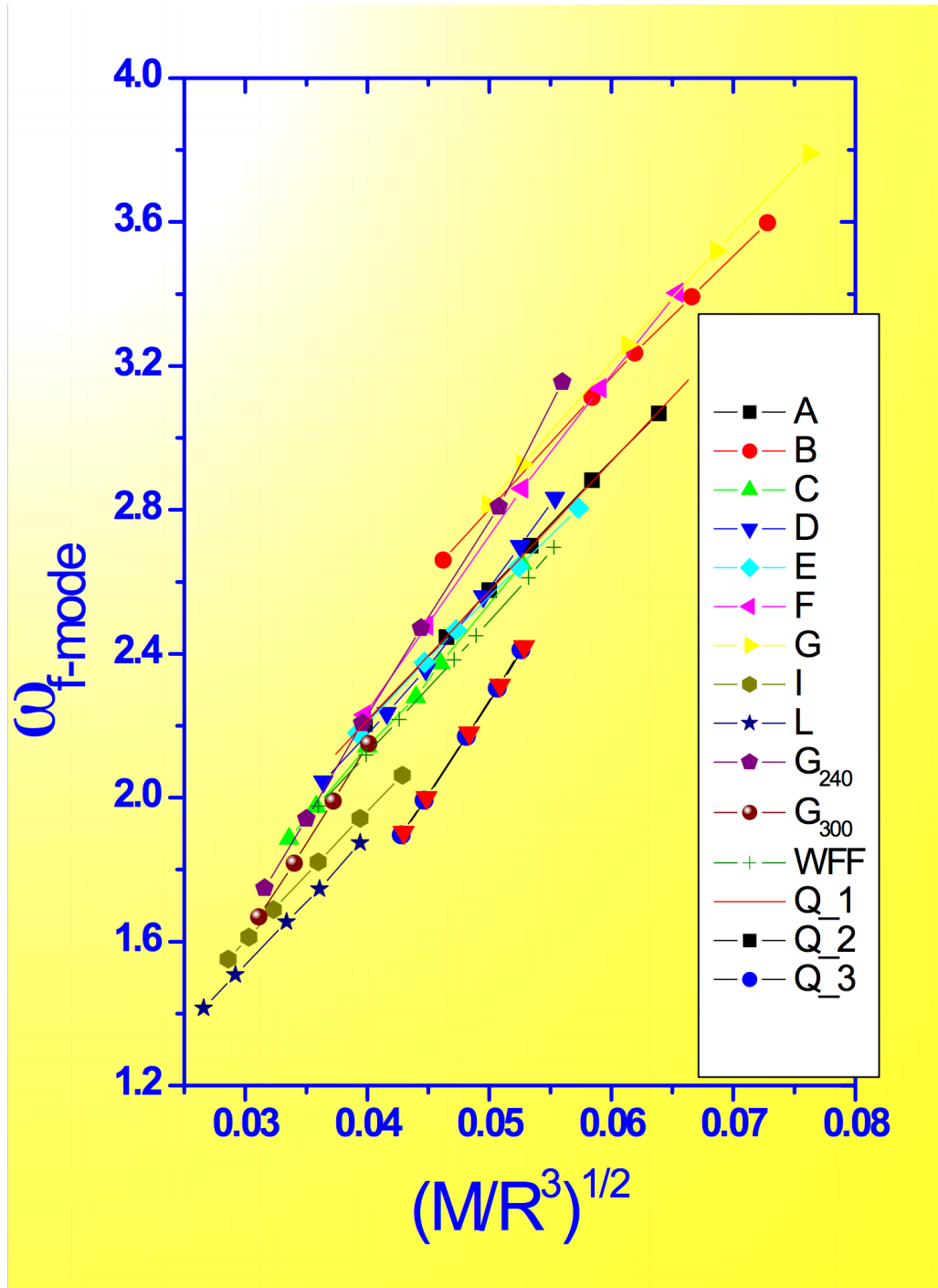
Vretinaris, NS & Bauswein (2019)

Clustering: *Affinity Propagation* (Scikit-Learn)

Classification via neural network: *Multi-layer Perceptron (MLP)* supervised learning



Quadrupole Frequencies for Nonrotating Stars



Empirical relations for GW asteroseismology:

$$\omega_f (\text{kHz}) \approx 0.78 + 1.637 \left(\frac{M_{1.4}}{R_{10}^3} \right)^{1/2}$$

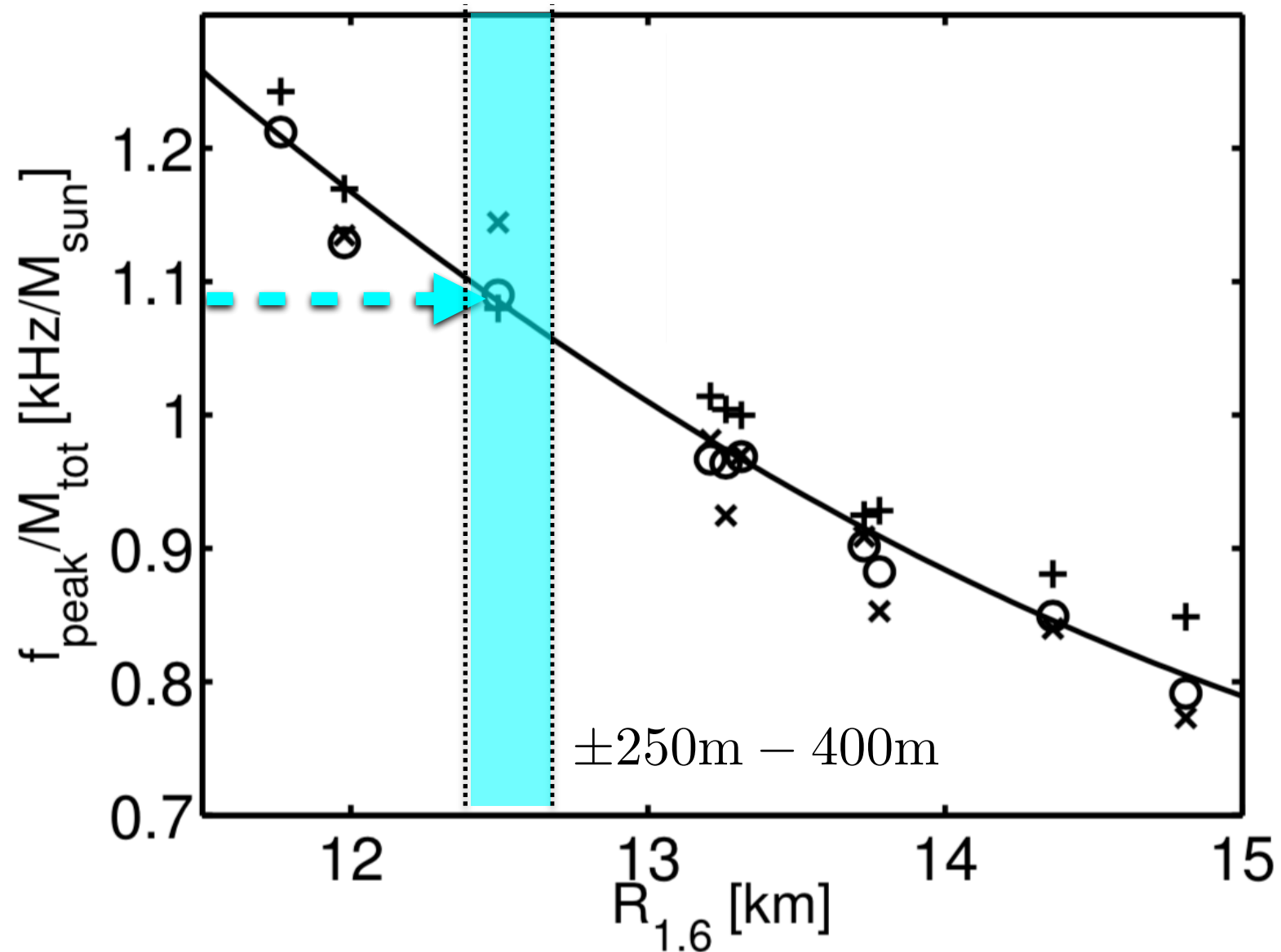
Andersson & Kokkotas (1998)

Recently extended to fast rotation by Krueger and Kokkotas (2019) !

Radius Determination from Post-Merger Signal

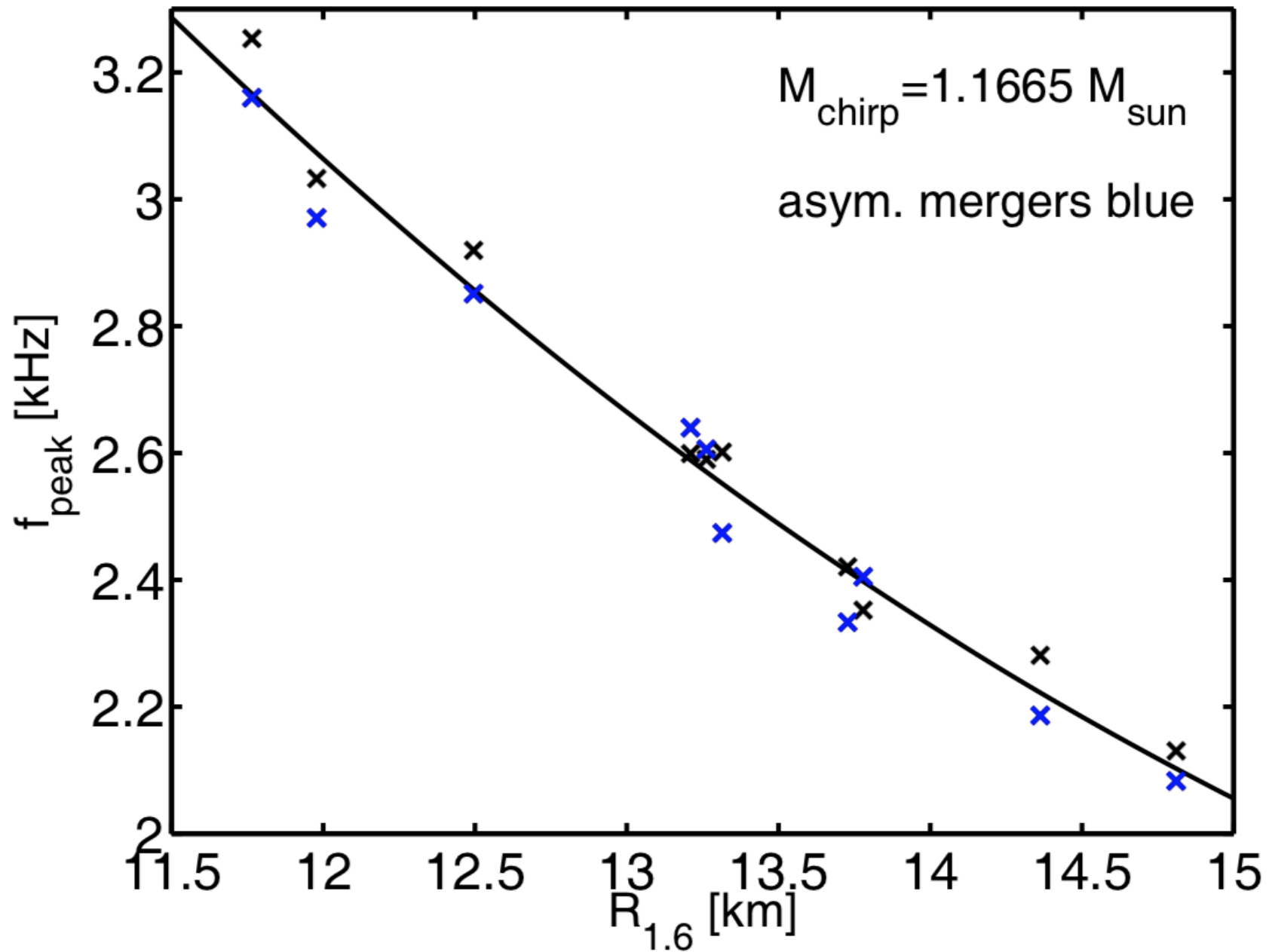
Bauswein et al. (2012)

Bauswein, NS, Janka (2016)



Asymmetric Mergers

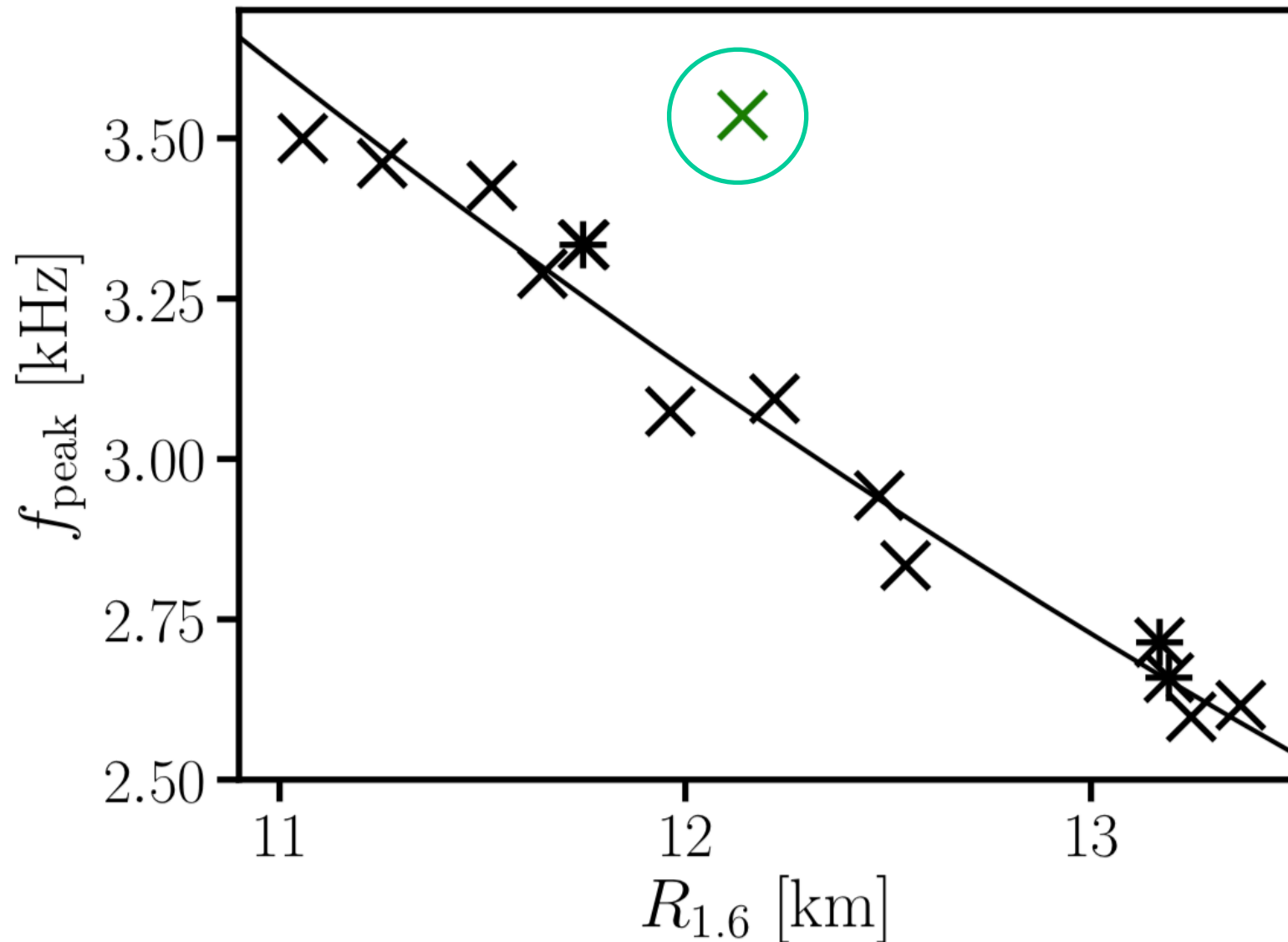
Bauswein, NS & Janka (2016)



EOS with Strong Phase Transition

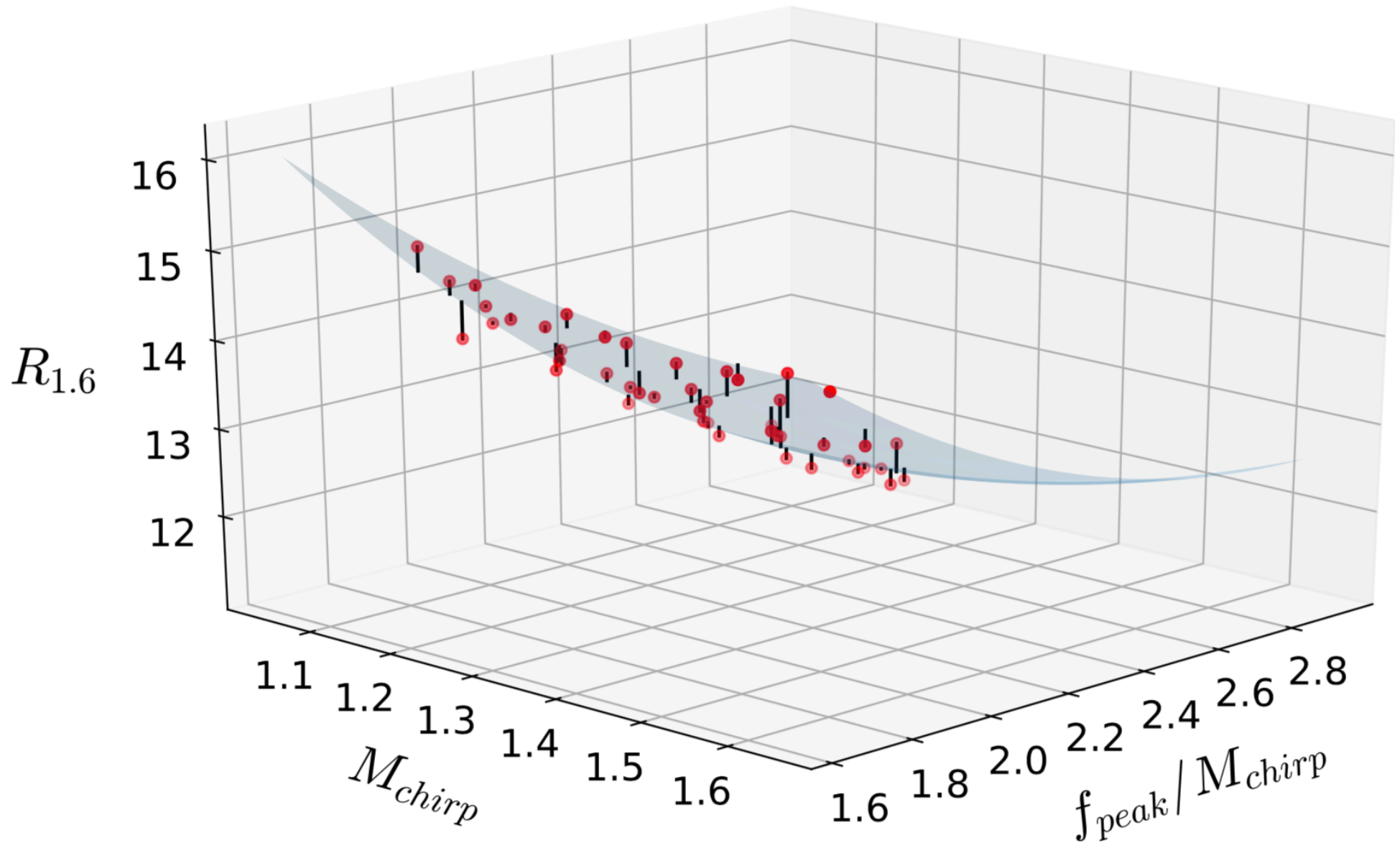
Bauswein et al. (2018)

A strong first-order phase transition will have a significant effect on radius extraction.



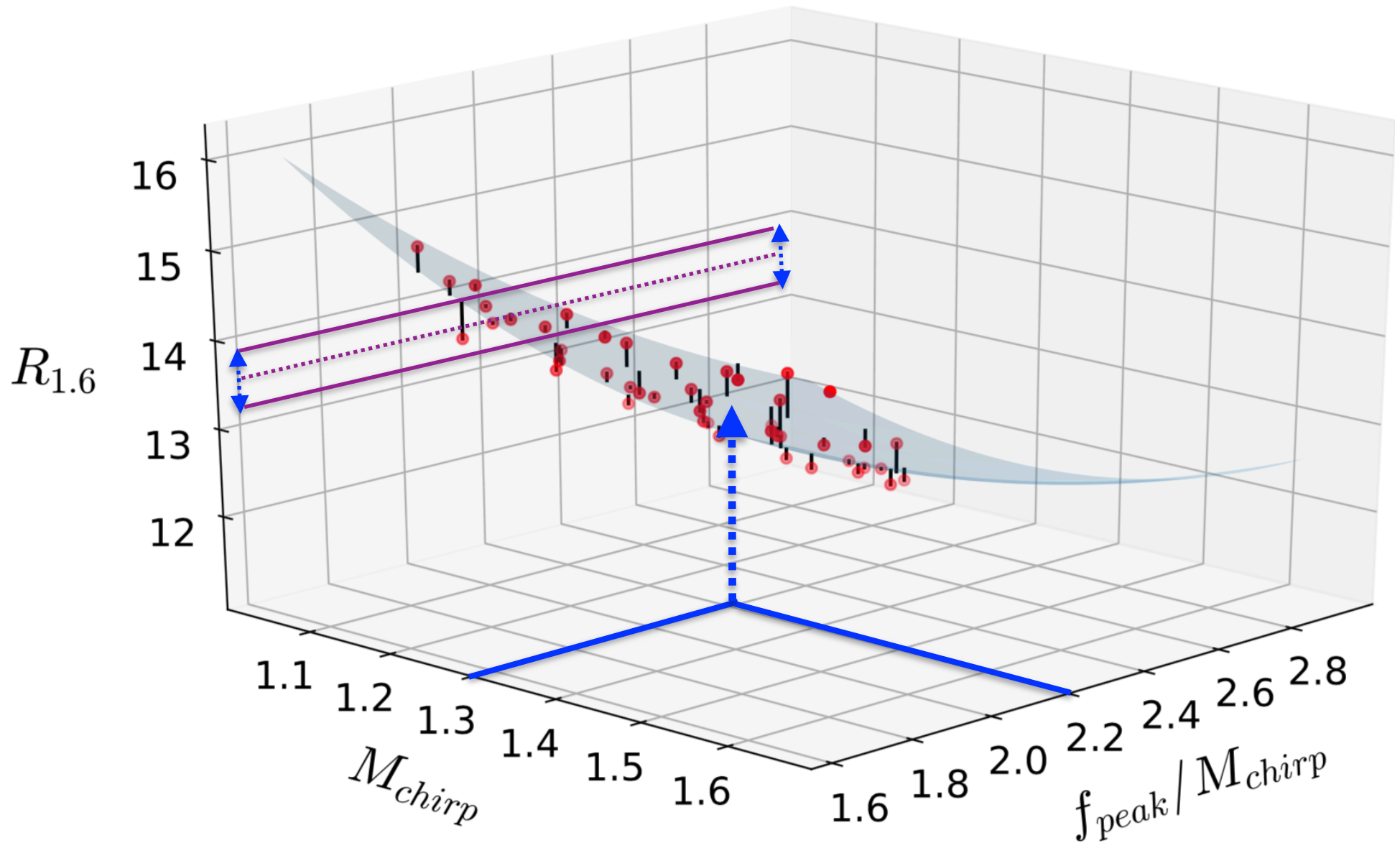
Optimized Empirical Relations

Vreinaris, NS & Bauswein (2019)



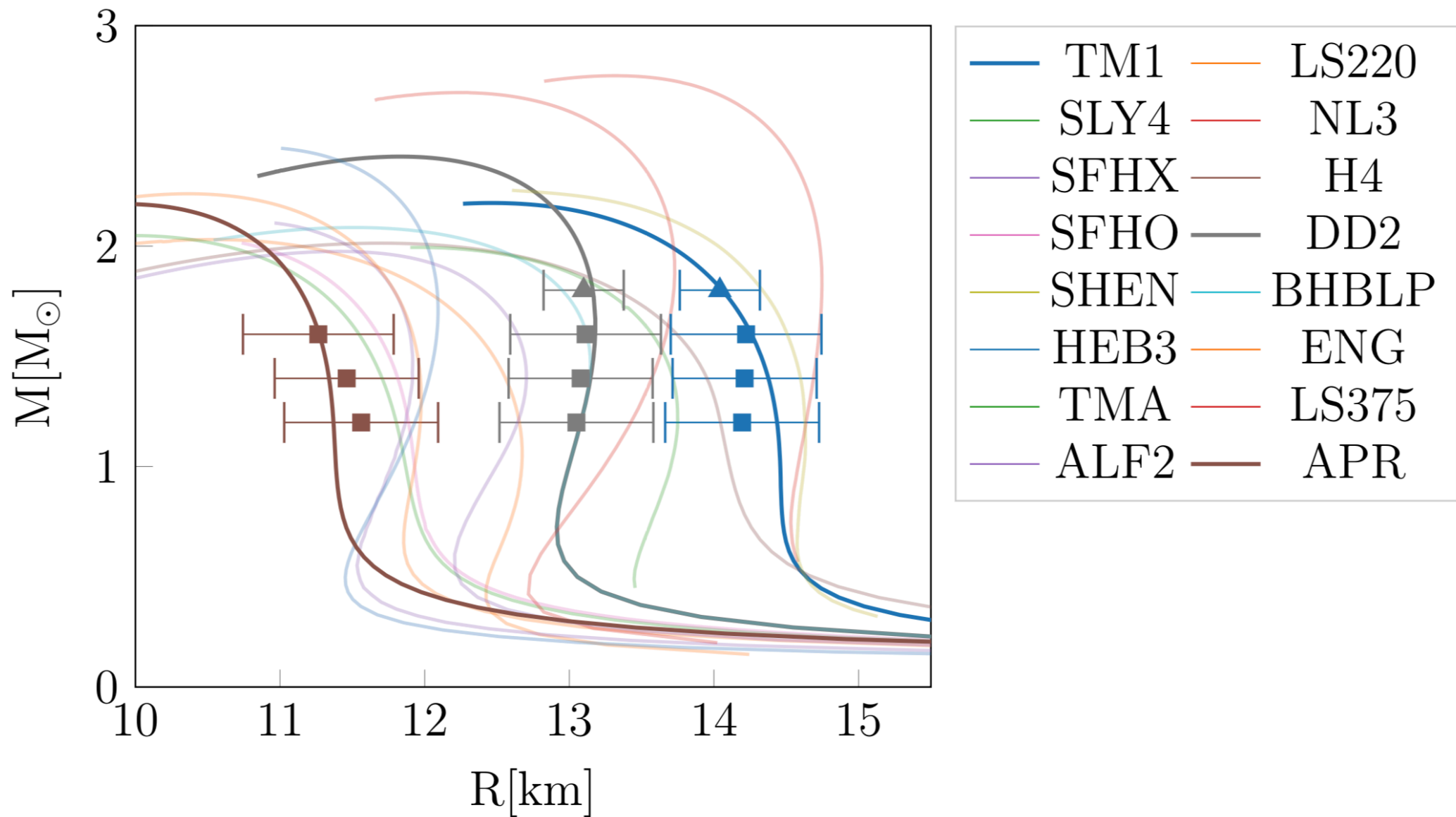
Optimized Empirical Relations

Vreinaris, NS & Bauswein (2019)



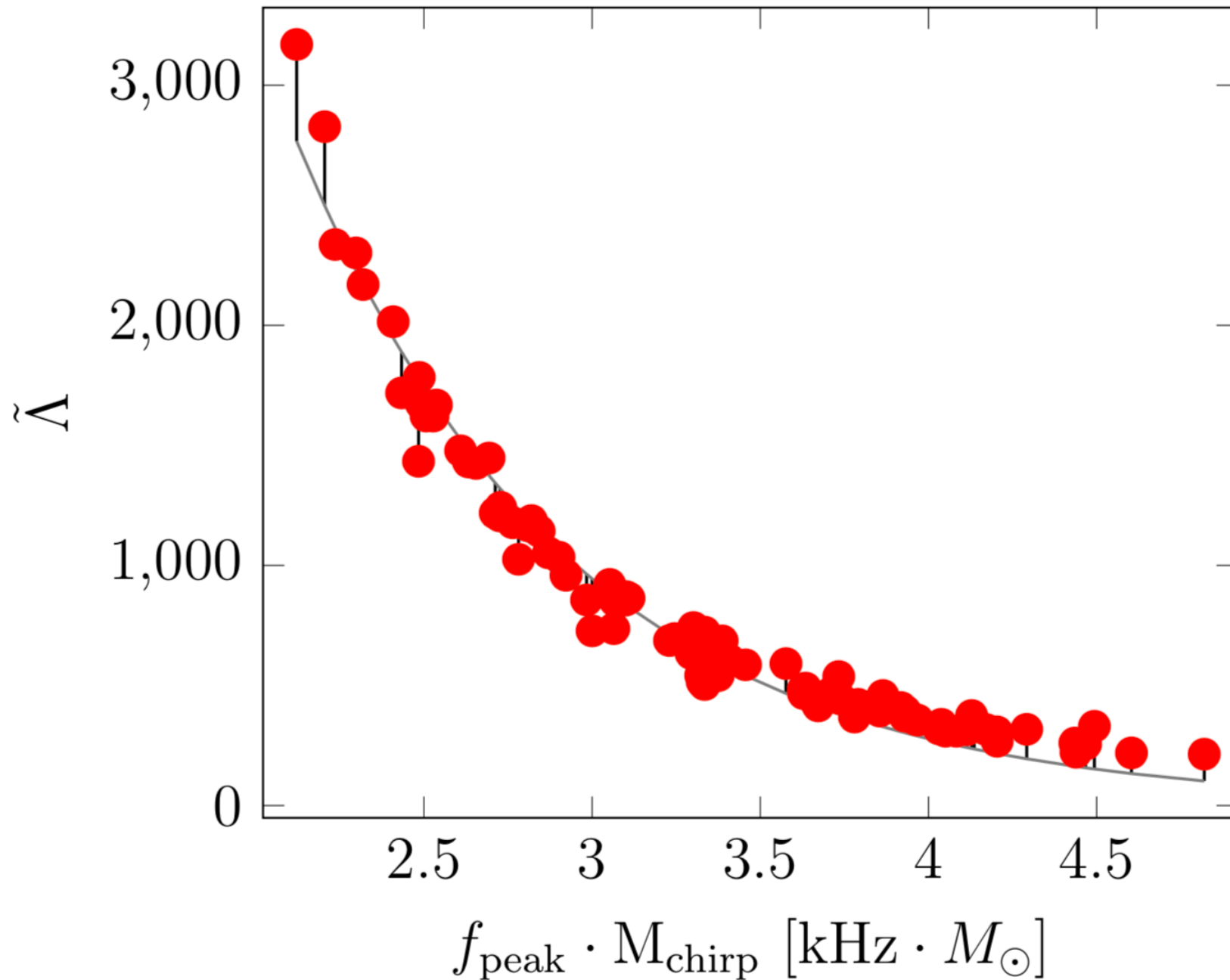
Reconstructing the M-R Diagram

Vretinaris, NS & Bauswein (2019)



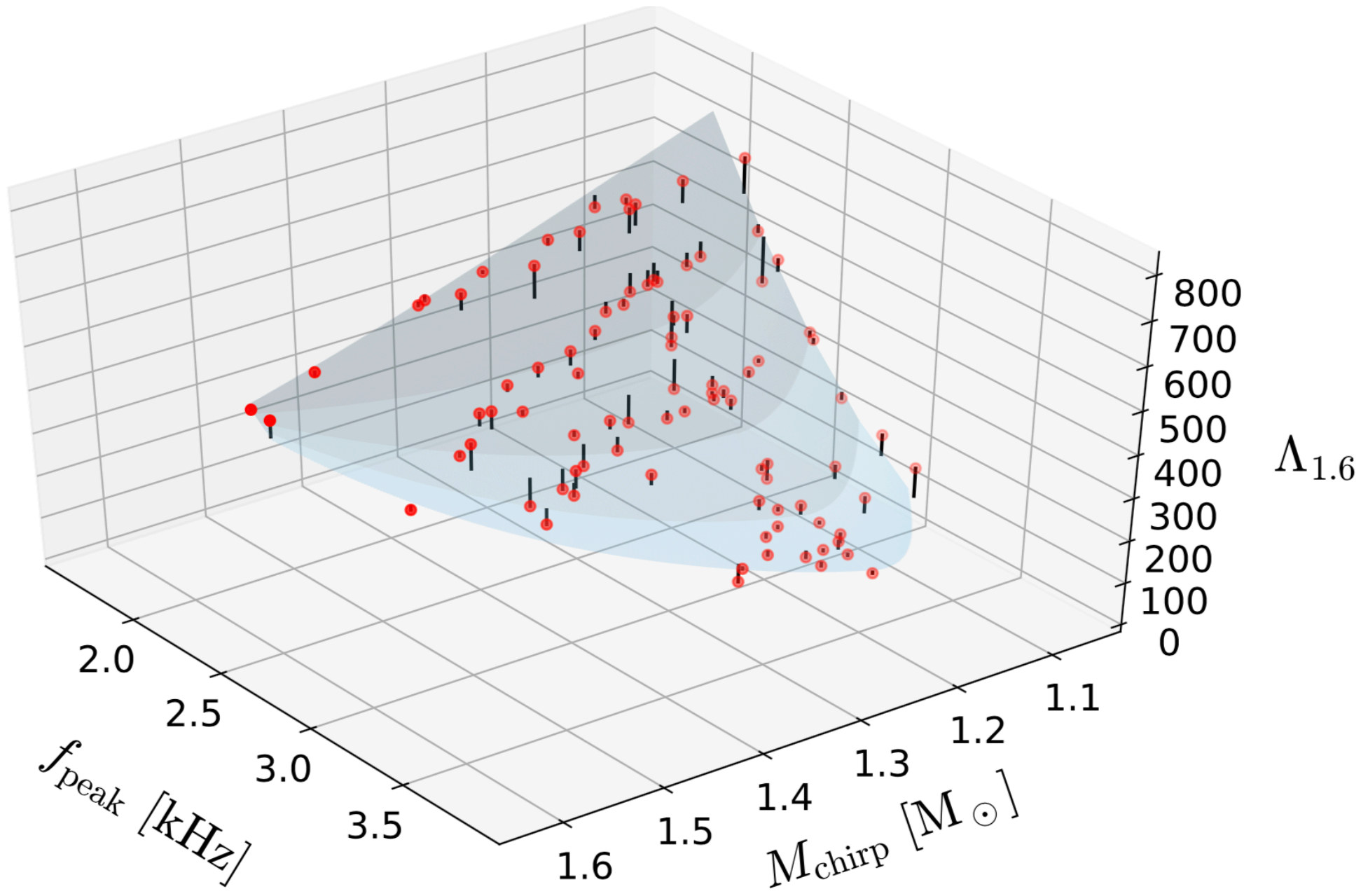
Λ through Post-merger Oscillations

Vretnaris, NS & Bauswein (2019)

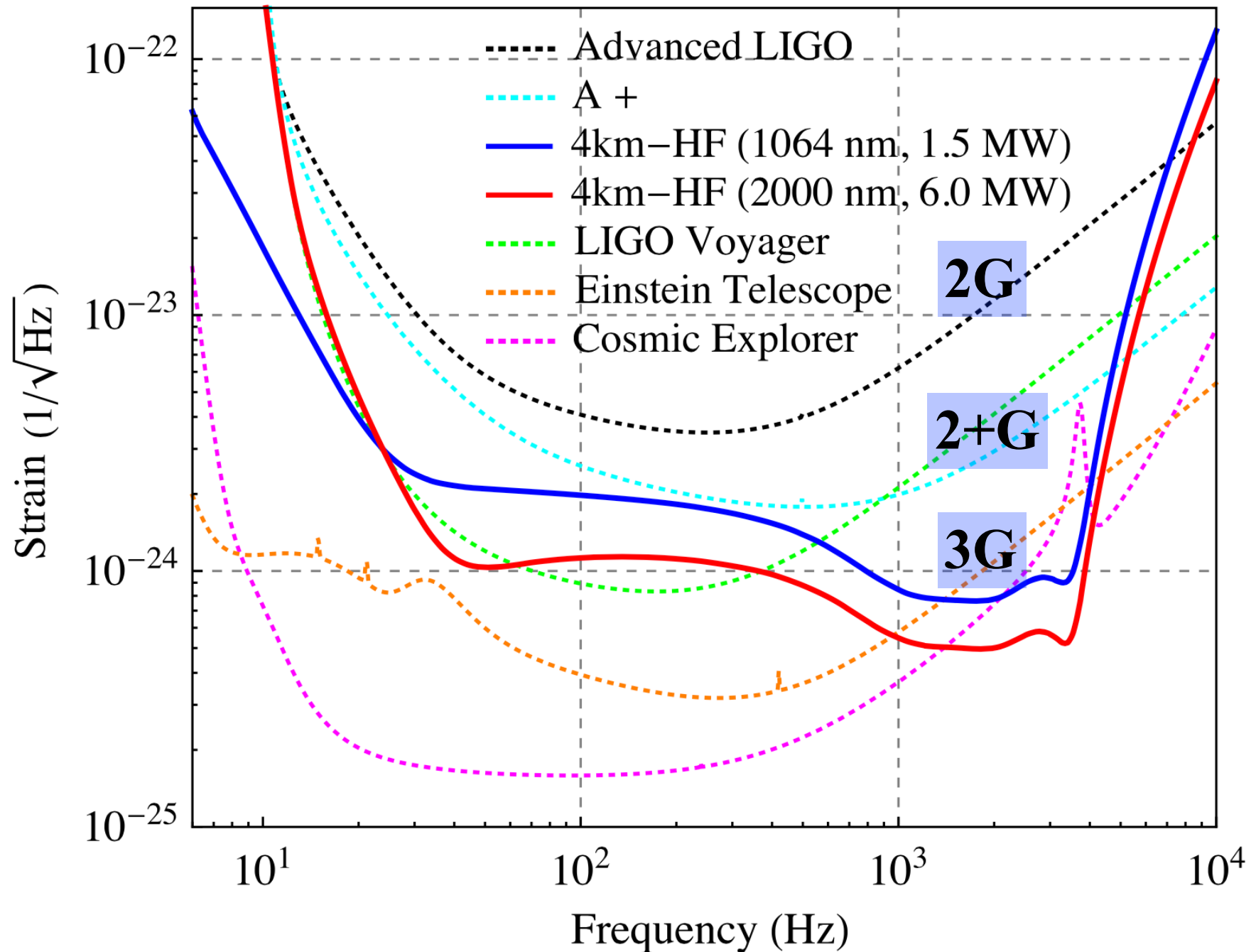


Λ through Post-merger Oscillations

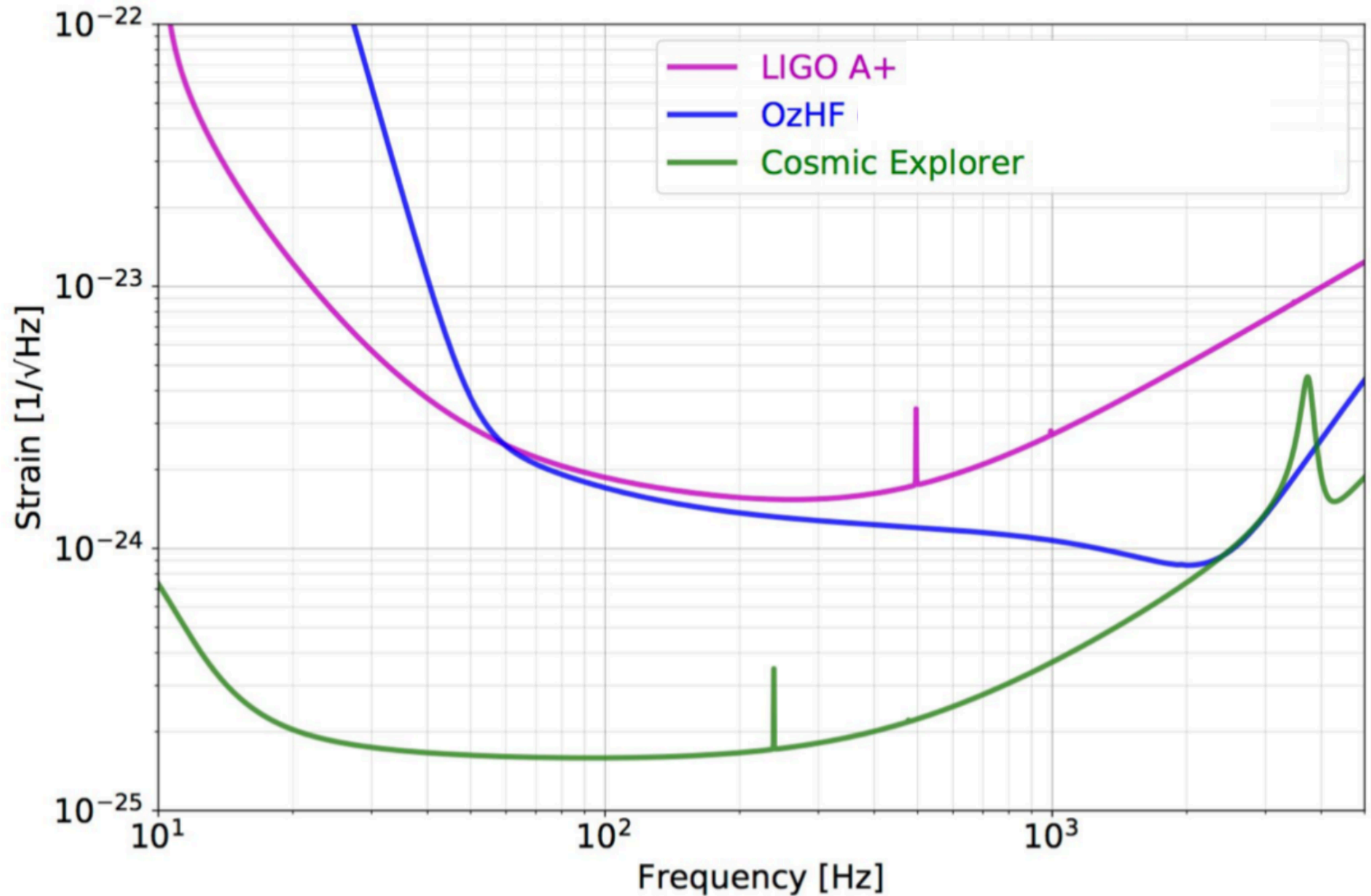
Vretnaris, NS & Bauswein (2019)



2G, 2+G & 3G Detectors



OzHF 2km High-Frequency GW Detector



Detectability of Post-Merger GWs

Clark, Bauswein, NS, Shoemaker (2016)

PCA template: extracts >80% of signal power.

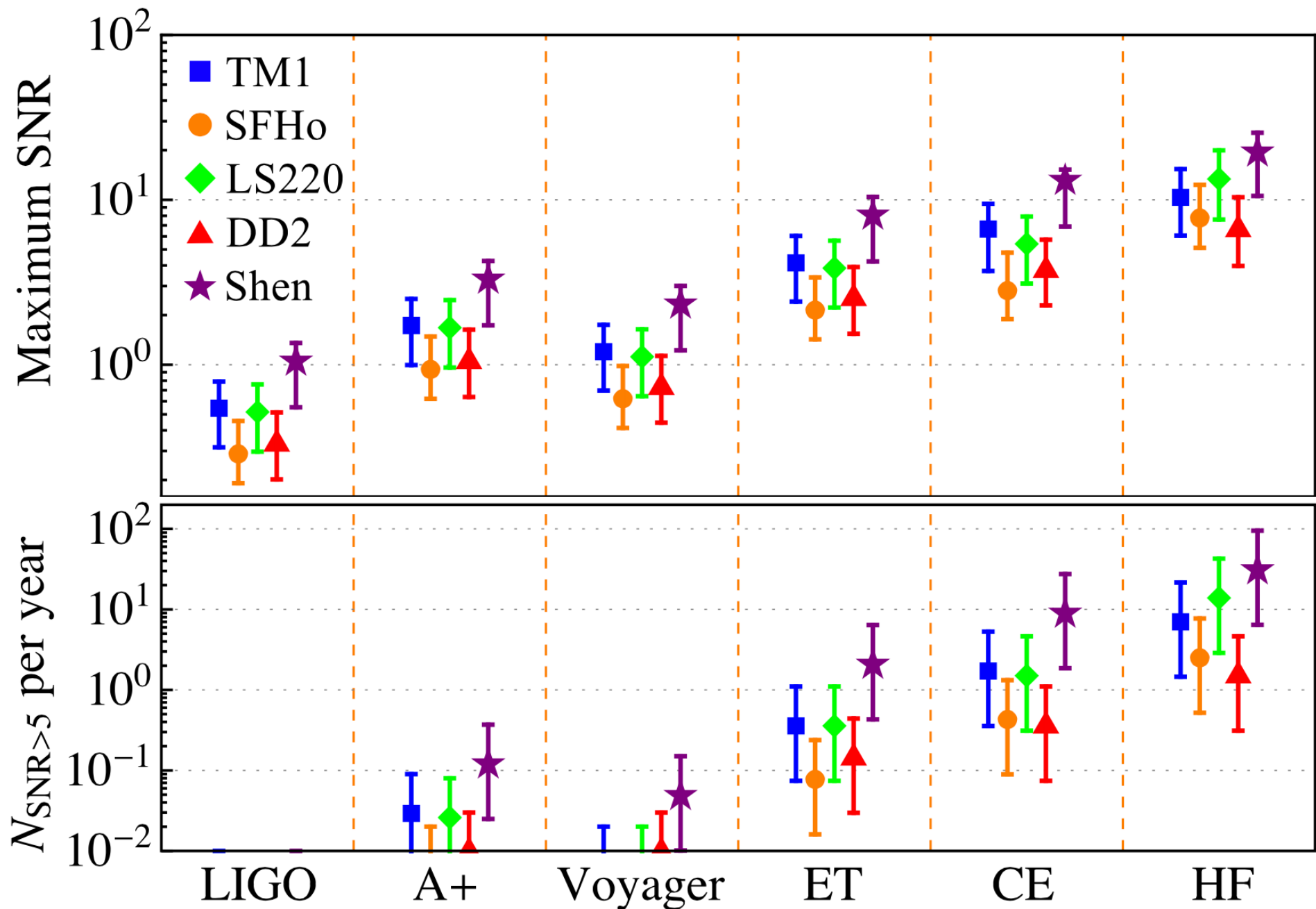
Single-detector detectability ($S/N > 5$, optimal orientation)

Instrument	SNR_{full}	SNR_{post}	D_{hor} [Mpc]	$\dot{\mathcal{N}}_{det}$ [year ⁻¹]	
aLIGO	2.99 ^{3.86} _{2.37}	1.48 ^{1.86} _{1.13}	29.89 ^{38.57} _{23.76}	0.01 ^{0.03} _{0.01}	2G
A+	7.89 ^{10.16} _{6.25}	4.19 ^{5.35} _{3.26}	78.89 ^{101.67} _{62.52}	0.13 ^{0.20} _{0.10}	2G+
LVoyager	14.06 ^{18.13} _{11.16}	7.28 ^{9.30} _{5.64}	140.56 ^{181.29} _{111.60}	0.41 ^{0.88} _{0.21}	
ET-D	26.65 ^{34.28} _{20.81}	12.16 ^{15.31} _{9.34}	266.52 ^{342.80} _{208.06}	2.81 ^{5.98} _{1.33}	3G
CE	41.50 ^{53.52} _{32.99}	20.52 ^{25.83} _{15.72}	414.62 ^{535.22} _{329.88}	10.59 ^{22.78} _{5.33}	

Improvements:

- Network of 5 detectors
- Stacking of several detections (e.g. Bose et al. 2018)
- Improved templates

3G Detectors

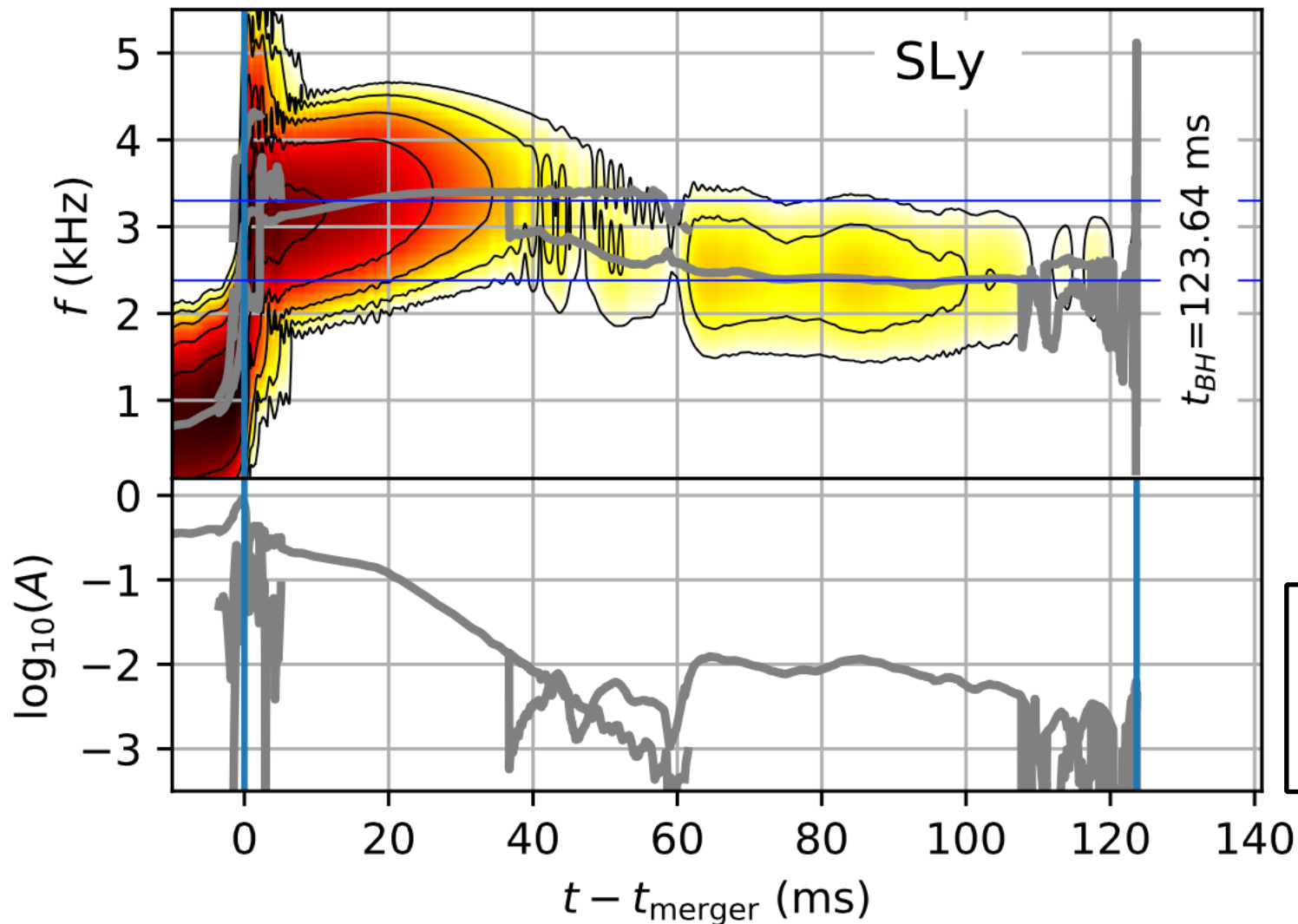


Inertial Modes!

(Physical Review Letters, 2018)

Convective Excitation of Inertial Modes in Binary Neutron Star Mergers

Roberto De Pietri,^{1,2} Alessandra Feo,^{3,2} José A. Font,^{4,5} Frank Löffler,^{6,7} Francesco Maione,^{1,2}
Michele Pasquali,^{1,2} and Nikolaos Stergioulas⁸



30 million
core hours
(PRACE)

Convective Instability

The local convective instability depends on the sign of the Schwarzschild discriminant

$$A_{\alpha} = \frac{1}{\varepsilon + p} \nabla_{\alpha} \varepsilon - \frac{1}{\Gamma_1 p} \nabla_{\alpha} p$$

where

$$\Gamma_1 := (\varepsilon + p)/p(dp/d\varepsilon)_s = (d \ln p / d \ln \rho)_s$$

is the adiabatic index.

$A_{\alpha} < 0$ convective stability

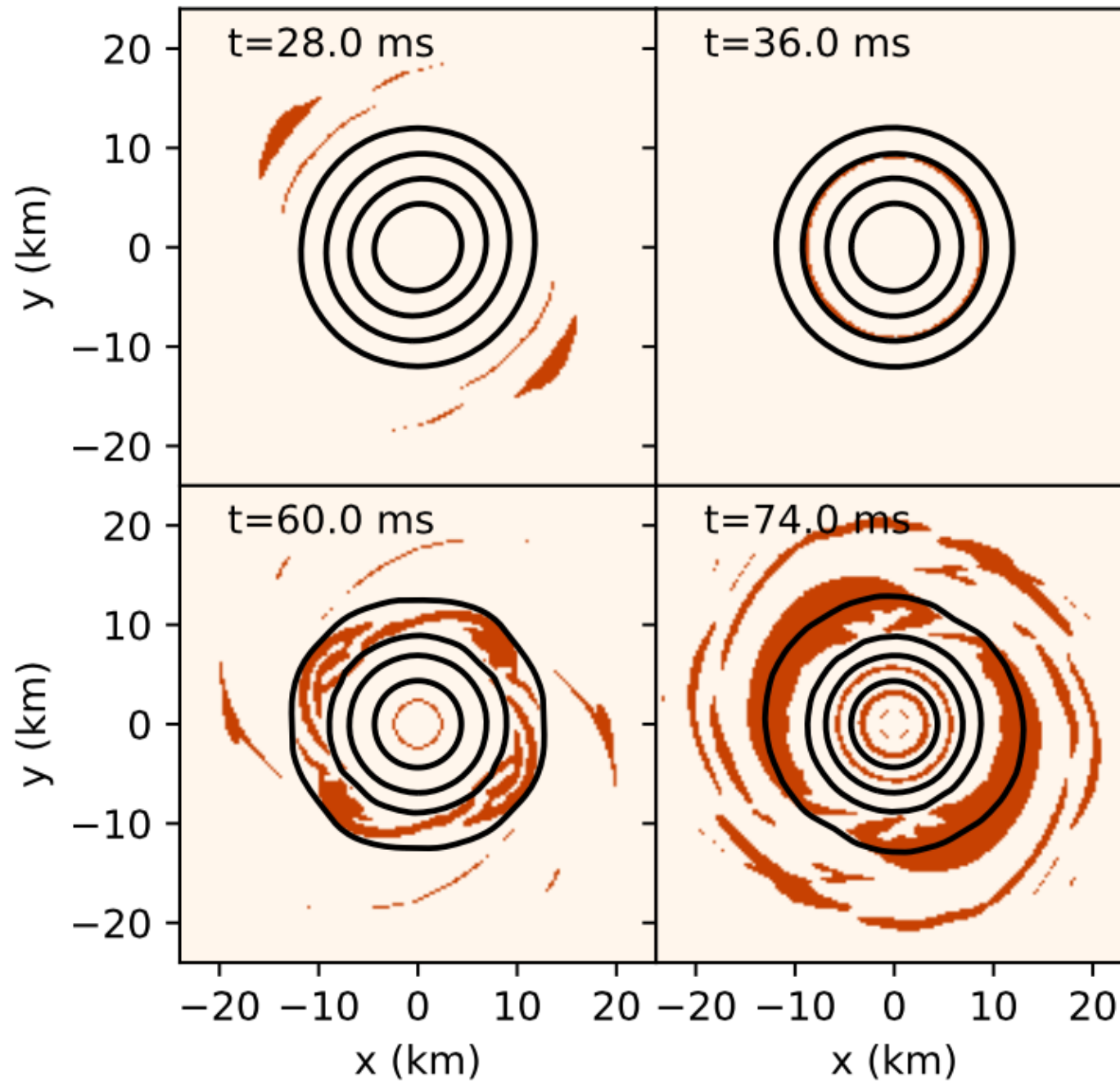
$A_{\alpha} > 0$ convective instability

For a piecewise-polytropic EOS with a thermal component, we find analytically:

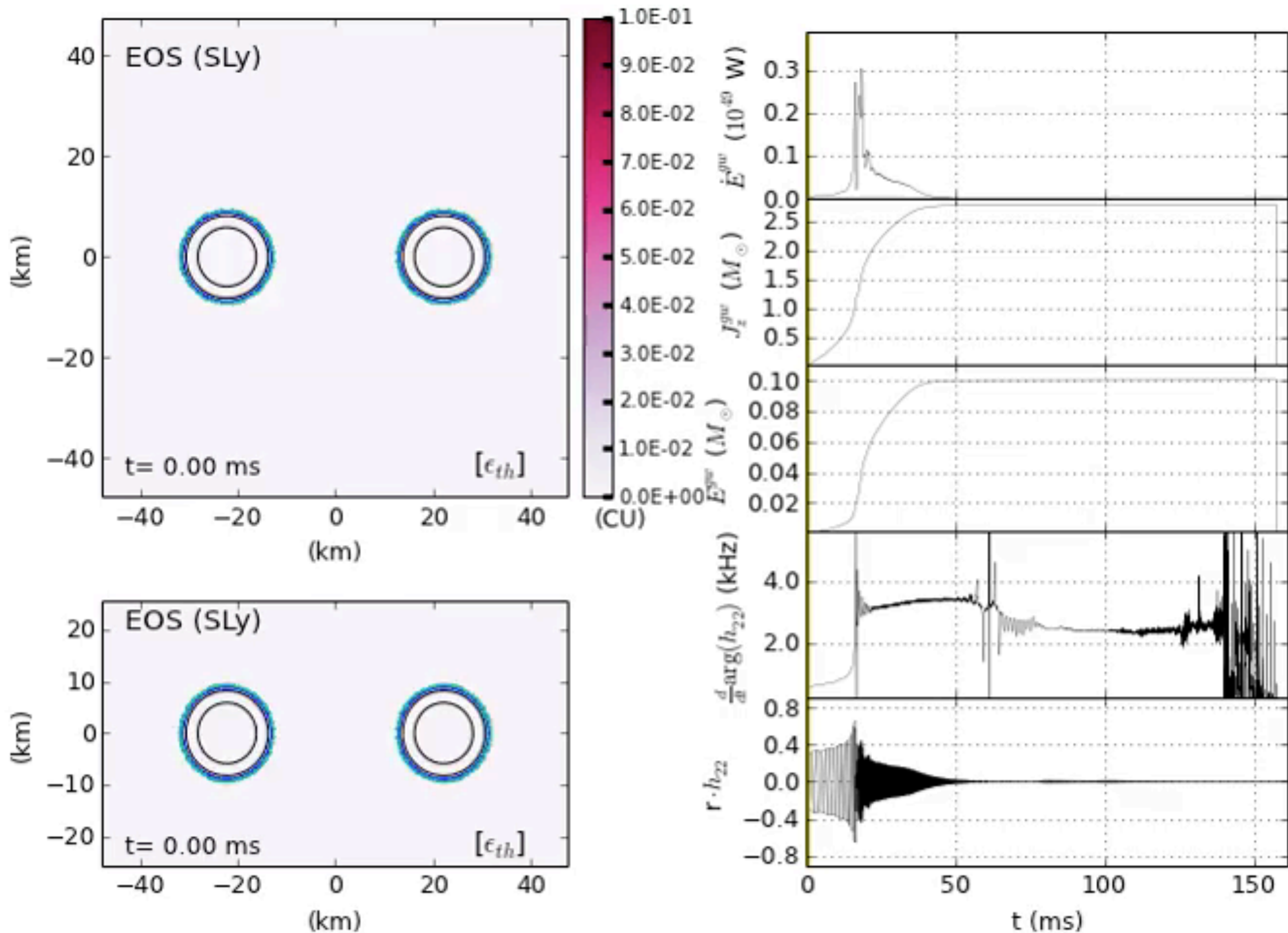
$$\Gamma_1 = \Gamma_{\text{th}} + (\Gamma_i - \Gamma_{\text{th}}) \frac{K_i \rho^{\Gamma_i}}{p}$$

Convective Instability

The sign of A_r in the equatorial plane:

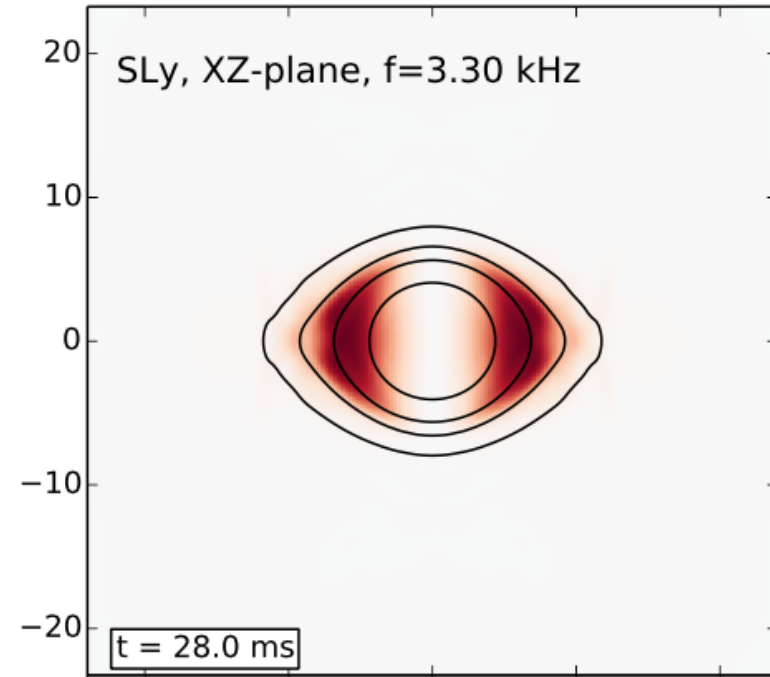
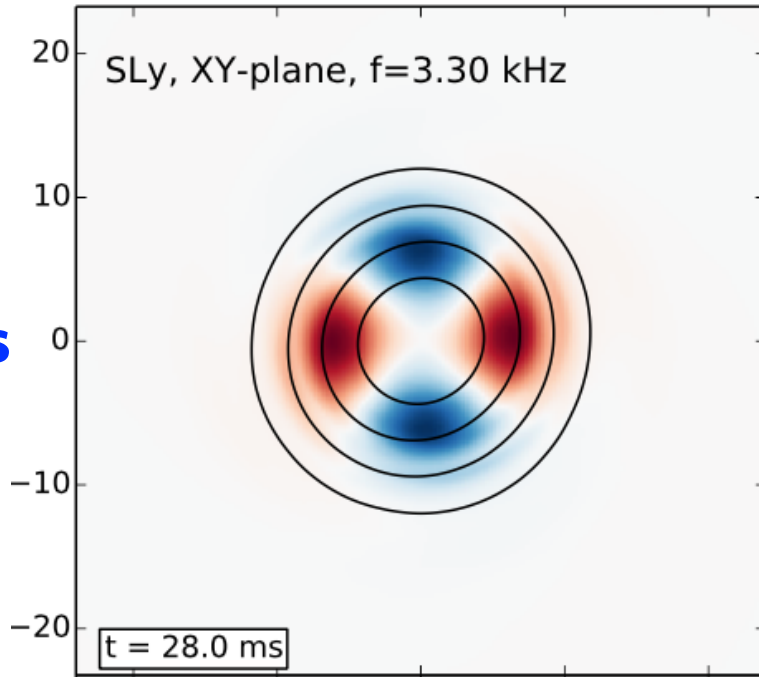


Thermal Evolution



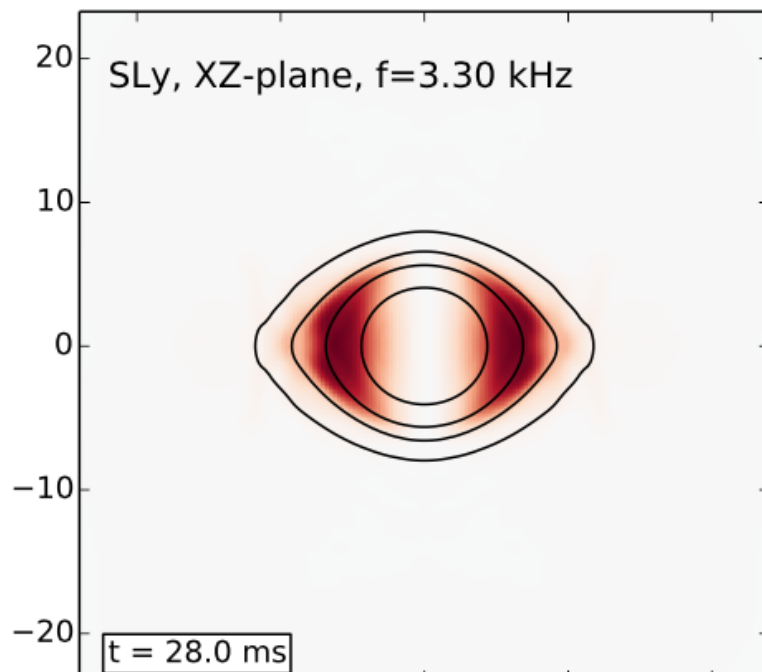
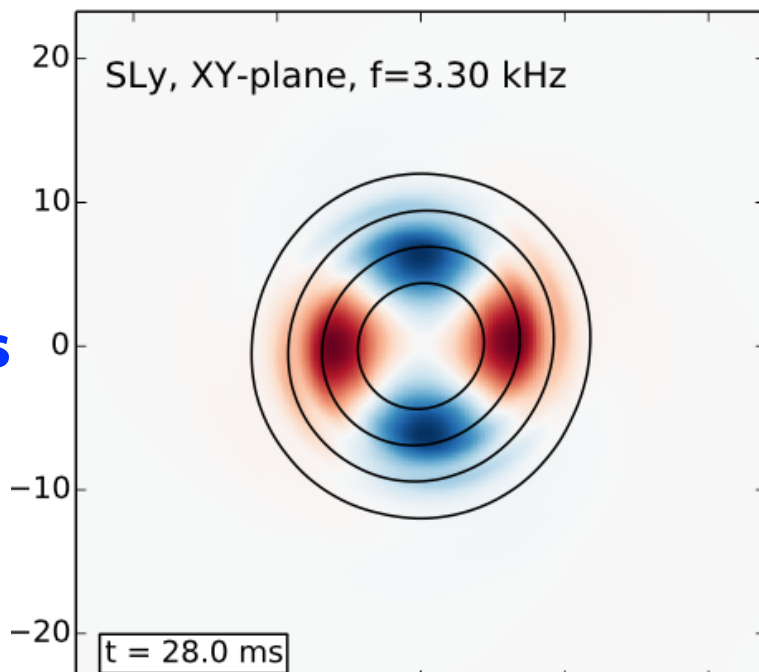
Oscillations

f-modes

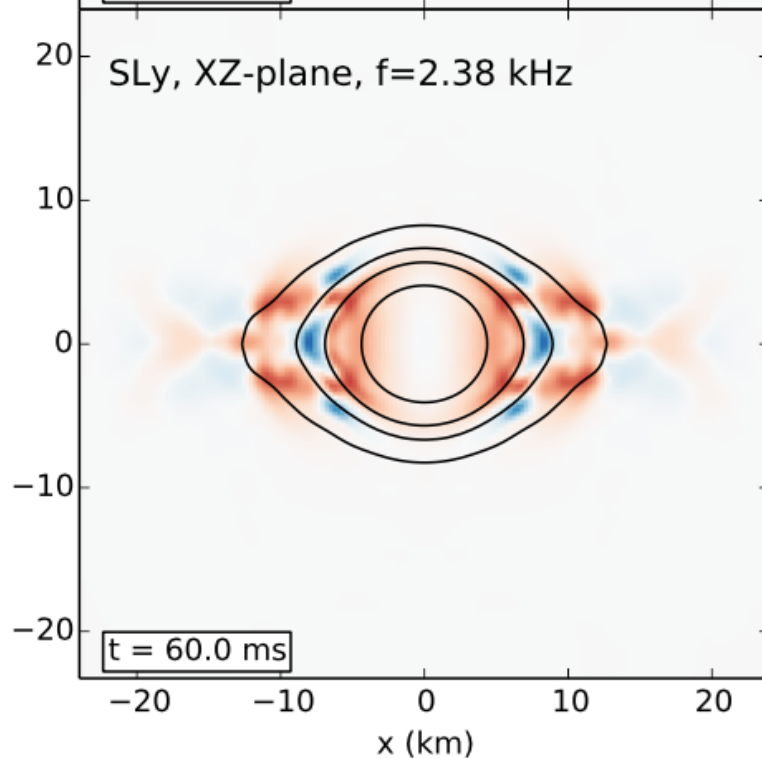
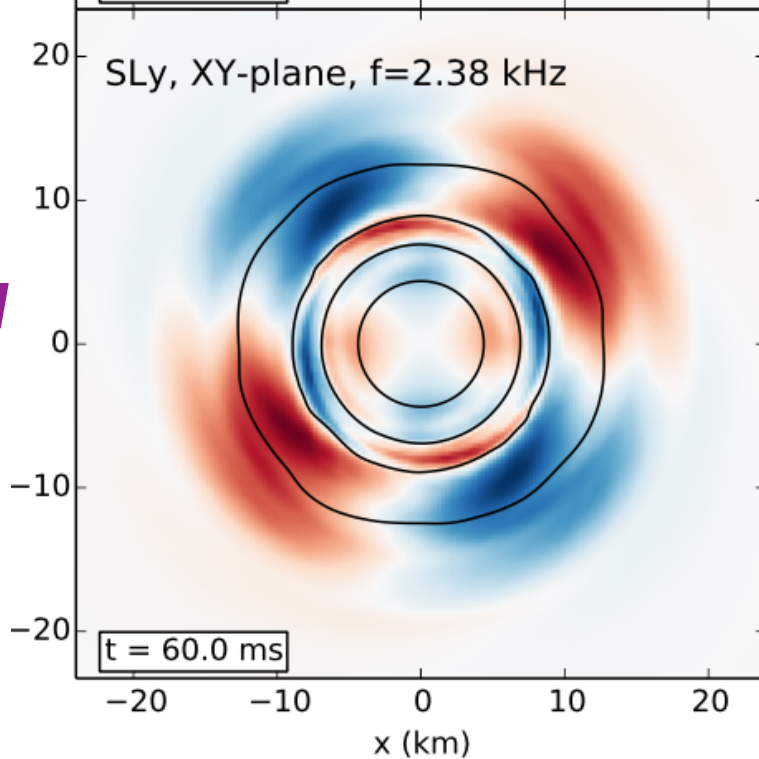


Oscillations

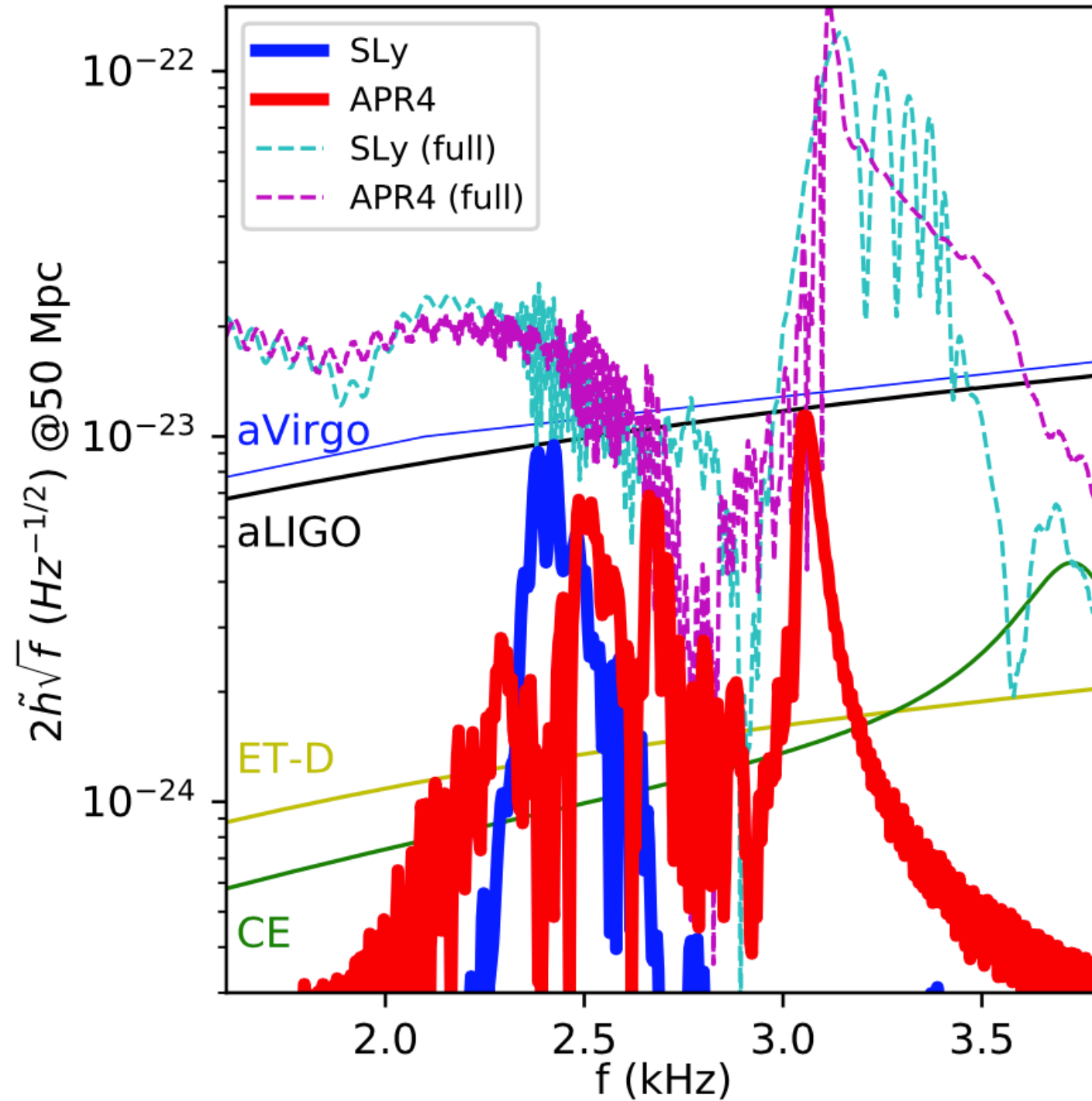
f-modes



inertial modes



Gravitational Wave Spectrum



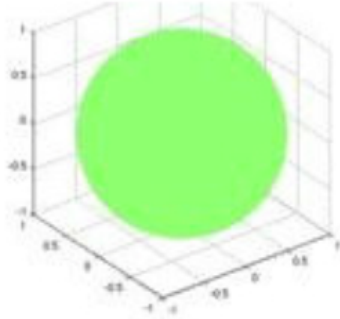
Summary

- Based on a set of minimal assumptions, we set a *strict* lower limit on neutron star radii of **10.7km** at $1.6M_{\text{sun}}$.
- Gravitational-wave asteroseismology can constrain the neutron star radius to within 0.4km with future observations.
- Principal Component Analysis (PCA) sufficient to reach >80% of optimal signal.
- We discovered *convective instabilities* and *inertial modes* that can probe the thermal part of the EOS.
- Once the EOS is well constrained, one can investigate departures from GR.

Supplementary material

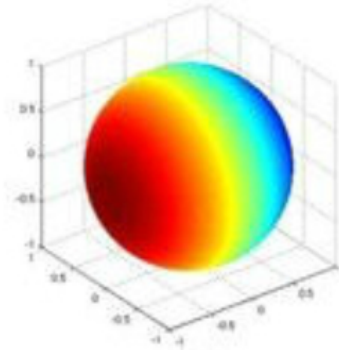
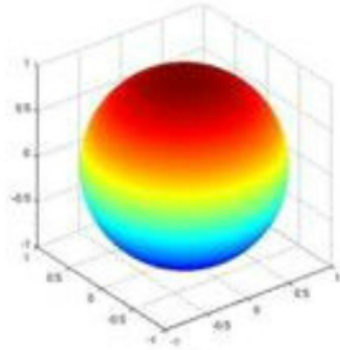
Nonradial Oscillations

$$\ell = 0$$



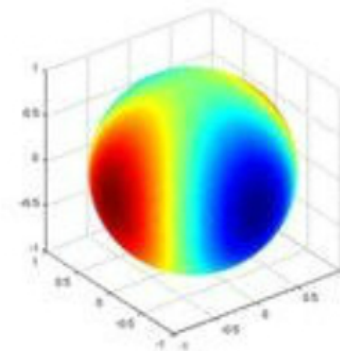
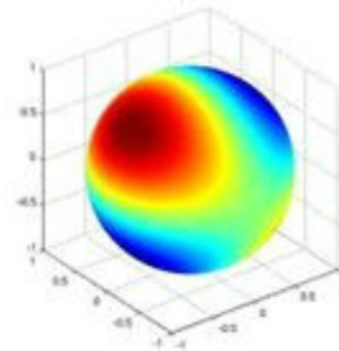
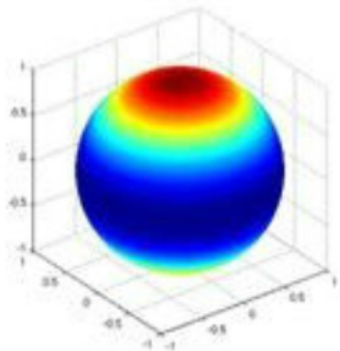
$$\cos(m\phi) P_{\ell}^m(\cos \theta)$$

$$\ell = 1$$



$l = m$ (sectoral)
 $l \neq m$ (tesseral)

$$\ell = 2$$



$$m = 0$$

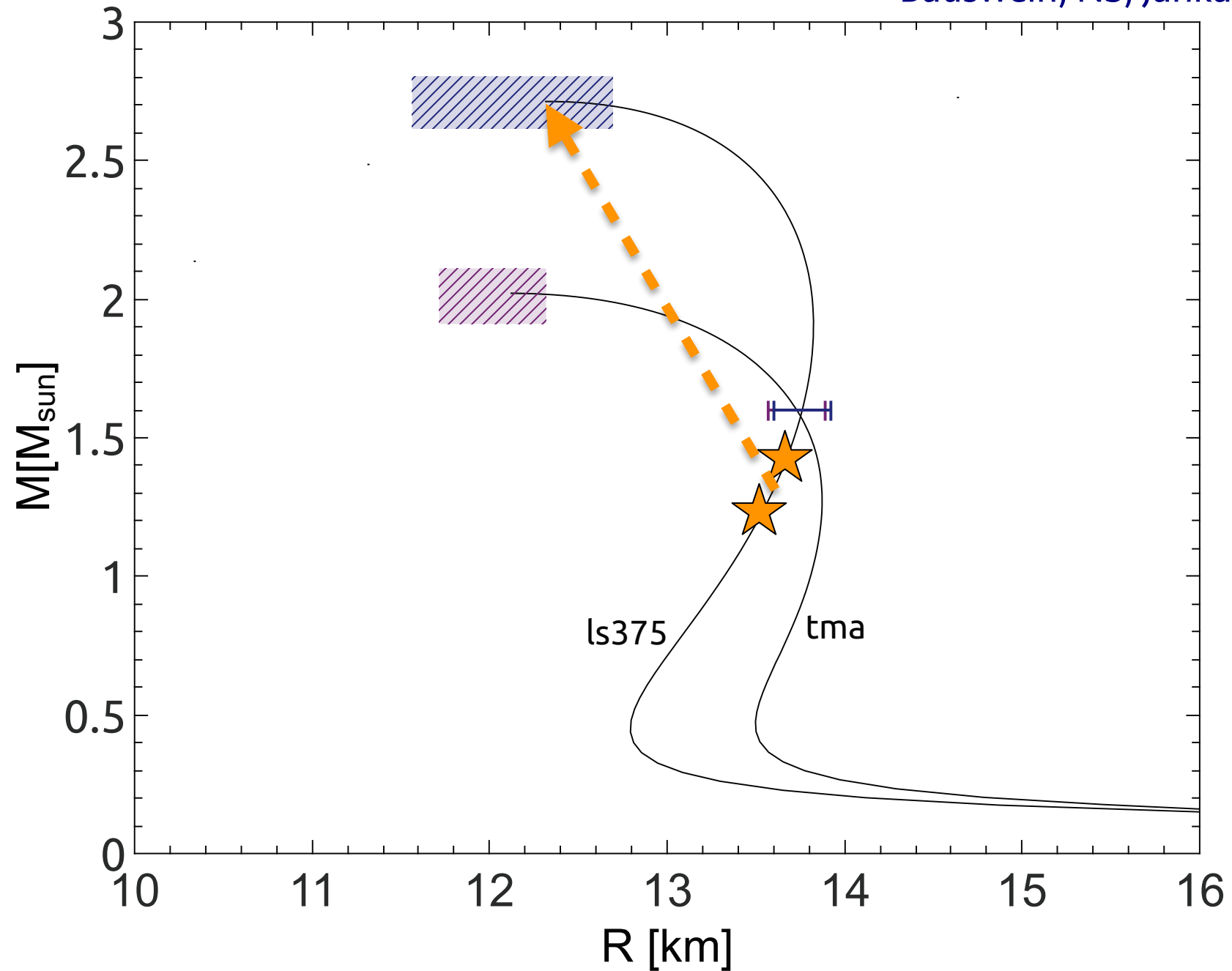
(zonal)

$$m = 1$$

$$m = 2$$

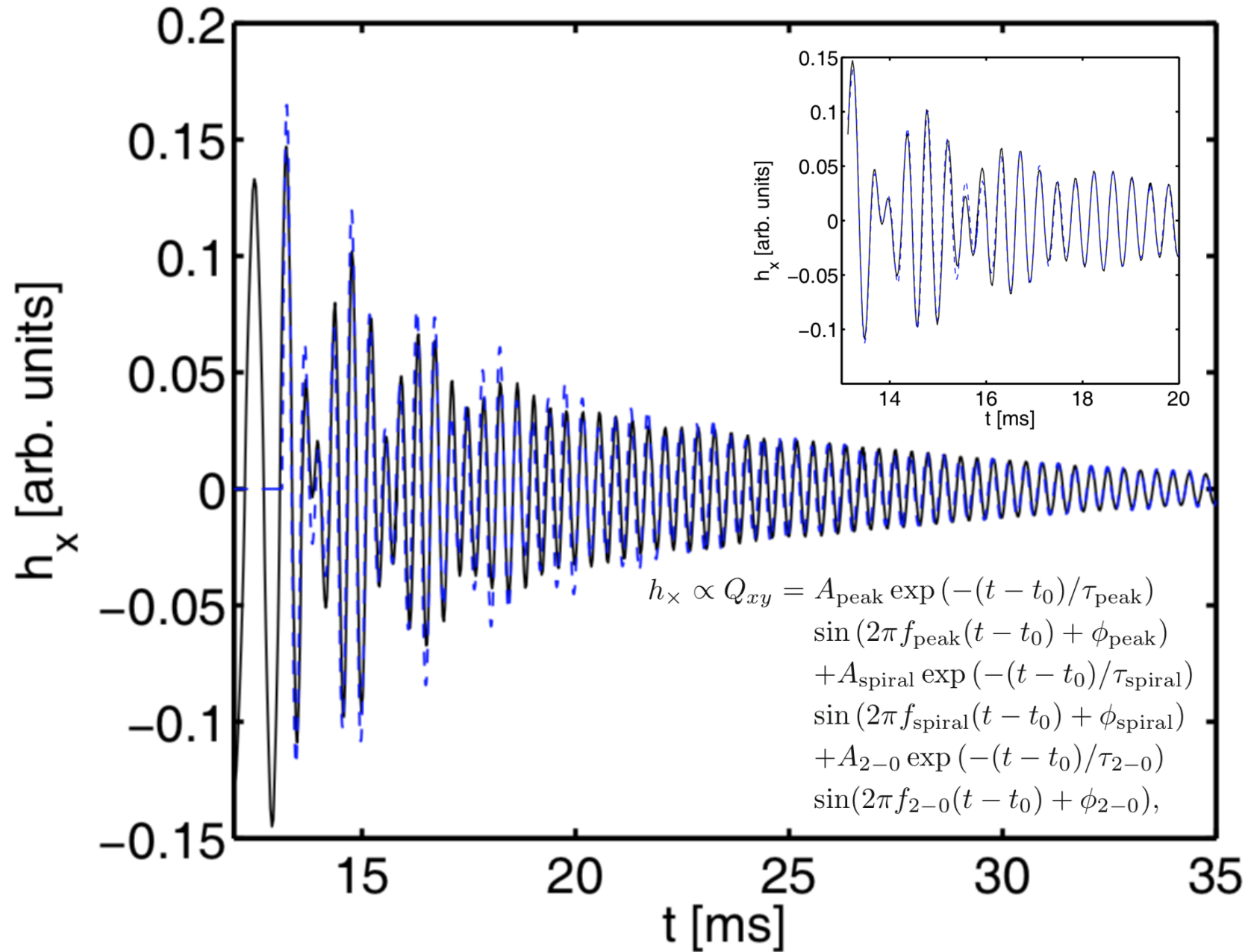
Breaking the EOS Degeneracy

Bauswein, NS, Janka (2014)

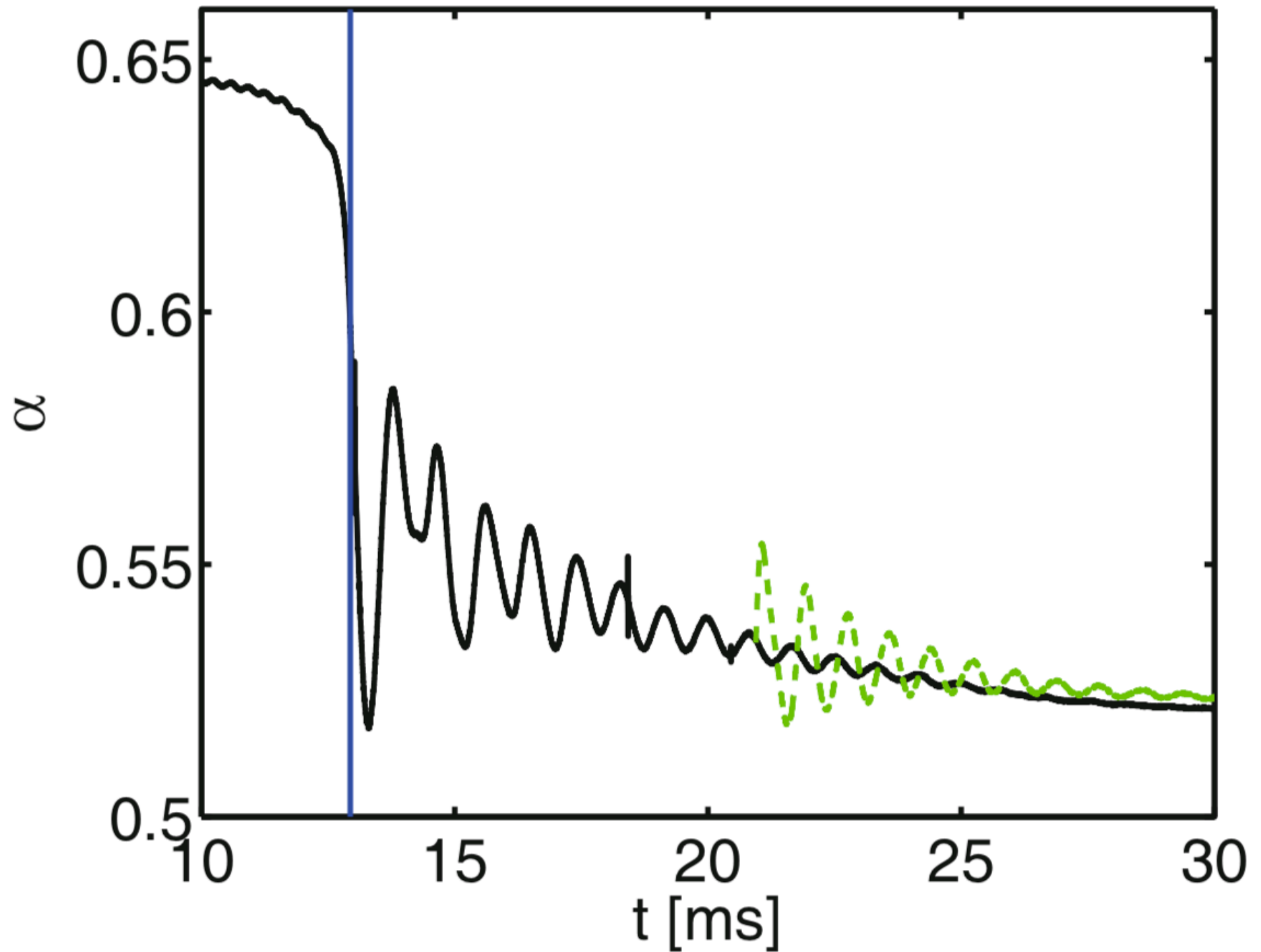


Analytic Model with Physical Parameters

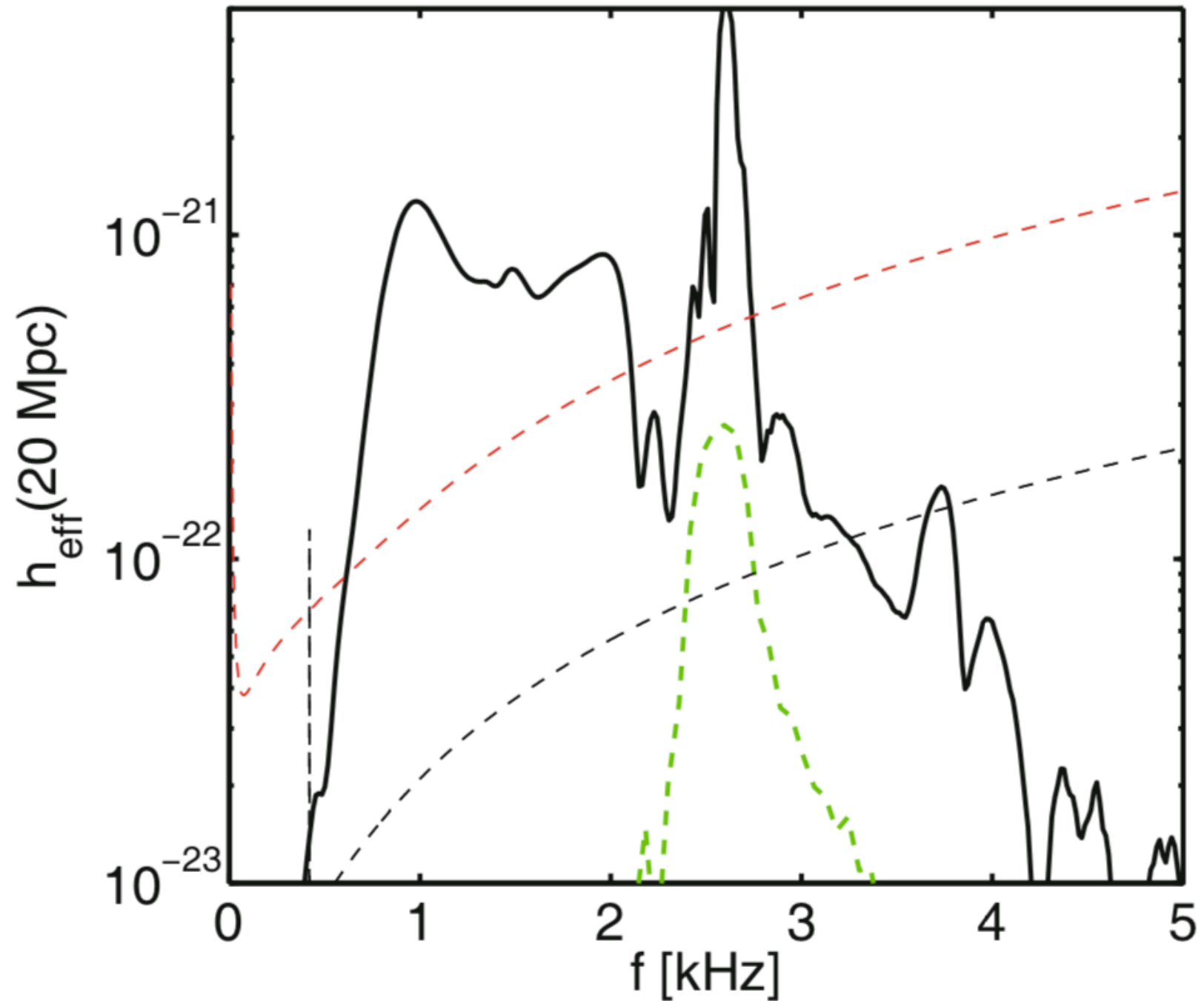
Bauswein, NS & Janka (2016)



Central Lapse Evolution



Late-time Excitation of m=2 Mode



Nonradial Oscillations of Neutron Stars

Main oscillation modes:

1. f -modes / p -modes

fluid modes restored by pressure

2. g -modes

restored by gravity/buoyancy in non-isentropic stars

3. inertial modes (r -modes)

restored by the Coriolis force in rotating stars

4. w -modes

spacetime modes (similar to black hole modes)

GW-detection: f -, p -, g -, r -modes : stable oscillations
instabilities

Spacetime Evolution

90's Nakamura, Oohara, Kojima / Shibata, Nakamura / Baumgarte, Shapiro

Definitions

$$\tilde{\gamma}_{ij} = e^{-4\phi} \gamma_{ij}$$

$$e^{4\phi} = \gamma^{1/3} \equiv \det(\gamma_{ij})^{1/3}$$

$$\tilde{A}_{ij} = e^{-4\phi} A_{ij} \quad A_{ij} = K_{ij} - \frac{1}{3} \gamma_{ij} K,$$

$$\tilde{\Gamma}^i := \tilde{\gamma}^{jk} \tilde{\Gamma}_{jk}^i = -\tilde{\gamma}^{ij}_{,j}$$

“1+log” lapse function

$$\partial_t \alpha = -2\alpha A$$

$$\partial_t A = \partial_t K$$

“Gamma-driver” shift condition

$$\partial_t \beta^i = B^i$$

$$\partial_t B^i = \frac{3}{4} \alpha \partial_t \tilde{\Gamma}^i - e^{-4\phi} \beta^i$$

Time evolution

$$\frac{d}{dt} \tilde{\gamma}_{ij} = -2\alpha \tilde{A}_{ij}, \quad \frac{d}{dt} = \partial_t - \mathcal{L}_\beta$$

$$\frac{d}{dt} \phi = -\frac{1}{6} \alpha K.$$

$$\frac{d}{dt} K = -\gamma^{ij} D_i D_j \alpha + \alpha \left[\tilde{A}_{ij} \tilde{A}^{ij} + \frac{1}{3} K^2 + \frac{1}{2} (\rho + S) \right],$$

$$\frac{d}{dt} \tilde{A}_{ij} = e^{-4\phi} [-D_i D_j \alpha + \alpha (R_{ij} - S_{ij})]^{TF}$$

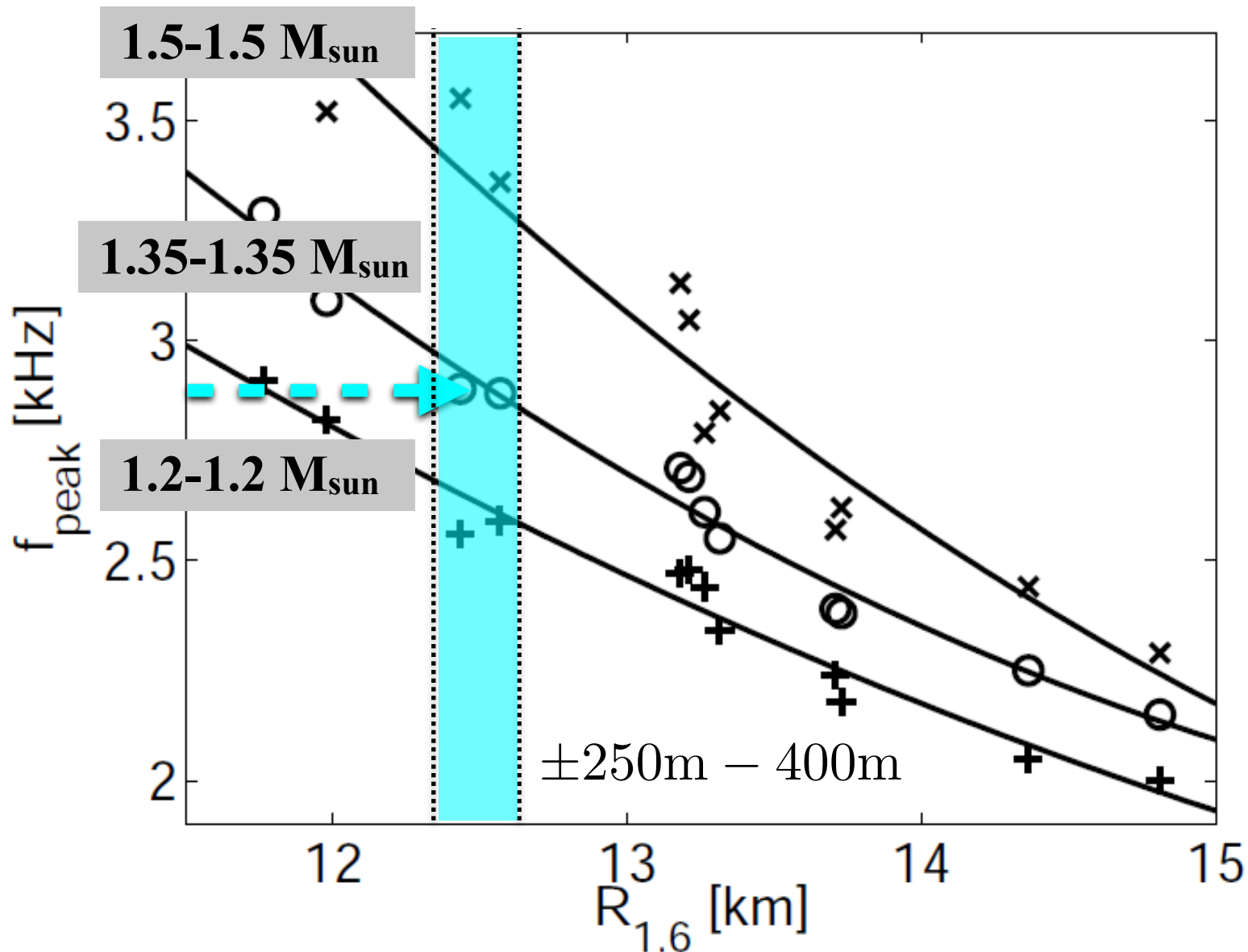
$$+ \alpha (K \tilde{A}_{ij} - 2 \tilde{A}_{il} \tilde{A}_j^l),$$

$$\begin{aligned} \frac{\partial}{\partial t} \tilde{\Gamma}^i = & -2 \tilde{A}^{ij} \alpha_{,j} + 2 \alpha \left(\tilde{\Gamma}_{jk}^i \tilde{A}^{kj} - \frac{2}{3} \tilde{\gamma}^{ij} K_{,j} - \tilde{\gamma}^{ij} S_{,j} + 6 \tilde{A}^{ij} \phi_{,j} \right) \\ & - \frac{\partial}{\partial x^j} \left(\beta^l \tilde{\gamma}^{ij}_{,l} - 2 \tilde{\gamma}^{m(j} \beta^i_{,m} + \frac{2}{3} \tilde{\gamma}^{ij} \beta^l_{,l} \right). \end{aligned}$$

Radius Determination from Post-Merger Signal

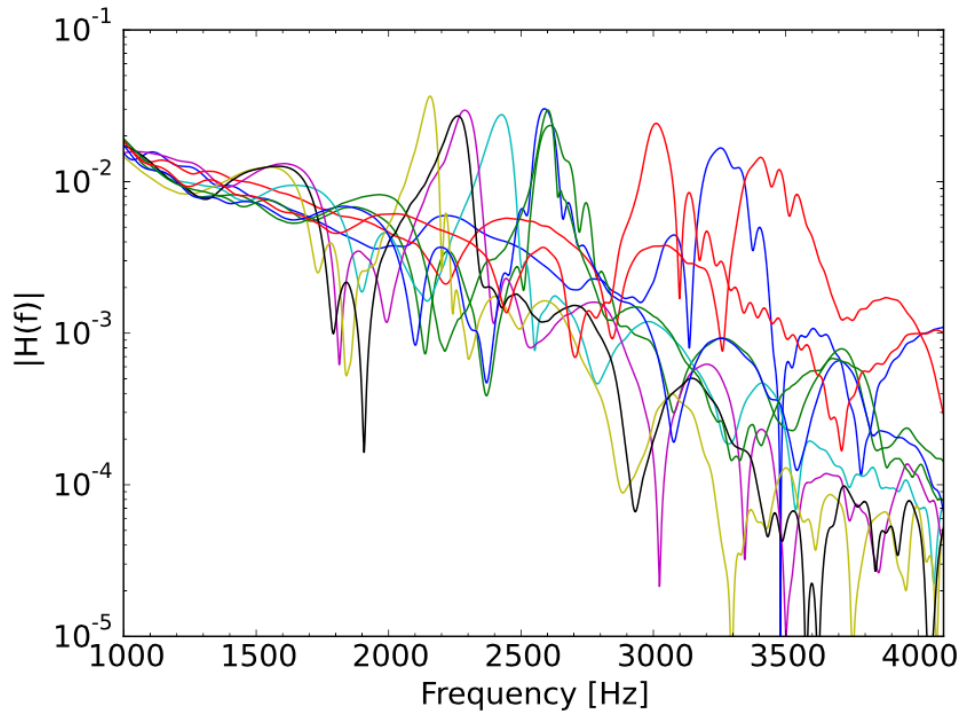
Bauswein, Janka, Hebeler & Schwenk (2012)

f_{peak} correlates very well with the radius @ 1.6 M_{sun} (M_{tot} is known from inspiral).

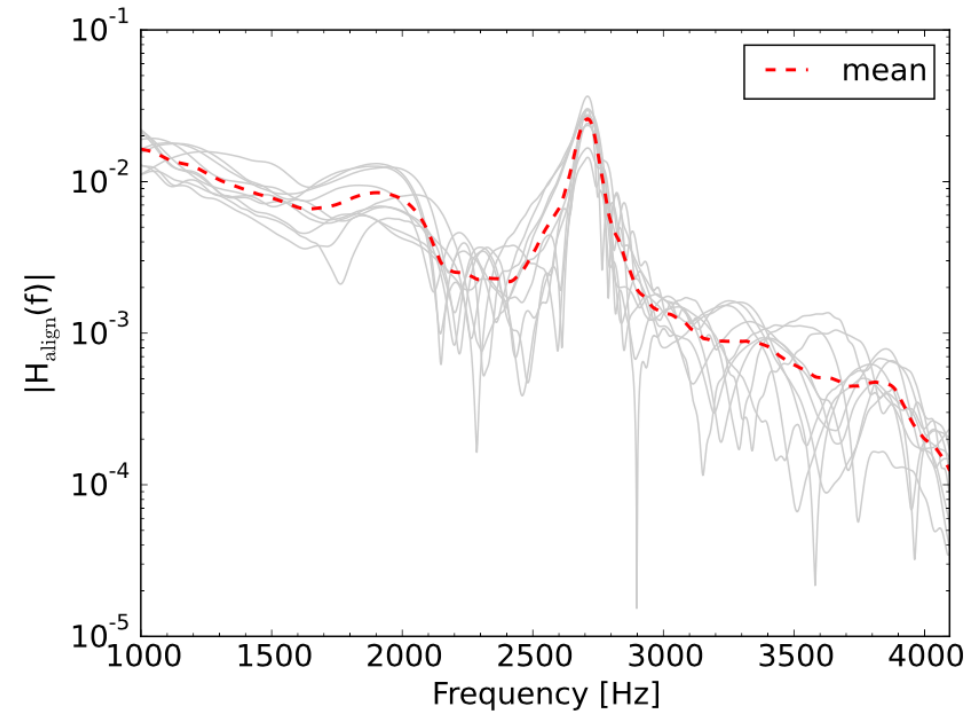


Principal Component Analysis (PCA)

Clark, Bauswein, NS, Shoemaker (2016)



Actual fft's for different models.



Rescaled to common reference model.

Our PCA template extracts **>90%** of signal power compared to only 40% when using simple burst analysis.