Constraints on post-Newtonian parameters from (no) monopolar GWs from pulsars

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November 14, 2019

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General introduction

Neutron stars

Neutron stars represent some of the most extreme objects in the universe; very high densities $\rho_c \gtrsim 10^{15}$ g cm^{-3}, magnetic fields B $\gtrsim 10^{15}$G, and rotation rates $\nu \gtrsim 700$ Hz.

Gravitational waves

In GR, GWs are generated by a rotating, biaxial object when the angle made between its angular momentum and symmetry axis vectors ('wobble angle') is non zero, or if there is some deviation in axisymmetry about its principal axis ('ellipticity').

– The exact character of the gravitational waves can be written in terms of the multipole moments, and depends on the properties of the internal fluid.

Some example mechanisms

$$g_{\mu\nu} = g^{(0)}_{\mu\nu} + h_{\mu\nu} \quad (1)$$

containing an equilibrium piece and a radiative piece.

Density asymmetries

– Modes which are restored by hydrostatic forces in the star; e.g. pressure for f-modes, or buoyancy for g-modes.
– Magnetic fields; Lorentz force is not spherically symmetric
– Accretion mounds; neutron stars in LMXB accrete matter

Velocity asymmetries

– Modes which are restored by rotational forces; e.g. inertial modes from Coriolois force.
– Pulsar glitches; rapid reconfigurations of vortex pinning sites can change angular distribution of star.

A brief treatise on multipole moments

The aim

The idea is that we want to write the field equations in a certain way such that they resemble Newtonian physics.

Introduce a particular representation for the metric $\bar{h}^{\alpha\beta} = -(-g)^{1/2}g^{\alpha\beta} + \eta^{\alpha\beta}$ with $\bar{h}^{\alpha\beta}_{,\beta} = 0$. Write

$$\Box_\eta \bar{h}^{\mu\nu} = -8\pi\tau^{\mu\nu} \qquad (2)$$

This object τ involves the stress-energy tensor, and various components of the metric in some complicated way (note: we haven't done anything fancy here!): $\tau = T + t_{LL}$, where the Landau-Lifshitz looks like : $t_{LL} = t_{LL}^{GR} + t_{LL}^{NGR}$.
Gauge condition implies conserved current $P^\mu = \int \tau^{\mu 0} d^3x$, which, using Stokes' theorem and energy-momentum conservation, implies that

$$dP^0/dt \equiv \dot{E}_{GW} = -R^2 \oint t_{LL}^{0j}\hat{n}_j d\Omega. \qquad (3)$$

Metric solutions

Using a Green's function to "solve" $\Box_\eta \bar{\mathrm{h}}^{\mu\nu} = -8\pi\tau^{\mu\nu}$, one can write

$$\bar{\mathrm{h}}^{\mu\nu} = 4\int \tau^{\mu\nu}(\mathrm{t} - \mathrm{v}_\mathrm{g}^{-1}|\mathrm{x}-\mathrm{x}'|, \mathrm{x}')|\mathrm{x}-\mathrm{x}'|^{-1}\mathrm{d}^3\mathrm{x}'. \tag{4}$$

Additionally, using a harmonic decomposition, this can be written

$$\bar{\mathrm{h}}^{\mu\nu} = 4\mathrm{d}^{-1}\sum_{\mathrm{m}=0}^{\infty}(\frac{1}{\mathrm{m}!})\frac{\partial^\mathrm{m}}{\partial \mathrm{t}^\mathrm{m}}\int \tau^{\mu\nu}(\mathrm{t}-\mathrm{d}, \mathrm{x}')(\hat{\mathrm{n}}\cdot\mathrm{x}')^\mathrm{m}\mathrm{d}^3\mathrm{x}' + \mathcal{O}(\mathrm{r}/\mathrm{d})^2. \tag{5}$$

In general, monopole terms are like $\mathrm{m} = 0$, dipole $\mathrm{m} = 1$, and quadrupoles $\mathrm{m} = 2$, etc.

However, because of the de Donder (Lorenz) gauge condition, one has

$$\partial_\mathrm{t}^2\,(\tau^{00}\mathrm{x}^\mathrm{i}\mathrm{x}^\mathrm{j}) = \partial_\mathrm{k}\partial_\mathrm{l}(\tau^{\mathrm{kl}}\mathrm{x}^\mathrm{i}\mathrm{x}^\mathrm{j}) - 2\partial_\mathrm{k}(\tau^{\mathrm{ik}}\mathrm{x}^\mathrm{j} + \tau^{\mathrm{kj}}\mathrm{x}^\mathrm{i}) + 2\tau^{\mathrm{ij}}, \tag{6}$$

so that monopole and quadrupole terms do not manifest in $\bar{\mathrm{h}}$.

Post-Newtonian expansions

As we saw, monopolar (and dipolar) GWs cannot radiate in general relativity. This can also be thought of via Birkhoff's theorem: spacetime surrounding spherically symmetric source is static.

Essentially: $\bar{\mathrm{h}}$ has no mono. or dip. terms, and thus $\mathrm{t_{LL}} \sim < \dot{\mathrm{h}}, \dot{\mathrm{h}} >$ has none either.

Post-Newtonian

A further simplification is then obtained by expanding terms (i.e. Landau-Lifshitz, $\bar{\mathrm{h}}$, the stress-energy tensor, ...) in powers of $\mathrm{c}^{-2\mathrm{n}}$, where n denotes the PN-order.

For a compact object with spin frequency ν and radius $\mathrm{R}_\star$, the contribution to the overall GW strain from each m-pole (and each PN!) scales with $(\nu \mathrm{R}_\star/\mathrm{c})^\mathrm{m}$; for a (mature) NS this $\nu \mathrm{R}_\star/\mathrm{c} \lesssim 10^{-2}$;

Modified gravity

However, if the theory of gravity of differs from GR, monopolar GWs are (generally) predicted to exist because extra fields are introduced which are not tensorial!

In a scalar-tensor theory, we have

$$\mathrm{t}_{\mathrm{LL}}^{\mathrm{GR}} + \mathrm{t}_{\mathrm{LL}}^{\mathrm{NGR}} \sim < \dot{\mathrm{h}}, \dot{\mathrm{h}} > + < \dot{\psi}, \dot{\mathrm{h}} > + \cdots . \tag{7}$$

The equation for the scalar-field ψ also looks like $\Box_\eta \psi = \mathrm{S}$, for some 'source' S; in Brans-Dicke

$$\Box_\eta \psi = \frac{8\pi}{3+2\omega} \mathrm{T}_\mu^\mu - (\psi_{;\alpha}^{;\alpha} - \Box_\eta \psi). \tag{8}$$

Because this equation is not tensorial, we cannot do the same trick to get rid of monopole terms in the harmonic expansion of ψ (Will 1993).

General post-Newtonian 'super-metric' (Will 1971)

Find all possible terms at post-Newtonian order, build a 'super metric' including all of these with some coefficients.

$$
\begin{aligned}
\mathrm{ds}^2 = & \mathrm{c}^2 \left(1 - \frac{2\mathrm{U}}{\mathrm{c}^2} + \beta \frac{2\mathrm{U}^2}{\mathrm{c}^4} - \frac{4\Phi}{\mathrm{c}^2} + \zeta \mathcal{A}\right) \mathrm{dt}^2 \\
& + \left(\frac{7}{2}\Delta_1 \mathrm{V_a} + \frac{1}{2}\Delta_2 \mathrm{W_a}\right) \mathrm{dtdx^a} - \left(1 + \frac{2\gamma \mathrm{U}}{\mathrm{c}^2}\right) \delta_{\mathrm{ab}} \mathrm{dx^a dx^b},
\end{aligned} \tag{9}
$$

γ and β: spatial curvature due to unit mass (e.g. light deflection), ζ and Σ: spatial curvature due to the radial and transverse components of kinetic energy and stress (preferred-frame effects), Δ_1 and Δ_2: dragging of inertial frames by unit momentum (violation of conservation of angular momentum), β_i (in Φ): spatial curvature due to unit kinetic energy, unit gravitational potential, unit internal energy, and unit pressure (violation of conservation of linear momentum).

Example: PPN values in Brans-Dicke theory

To find the PPN values in the theory set

$$g_{\alpha\beta} = \eta_{\alpha\beta} + h_{\alpha\beta}; \psi = \psi_0 + \varphi. \tag{10}$$

To lowest order, the equations of the theory coupled to some perfect-fluid matter source, allows us to find the spatial components of the metric:

$$g_{ij} \sim \delta_{ij} \left(1 + \frac{2U}{c^2} \frac{1+\omega}{2+\omega} \right); \tag{11}$$

Comparing this with the general PPN-metric, we have that

$$\gamma_{BD} = \frac{1+\omega}{2+\omega} \tag{12}$$

Similarly for other parameters; for Lagrangian theories, many of the PPN parameters vanish (e.g. no violations of energy and angular momentum conservation).

Brans-Dicke theory: $\gamma = \frac{1+\omega}{2+\omega}$

The details are rather involved, but we are able to find that

$$\dot{\mathrm{E}}_{\mathrm{GW}}^{\mathrm{ST}} = \frac{1536\pi^6 \mathrm{G}\nu^2}{\mathrm{c}^5} \left\{ \frac{2(1+\gamma)}{15} \nu^4 \epsilon^2 \mathrm{I}_0^2 \right. \\ \left. + \pi^2 (1-\gamma) \left\{ \int \mathrm{d}r\mathrm{d}\theta \delta\rho(\mathrm{r},\theta) \mathrm{r}^2 \sin\theta \left[\mathrm{r}^2\nu^2 + (6\gamma - 2)\, \mathrm{U}(\mathrm{r}) \right] \right\}^2 \right., \tag{13}$$

In general, the ratio of the monopole to quadrupole contributions to (13) can be estimated as

$$\frac{\dot{\mathrm{E}}_{\mathrm{GW}}^{\mathrm{mono}}}{\dot{\mathrm{E}}_{\mathrm{GW}}^{\mathrm{quad}}} \sim 10^{10} (1-\gamma) \left(\frac{\mathcal{C}}{0.2} \right)^2 \left(\frac{\nu}{100 \text{ Hz}} \right)^{-4} \left(\frac{\mathrm{R}_\star}{10^6 \text{ cm}} \right)^{-4}, \tag{14}$$

implying that monopolar radiation is likely to dominate even for rapidly rotating stars $\nu \lesssim 1$ kHz unless $1-\gamma$ is sufficiently small.

Magnetised PPN NSs

Using the general PPN super-metric, equations of motion for a perfect fluid coupled to an electromagnetic field gives rise to a system of higher-order MHD equations, i.e. PPN-MHD. Continuity, for example,

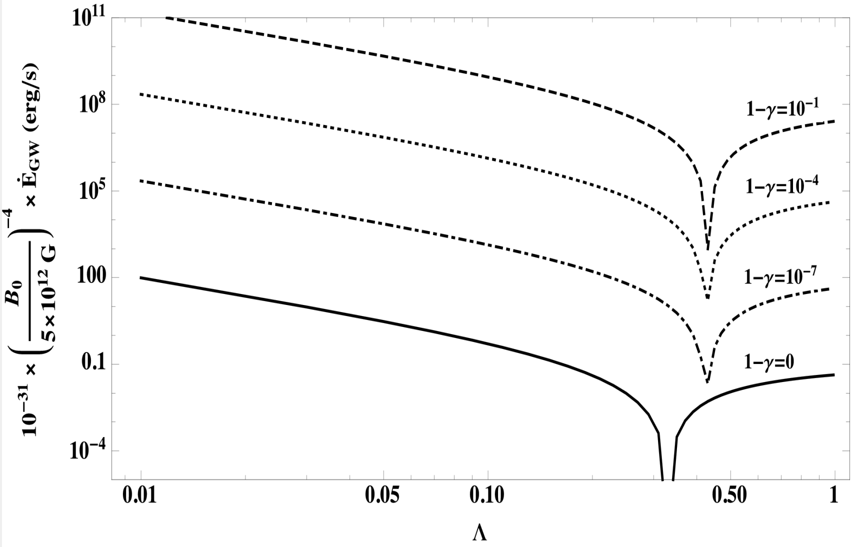
$$0 = \frac{\partial \rho^\star}{\partial \mathrm{t}} + \nabla \cdot (\rho^\star \mathrm{v}) \, , \tag{15}$$

where

$$\rho^\star = \rho \left(1 + \frac{1}{2} \frac{\mathrm{v}^2}{\mathrm{c}^2} + \frac{3\gamma \mathrm{U}}{\mathrm{c}^2} \right) . \tag{16}$$

We use a mixed poloidal-toroidal dipole field model, which has an energy ratio Λ (see paper for details).

Magnetised stars and GW energy



Pulsar	$B_\star$ (10^{12} G)	ν (Hz)	Observational limit $\dot{E}_{GW}^{95\%}$ (10^{35} erg/s)	$1-\gamma$ ($\Lambda = 1$)	$1-\gamma$ ($\Lambda = 10^{-2}$)
J0205+6449	3.60	15.2	17.2	$\leq 1.2 \times 10^{-1}$	$\leq 1.3 \times 10^{-5}$
J0534+2200 (Crab)	3.75	29.7	9.61	$\leq 8.4 \times 10^{-3}$	$\leq 1.5 \times 10^{-6}$
J0835-4510 (Vela)	3.62	11.2	0.617	$\leq 4.2 \times 10^{-3}$	$\leq 8.0 \times 10^{-7}$
J1302-6350	0.337	20.9	8.30	–	$\leq 5.2 \times 10^{-2}$
J1809-1917	1.46	12.1	110	–	$\leq 4.7 \times 10^{-3}$
J1813-1246	0.956	20.8	3.15	–	$\leq 2.6 \times 10^{-4}$
J1826-1256	3.74	9.05	161	–	$\leq 3.4 \times 10^{-4}$
J1928+1746	0.974	14.6	147	–	$\leq 2.4 \times 10^{-2}$
J1952+3252 (CTB 80)	0.472	25.3	7.66	–	$\leq 2.1 \times 10^{-2}$
J2043+2740	3.87	10.4	23.1	–	$\leq 2.7 \times 10^{-5}$
J2229+6114	2.03	19.4	5.12	–	$\leq 2.3 \times 10^{-5}$

The fourth column shows the observational upper limits from a Bayesian analysis on at the 95% confidence level (Abbott et al. 2017). The fifth and sixth columns shows bounds on $1-\gamma$ obtained by requiring that $\dot{E}_{GW}^{ST} \leq \dot{E}_{GW}^{95\%}$.

Comparison on limits

Experiment	Constraint
Precession of Mercury	$-3.0 \times 10^{-3} < 2\gamma^{\mathrm{PPN}} - \beta^{\mathrm{PPN}} - 1 < 3.0 \times 10^{-3}$
Lunar Laser Ranging (Nordtvedt effect)	$-1.7 \times 10^{-3} < 4\beta^{\mathrm{PPN}} - \gamma^{\mathrm{PPN}} - 3 < 0.3 \times 10^{-3}$
Very Long Baseline Interferometry	$-4.0 \times 10^{-4} < \gamma^{\mathrm{PPN}} - 1 < 4.0 \times 10^{-4}$
Cassini tracking (grav. redshift)	$-0.2 \times 10^{-5} < \gamma^{\mathrm{PPN}} - 1 < 4.4 \times 10^{-5}$
Timing of PSR B1913+16	$\beta^{\mathrm{PPN}} - 1 < 1.1 \times \left(\gamma^{\mathrm{PPN}} - 1\right)$

For conservative, purely poloidal models with characteristic field strength given by the spindown minimum, upper limits for the Vela pulsar yield $1 - \gamma \lesssim 4.2 \times 10^{-3}$. For models containing a strong toroidal field housing $\sim 99\%$ of the internal magnetic energy, we obtain the bound $1 - \gamma \lesssim 8.0 \times 10^{-7}$.

$\Longrightarrow$ comparable limits with Solar system experiments!