

# Quick User Guide for RELXILL\_NK 1.3.2

## I. INTRODUCTION

RELXILL\_NK is a reflection model to test the Kerr nature of astrophysical black holes [1, 2]. It is an extension of the RELXILL package [3, 4], developed by Thomas Dauser and Javier Garcia, to non-Kerr spacetimes. The spacetime geometry is described by a parametric black hole metric in which some “deformation parameters” quantify possible deviations from the Kerr background. From the comparison of X-ray data of astrophysical black holes with the theoretical predictions of RELXILL\_NK we can measure the value of these deformation parameters and check whether they vanish, as it is required in the Kerr metric.

RELXILL\_NK v 1.3.2 is based on RELXILL v 1.2.0 and employs the Johannsen metric with the deformation parameters  $\alpha_{13}$  and  $\alpha_{22}$  [5]. As RELXILL, RELXILL\_NK could be readily used in common X-ray data analysis tools such as ISIS, XSPEC, or SHERPA. However, it has only been tested for XSPEC.

## II. DISK-CORONA MODEL

The astrophysical set-up is shown in Fig. 1. A black hole (the black filled circle in Fig. 1) is surrounded by a geometrically thin and optically thick accretion disk (the two gray regions on the left and right of the black hole in Fig. 1). At any point of the accretion disk, we can define an effective temperature, which increases as we approach the black hole. The thermal emission (red arrows) of the inner part of the accretion disk is in the soft X-ray band for  $\sim 10 M_\odot$  black holes accreting at  $\sim 10\%$  of the Eddington limit, and shifts to the optical/UV band for supermassive black holes of  $10^6 - 10^9 M_\odot$ . The corona is a hotter ( $\sim 100$  keV), usually compact and optically thin, medium close to the black hole (the yellow star in Fig. 1). As shown in Fig. 2, there are several possibilities for the nature and morphology of the corona. In the lamppost model, the corona is a point-like source above the black hole and along its spin axis. Such a configuration could be realized by the base of the jet. The atmosphere above the accretion disk can instead be responsible for a sandwich corona. The accretion flow between the accretion disk and the black hole can create a spherical or toroidal corona. The thermal photon from the accretion disk can have inverse Compton scattering off the free electrons in the corona and generate a power law component (blue arrows). A fraction of the power law component can illuminate the disk. As a

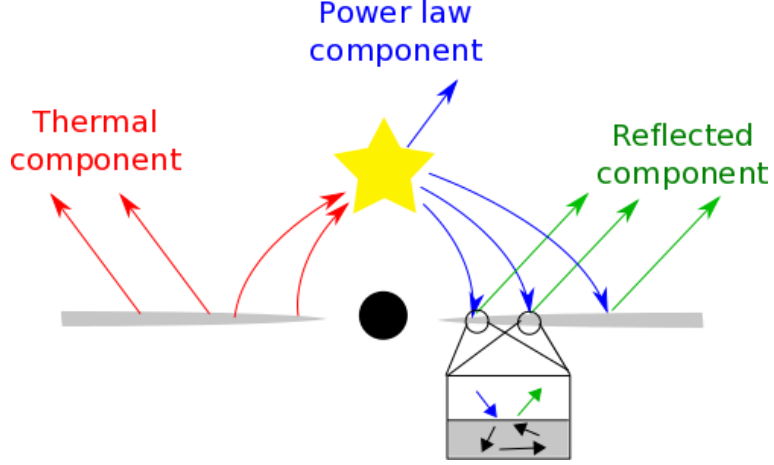


FIG. 1: Disk-corona model. The black hole is surrounded by a thin accretion disk with a multi-color blackbody spectrum (red arrows). Some thermal photons from the disk have inverse Compton scattering off free electrons in the corona, producing a power-law component (blue arrows). The latter also illuminates the disk, generating a reflection component (green arrows).

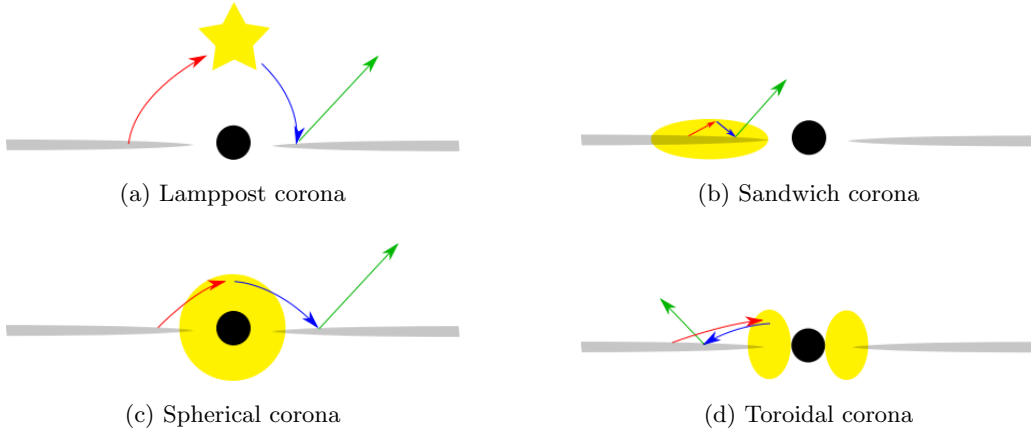


FIG. 2: Examples of possible corona geometries: lamppost geometry (top left panel), sandwich geometry (top right panel), spherical geometry (bottom left panel), and toroidal geometry (bottom right panel).

result of the interactions between the X-ray photons of the power law component with the gas of the accretion disk, we get a reflection component (green arrows). The reflection spectrum at the emission point in the rest-frame of the gas of the disk is only determined by atomic physics. The photons of the reflection component have then to propagate in the strong gravitational field of the black hole and they thus experience relativistic effects (gravitational redshift, Doppler boosting,

light bending) before being detected by our instruments.

RELXILL\_NK can be employed to describe the reflection and the power law components.

### III. INSTALLATION

RELXILL\_NK can be installed as RELXILL. You can compile the model with the commands

```
chmod u+x ./compile_relxill.csh
./compile_relxill.sh
```

To upload the model, the command is

```
lmod relxill .
```

Note the dot . at the end of the line.

### IV. FLAVORS

RELXILL\_NK v 1.3.2 has the same “Kerr” flavors as RELXILL v 1.2.0 (in bracket we write the list of the parameters for every flavor):

1. RELLINE ( $E_{\text{line}}, q_{\text{in}}, q_{\text{out}}, R_{\text{br}}, a_*, i, R_{\text{in}}, R_{\text{out}}, z, l$ )
2. RELCONV ( $q_{\text{in}}, q_{\text{out}}, R_{\text{br}}, a_*, i, R_{\text{in}}, R_{\text{out}}, l$ )
3. RELLINE\_LP ( $E_{\text{line}}, h, a_*, i, R_{\text{in}}, R_{\text{out}}, z, l, \Gamma$ )
4. RELCONV\_LP ( $h, a_*, i, R_{\text{in}}, R_{\text{out}}, l, \Gamma$ )
5. RELXILL ( $q_{\text{in}}, q_{\text{out}}, R_{\text{br}}, a_*, i, R_{\text{in}}, R_{\text{out}}, z, \Gamma, \log \xi, A_{\text{Fe}}, E_{\text{cut}}, R_{\text{f}}$ )
6. RELXILL\_LP ( $h, a_*, i, R_{\text{in}}, R_{\text{out}}, z, \Gamma, \log \xi, A_{\text{Fe}}, E_{\text{cut}}, R_{\text{f}}$ )
7. XILLVER ( $\Gamma, A_{\text{Fe}}, E_{\text{cut}}, \log \xi, z, i, R_{\text{f}}$ )
8. RELXILLD ( $q_{\text{in}}, q_{\text{out}}, R_{\text{br}}, a_*, i, R_{\text{in}}, R_{\text{out}}, z, \Gamma, \log \xi, A_{\text{Fe}}, \log N_{\text{e}}, R_{\text{f}}$ )
9. RELXILL\_LP ( $h, a_*, i, R_{\text{in}}, R_{\text{out}}, z, \Gamma, \log \xi, A_{\text{Fe}}, \log N_{\text{e}}, R_{\text{f}}$ )
10. XILLVERD ( $\Gamma, A_{\text{Fe}}, \log N_{\text{e}}, \log \xi, z, i, R_{\text{f}}$ )

11. RELXILLCP ( $q_{\text{in}}, q_{\text{out}}, R_{\text{br}}, a_*, i, R_{\text{in}}, R_{\text{out}}, z, \Gamma, \log \xi, A_{\text{Fe}}, kT_{\text{e}}, R_{\text{f}}$ )
12. RELXILLLP (  $h, a_*, i, R_{\text{in}}, R_{\text{out}}, z, \Gamma, \log \xi, A_{\text{Fe}}, kT_{\text{e}}, R_{\text{f}}$ )
13. XILVERCP ( $\Gamma, A_{\text{Fe}}, kT_{\text{e}}, \log \xi, z, i, R_{\text{f}}$ )

In addition, RELXILL\_NK v 1.3.2 has the following “non-Kerr” flavors:

14. RELXILL\_NK ( $q_{\text{in}}, q_{\text{out}}, R_{\text{br}}, a_*, i, R_{\text{in}}, R_{\text{out}}, z, \Gamma, \log \xi, A_{\text{Fe}}, E_{\text{cut}}, R_{\text{f}}, \text{defpar\_type}, \text{defpar\_value}$ )
15. RELLINE\_NK ( $E_{\text{line}}, q_{\text{in}}, q_{\text{out}}, R_{\text{br}}, a_*, i, R_{\text{in}}, R_{\text{out}}, z, l, \text{defpar\_type}, \text{defpar\_value}$ )
16. RELXILLCP\_NK ( $q_{\text{in}}, q_{\text{out}}, R_{\text{br}}, a_*, i, R_{\text{in}}, R_{\text{out}}, z, \Gamma, \log \xi, A_{\text{Fe}}, kT_{\text{e}}, R_{\text{f}}, \text{defpar\_type}, \text{defpar\_value}$ )
17. RELXILLLP\_NK ( $h, a_*, i, R_{\text{in}}, R_{\text{out}}, z, \Gamma, \log \xi, A_{\text{Fe}}, E_{\text{cut}}, R_{\text{f}}, \text{defpar\_type}, \text{defpar\_value}$ )
18. RELLINE\_NK ( $E_{\text{line}}, h, a_*, i, R_{\text{in}}, R_{\text{out}}, z, l, \Gamma, \text{defpar\_type}, \text{defpar\_value}$ )
19. RELCONV\_NK ( $q_{\text{in}}, q_{\text{out}}, R_{\text{br}}, a_*, i, R_{\text{in}}, R_{\text{out}}, l, \text{defpar\_type}, \text{defpar\_value}$ )
20. RELXILLD\_NK ( $q_{\text{in}}, q_{\text{out}}, R_{\text{br}}, a_*, i, R_{\text{in}}, R_{\text{out}}, z, \Gamma, \log \xi, A_{\text{Fe}}, \log N_{\text{e}}, R_{\text{f}}, \text{defpar\_type}, \text{defpar\_value}$ )
21. RELXILLLP\_NK ( $h, a_*, i, R_{\text{in}}, R_{\text{out}}, z, \Gamma, \log \xi, A_{\text{Fe}}, kT_{\text{e}}, R_{\text{f}}, \text{defpar\_type}, \text{defpar\_value}$ )
22. RELXILLLPD\_NK ( $h, a_*, i, R_{\text{in}}, R_{\text{out}}, z, \Gamma, \log \xi, A_{\text{Fe}}, \log N_{\text{e}}, R_{\text{f}}, \text{defpar\_type}, \text{defpar\_value}$ )

To use a flavor, the XSPEC command is

mo [name of the flavor]

Among these 22 flavors, there are non-relativistic models, Kerr and non-Kerr convolution models, and Kerr and non-Kerr relativistic models. For non-relativistic model, we mean that the reflection spectrum is that at the emission point in the rest-frame of the gas (only atomic physics is involved). For relativistic model, we mean that it is assumed that the emission comes from a thin accretion disk (with some intensity profile) and the output spectrum is that observed far from the source, so it includes all the relativistic effects. The convolution models are those models that transform non-relativistic models into relativistic models.

The RELLINE family describes relativistic models in which the spectrum at the emission point is supposed to be a single line at the energy  $E_{\text{line}}$ . The flavors of this family do not include the spectrum from the corona.

The spectrum from the corona is either described by a power law component with an exponential cut-off (default case) or by a thermalized Compton spectrum (the flavors with CP). In the former case, the spectrum is described by the photon index  $\Gamma$  and the high energy cut-off  $E_{\text{cut}}$ . In the latter case we have the photon index  $\Gamma$  and the electron temperature of the corona  $kT_e$ . Note that every flavor has one normalization constant, but there are two components, i.e. the spectrum of the corona and the reflection spectrum. The relative intensity is regulated by the reflection fraction  $R_f$ . If we set  $R_f = -1$ , we remove the spectrum of the corona.

By default, the non-relativistic reflection spectrum is calculated assuming an electron density of  $10^{15}$  electrons/cm<sup>3</sup>. In the flavors with D, the electron density of the medium,  $N_e$ , can vary from  $10^{15}$  to  $10^{19}$  electrons/cm<sup>3</sup>, but it is assumed  $E_{\text{cut}} = 300$  keV.

The XILLVER family describes non-relativistic reflection spectra, so the parameters of the flavors are those of the spectrum of the corona and of the medium emitting the reflection spectrum. There are thus no parameters related to the spacetime metric or the accretion disk, and the model can be used even to describe the reflection spectrum of generic material far from the black hole.

The RELCONV family describes the convolution models. They require as input the spectrum at the emission point in the accretion disk and give as output the spectrum far from the source. The parameters of these flavors are those of the spacetime metric and of the intensity profile of the accretion disk.

The RELXILL family describes relativistic reflection spectra. For example, RELXILL is roughly equivalent to RELCONV\*XILLVER (there is some minor difference related to the inclination angles at the emission points), RELXILLD is roughly equivalent to RELCONV\*XILLVERD, RELXILLP is roughly equivalent to RELCONV\_LP\*XILLVER, etc.

The intensity profile of the accretion disk is either modeled with a broken power law (default case) or with the emissivity profile expected in the lamppost geometry (the flavors with LP). In both cases, the emission region is defined by the inner edge,  $R_{\text{in}}$ , and the outer edge,  $R_{\text{out}}$ , of the accretion disk. If  $R_{\text{in}}$  and  $R_{\text{out}}$  assume positive values, they are measured in units of the gravitational radius  $r_g = G_N M / c^2$ , where  $G_N$  is Newton's gravitational constant,  $M$  is the black hole mass, and  $c$  is the speed of light. If they assume negative values, they are measured in units of the radius of the innermost stable circular orbit (ISCO). If we set  $R_{\text{in}} = -1$ , we assume that the inner edge of the accretion disk is at the ISCO radius.  $R_{\text{out}}$  is usually set at some large value, so

its exact value is not important because the emission is strongly suppressed at large radii. If the intensity profile is described by a broken power law, we have three parameters: the inner emissivity index  $q_{\text{in}}$ , the outer emissivity index  $q_{\text{out}}$ , and the breaking radius  $R_{\text{br}}$ . In the lamppost models, the corona is described by a point-like source just above the black hole and the emissivity only depends on the height of the corona  $h$ .

$i$  is the angle between the line of sight of the distant observer and the normal of the emitting medium. In the XILLVER family, the value of  $i$  has a very weak impact on the reflection spectrum, so it may be frozen to the default value. In all the other cases, its value is important because it determines the Doppler blueshift/redshift of the spectrum.

In the Kerr models, the spacetime geometry is only regulated by the black hole spin  $a_*$ , because the black hole mass  $M$  only sets the size of the system. In the non-Kerr models, we have also `defpar_type` and `defpar_value`, which are described in the next section.

The medium emitting the reflection spectrum is characterized by the ionization parameter  $\log \xi$  and the iron abundance  $A_{\text{Fe}}$ .  $\xi$  is measured in units of  $\text{erg cm s}^{-1}$ .  $A_{\text{Fe}}$  is measured in units of the Solar iron abundance. Note that only the iron abundance is variable, while all other elements are assumed to have the Solar abundance for simplicity because their impact on the reflection spectrum is weaker.

Lastly,  $z$  is the redshift of the source. In the case of stellar-mass black holes in our Galaxy, it is usually set to 0 (their relative motion is too low). In the case of supermassive black holes in other galaxies, it is usually frozen to the value measured from other observations. The parameter  $l$  can assume the integer values 0, 1, and 2.  $l = 0$  corresponds to isotropic emission.  $l = 1$  is for limb-darkening emission.  $l = 2$  is for limb-brightening emission.

## V. DEFORMATION PARAMETERS

The non-Kerr flavors are equivalent to their Kerr counterpart with two more parameters: `defpar_type` and `defpar_value`.

With `defpar_type` you choose the deformation parameter. If `defpar_type` is 1, the model employs the Johannsen metric with the deformation parameter  $\alpha_{13}$  and all other deformation parameters vanish. If `defpar_type` is 2, we have the Johannsen metric with the deformation parameter  $\alpha_{22}$  and all other deformation parameters vanish.

`defpar_value` stands for the “scaled” deformation parameter (scaled  $\alpha_{13}$  if `defpar_type`=1 and scaled  $\alpha_{22}$  if `defpar_type`=2). It ranges from  $-1$  to  $1$  (from  $-0.95$  to  $0.95$  in the lamppost flavors).

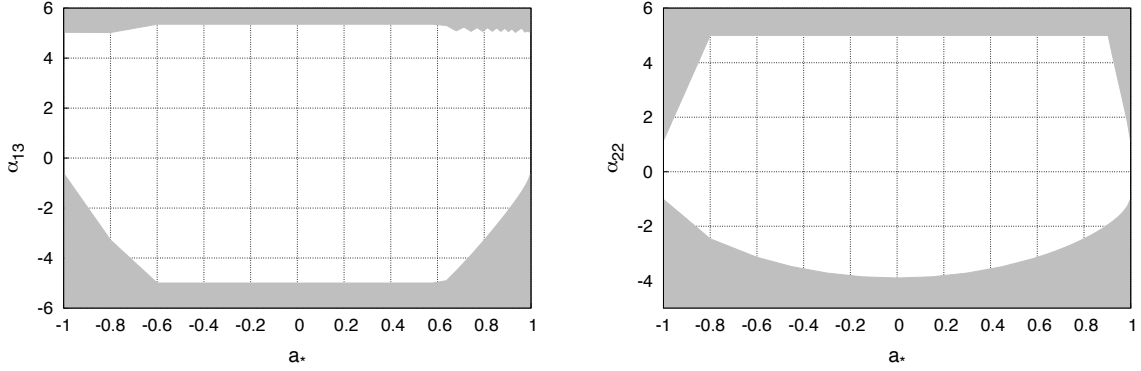


FIG. 3: The white regions are the parameter space of  $a_*$  and of the actual deformation parameters covered by RELXILL\_NK v 1.3.2. The grayed regions are excluded.

To get the corresponding value of the actual deformation parameter, you need to “unscale” it. The range of the actual deformation parameters is shown in Fig. 3. This unscaling can be achieved with the Python scripts `unscale.py` and `unscale_batch.py`. If you want to transform a scaled value into the actual value, you can use `unscale.py` as follows

```
python unscale.py [defpar_type] [spin] [defpar_value]
```

where you have to write the value of the type of the deformation parameter in `[defpar_type]` (1 for  $\alpha_{13}$ , 2 for  $\alpha_{22}$ ), the value of the spin parameter in `[spin]`, and the value of the scaled deformation parameter in `[defpar_value]`. If you want to recover actual deformation values for a bunch of scaled deformation parameters (as obtained from `steppar`), you can use `unscale_batch.py`

```
python unscale_batch.py [defpar_type] [input file] [output file]
```

Ensure that the input file only contains the data and does not include the header information from `steppar`.

## VI. SUPPORT

For questions, comments, and suggestions about RELXILL\_NK, you can email us at

`relxill_nk@fudan.edu.cn`

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